Aggregation bias in investment and capital

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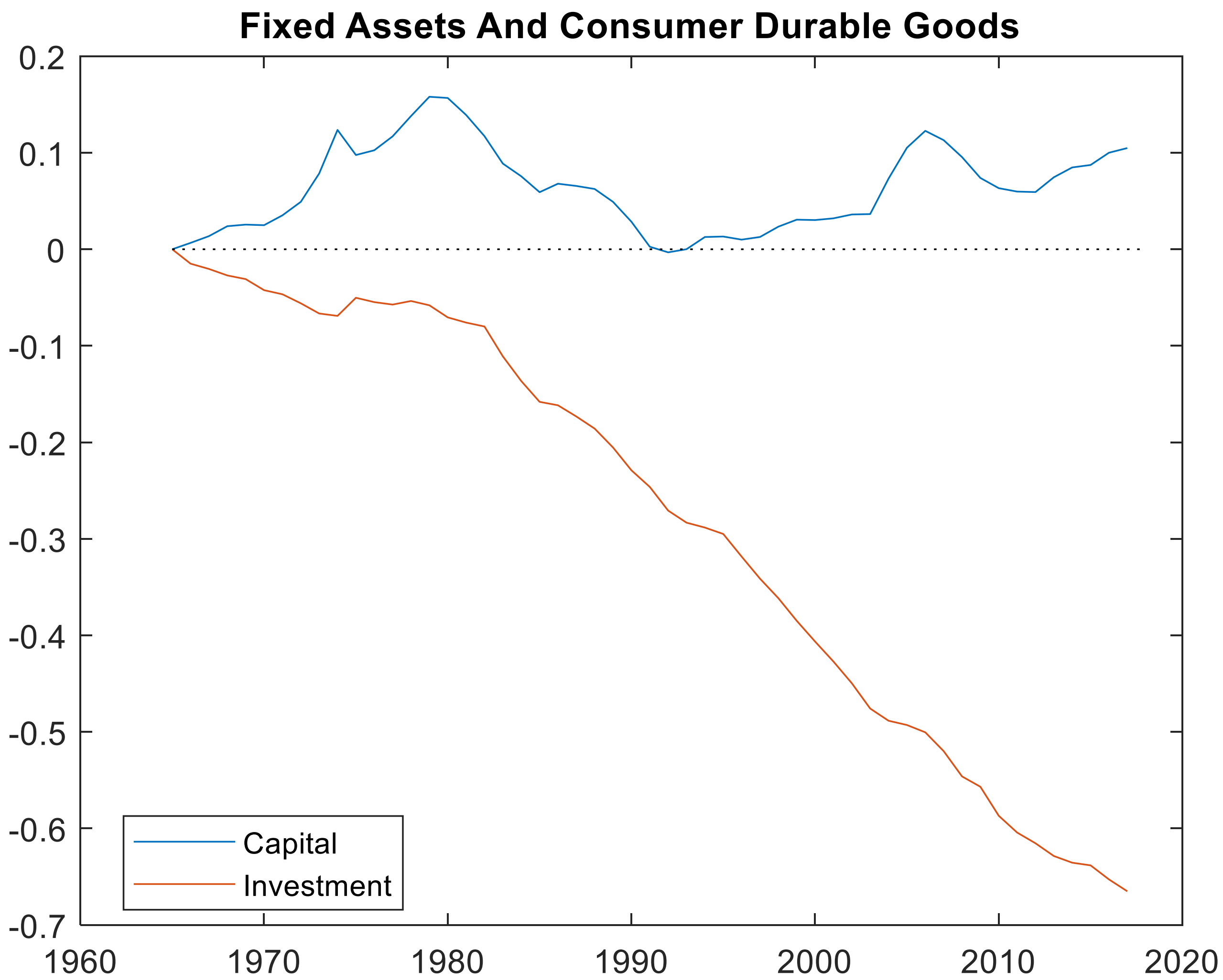
Overview

- Observed (US BEA) prices of **capital** are **increasing** in units of consumption.
- Observed (US BEA) prices of **investment** are **falling** in units of consumption.

- The BEA calculates these prices using the Fisher Index to aggregate across capital types and prices.
  - For non-durable goods, this is (roughly) the right thing to do. See e.g. Diewert (1993).
  - For durable goods this may lead to biases. See e.g. Jorgenson & Griliches (1972), Diewert & Lawrence (2000) for discussion of biases relating to depreciation schedules.

- We explain the gap between observed relative capital price growth and observed relative investment price growth through a new measurement bias.
  - The investment bundle is more skewed towards new varieties than the capital bundle.
  - New varieties experience faster productivity growth rates, so have faster falling prices.
- Correctly measured …
  - Capital and investment prices are identical. (Definition of investment units!)
  - US GDP (investment) growth is $\approx$ 0.10 (0.47) percentage points per year lower.
  - Investment specific technological change only explains 6.9% of aggregate growth.

Capital & inv. prices, units of cons. goods



Average age (2017): 21.7 years

Digression: Individual good price measurement

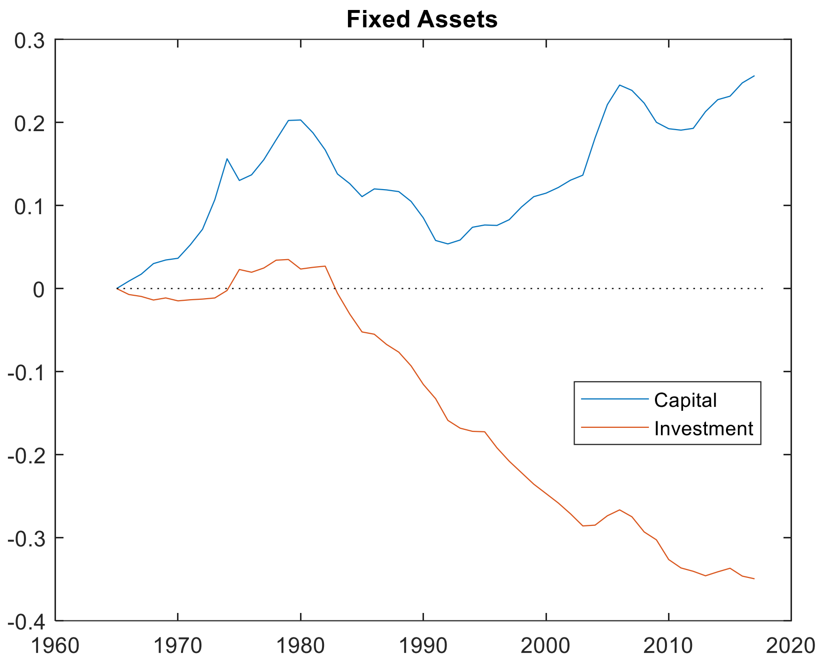
- This is data from the US BEA Fixed Asset tables.
  - Note that this data enters into standard (NIPA) national accounts via CFC.

- The literature from the 90s was concerned with mismeasurement of individual investment goods.
  - Gordon (1990) produced a new measure which was partly extended by Greenwood, Hercowitz & Krusell (1997) (GHK), Cummins & Violante (2002) and Basu, Fernald, Fisher & Kimball (2013).

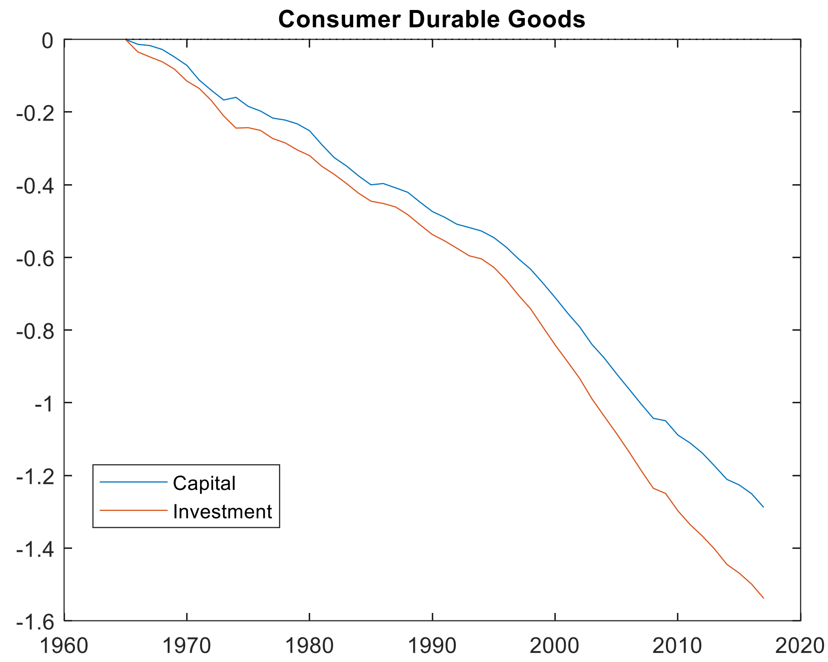
- Since then the national accounts have been thoroughly revised, and the problems raised by Gordon are mostly solved.
  - In part using Gordon's methodology.

- For the duration, we take the NIPA measures of the prices of individual goods as correct.
  - Our issue is entirely one of aggregation.

Capital & inv. prices, units of cons. goods

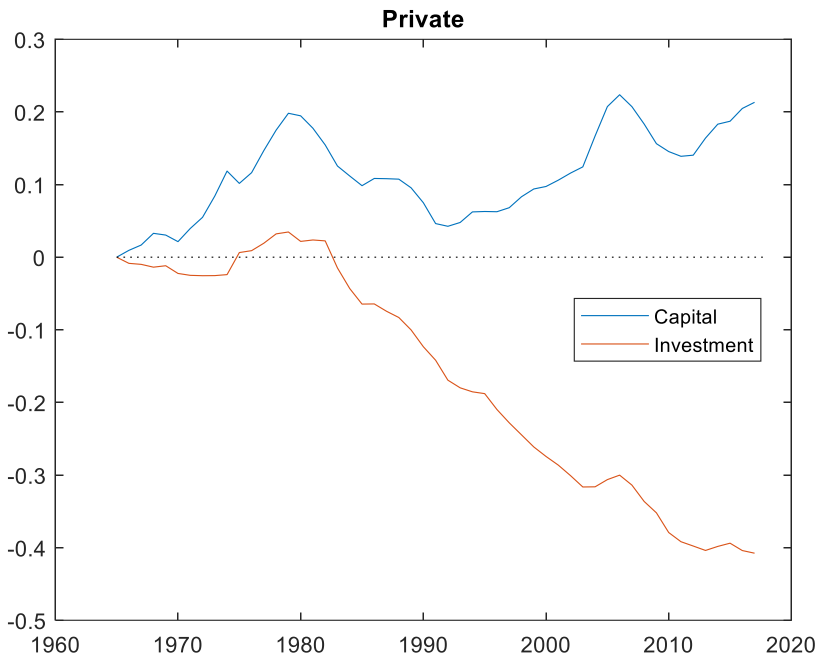


Average age (2017): 23.2 years

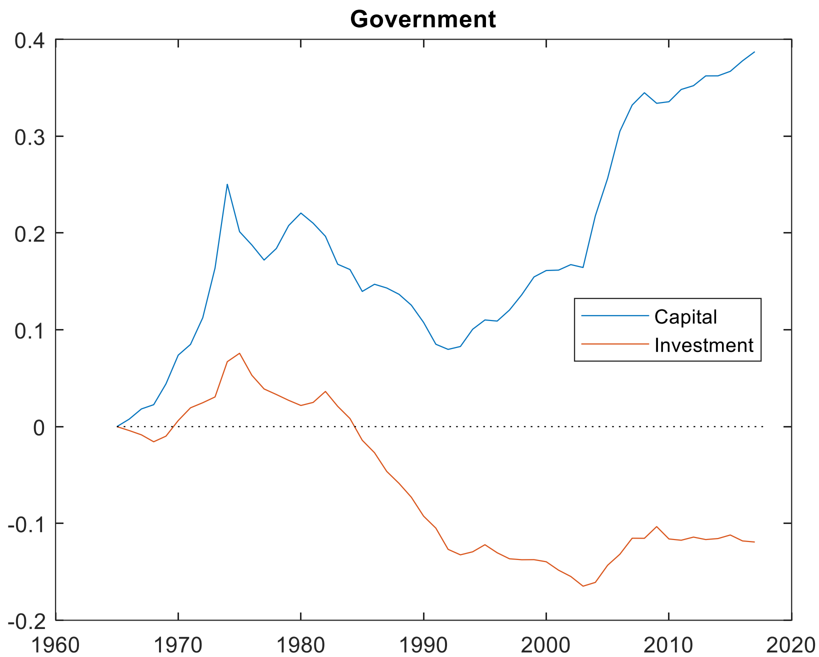


Average age (2017): 4.5 years

Capital & inv. prices, units of cons. goods

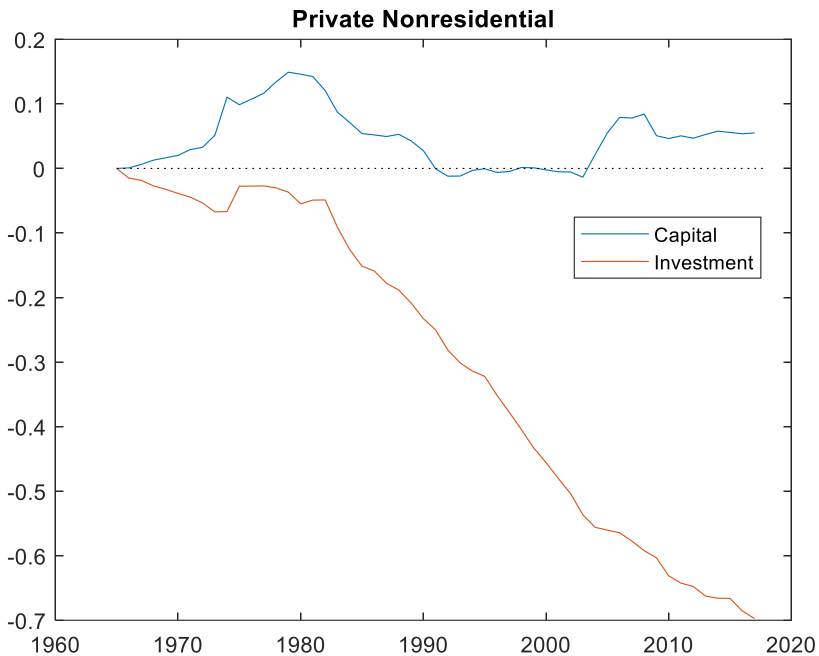


Average age (2017): 22.8 years

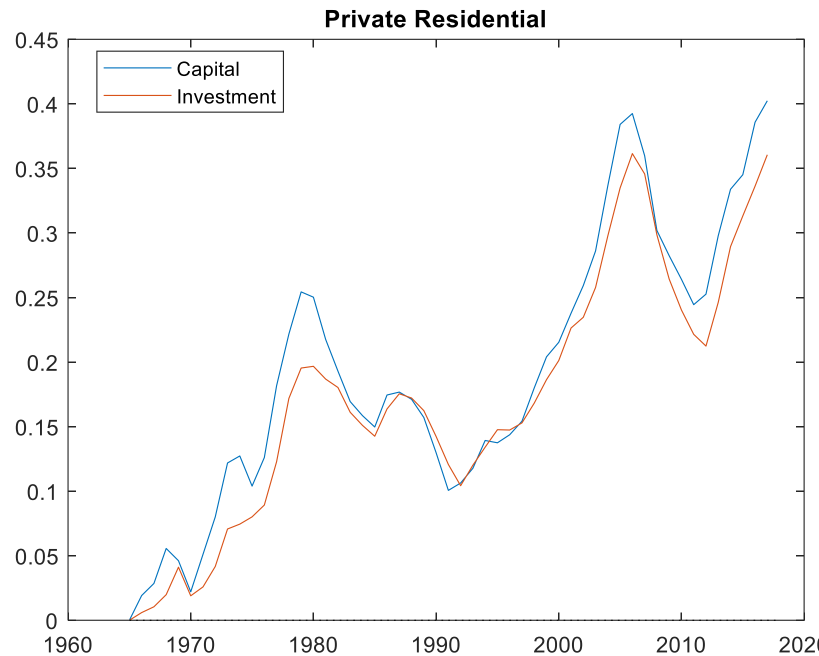


Average age (2017): 24.3 years

Capital & inv. prices, units of cons. goods

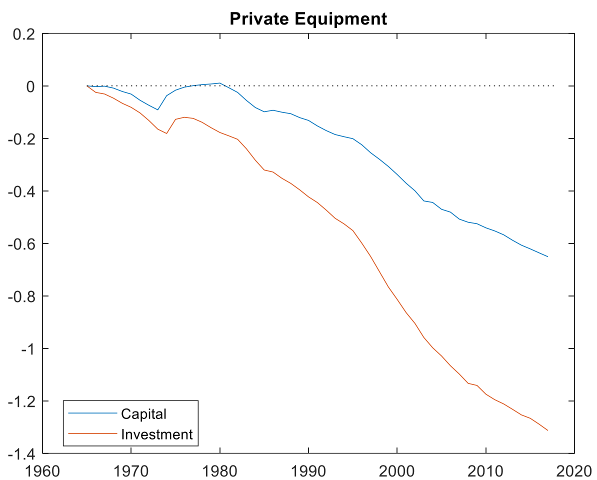


Average age (2017): 16.0 years

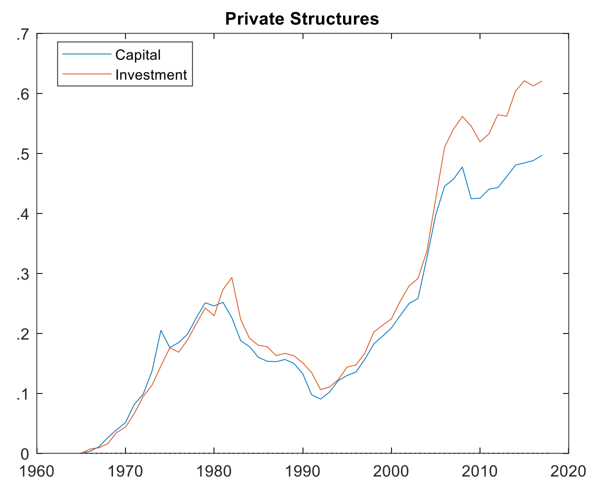


Average age (2017): 30.6 years

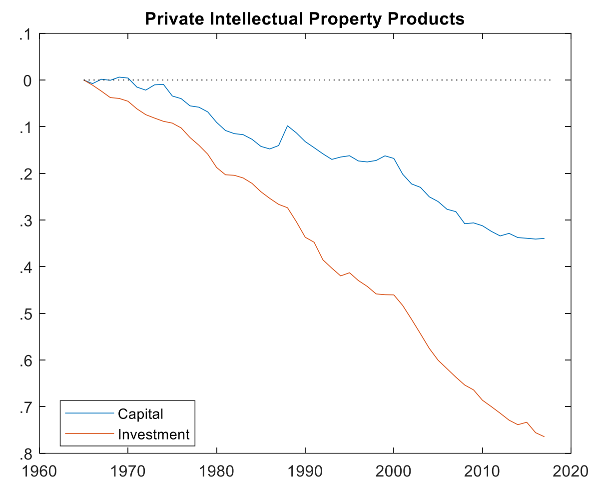
Capital & inv. prices, units of cons. goods



Average age (2017): 7.0 years

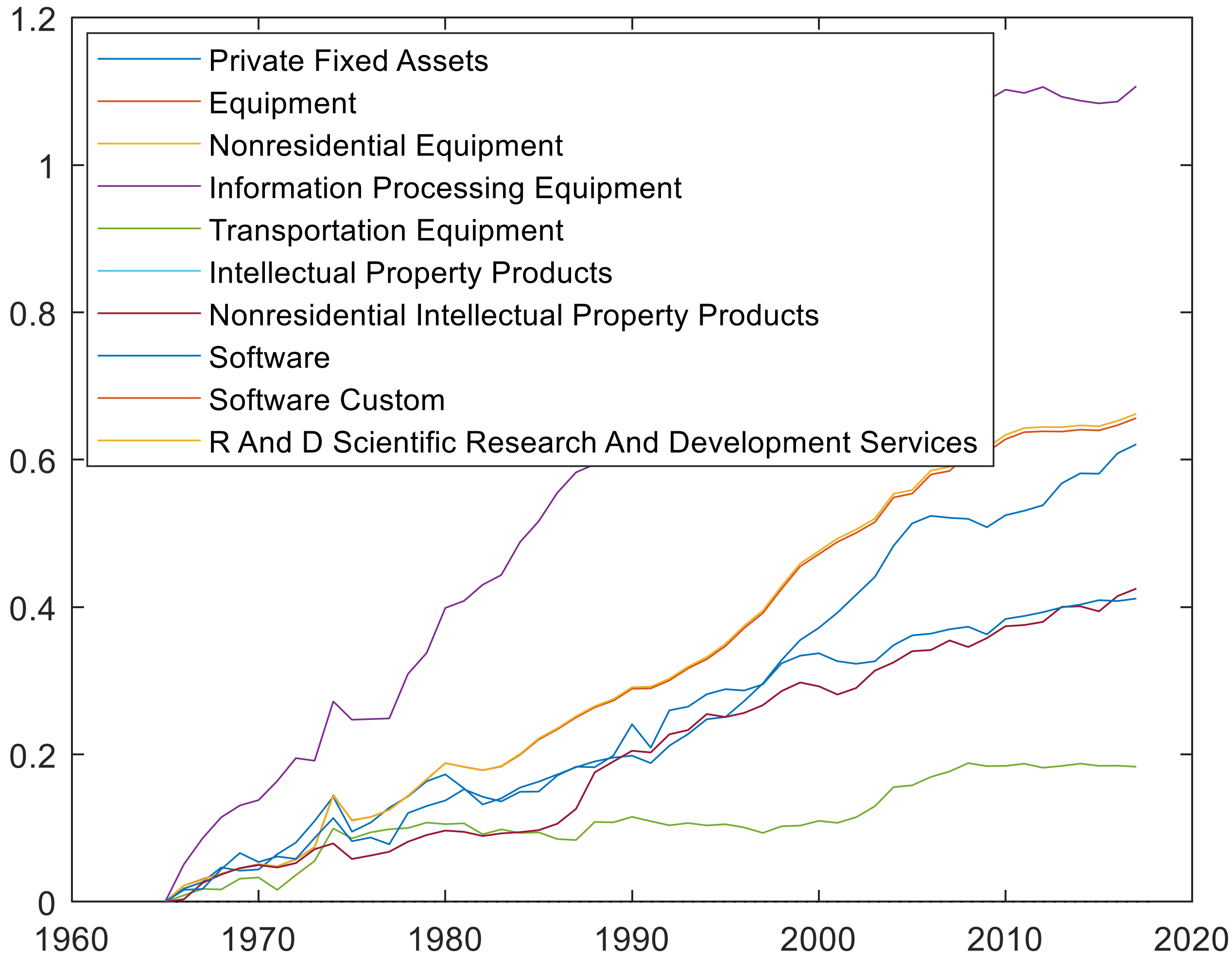


Average age (2017): 22.9 years



Average age (2017): 4.5 years

Price of capital in units of inv. by type



Types are plotted for which the graph has a slope statistically significantly different from 0 at 10% (HAC adjusted)

Lessons from these graphs

- Across all but one of these broad categories, the price of capital is growing faster than the price of investment.
- The price of fixed assets in units of consumption goods is increasing.

- The categories with the highest average ages have experienced the fastest increases in capital prices.
  - Only the categories with average ages below 10 years have experienced capital price declines.

- Particularly for capital types with relatively low shelf lives, the existing capital stock appears more valuable than you would expect from investment prices.
- Intuition:
  - For a capital type with high product turn-over, producing old varieties soon becomes relatively costly.
  - If new varieties are not perfect substitutes for old varieties, then the stock of old varieties can become valuable.

Additional related Literature

- **Cambridge capital controversy:** Robinson, Sraffa, Samuelson, Solow, etc.
- **Vintage capital:** See e.g. Boucekkine, de la Croix & Licandro (2008; 2011).
  - Note that unlike the standard vintage capital literature, in the model here it will be possible to produce old varieties as well as new ones.
- **Relative price of investment & investment specific technological change:** Greenwood, Hercowitz & Krusell (1997), Krussell (1998), Licandro, Ruiz-Castillo & Duran (2002), Justiniano, Primiceri & Tambalotti (2011).
- **Biases in the national accounts:** Broda & Weinstein (2006; 2010), Redding & Weinstein (2016).
- **Micro capital prices:** Lanteri (2018).
- **Variety specific growth:** Adam & Weber (2019).

Capital in the model

- The aggregate capital good is produced by a perfectly competitive industry with the technology:

$$K(t) = \left[\int_{-\infty}^{t} K_s(t)^{\frac{1}{1+\lambda}}\, ds\right]^{1+\lambda}.$$

- Note: the set of varieties/vintages evolves exogenously.

- Capital producers choose demand for specific vintages to maximise their profits (zero in equilibrium):

$$R(t)K(t) - \int_{-\infty}^{t} R_s(t)K_s(t)\, ds\,.$$

- $R(t)$ and $R_s(t)$ are aggregate and vintage s rental rates, respectively.

- In equilibrium:

$$K_s(t) = K(t)\left(\frac{R_s(t)}{R(t)}\right)^{-\frac{1+\lambda}{\lambda}}, \qquad R(t) = \left[\int_{-\infty}^{t} R_s(t)^{-\frac{1}{\lambda}}\, ds\right]^{-\lambda}.$$

Growth in rental rates

- For now we assume $R_t(t)K_t(t) = 0$.
    - This will be true in the model as new varieties will start off with zero productivity.
    - Removes biases coming from love of (new) varieties.
    - See e.g. Broda & Weinstein (2010).

- Then, we have:

$$\frac{\dot{R}(t)}{R(t)} = \frac{\int_{-\infty}^{t} R_s(t)K_s(t)\frac{\dot{R}_s(t)}{R_s(t)}\,ds}{\int_{-\infty}^{t} R_s(t)K_s(t)\,ds}.$$

Digression: Price indices in continuous time

- The BEA use Fisher aggregators, for which (for an arbitrary good):

$$\frac{P(t)}{P(t-\zeta)} = \sqrt{\frac{\int_{-\infty}^{t} P_s(t) Y_s(t)\, ds}{\int_{-\infty}^{t-\zeta} P_s(t-\zeta) Y_s(t)\, ds} \frac{\int_{-\infty}^{t-\zeta} P_s(t) Y_s(t-\zeta)\, ds}{\int_{-\infty}^{t-\zeta} P_s(t-\zeta) Y_s(t-\zeta)\, ds}}.$$

- Thus if $P_t(t) Y_t(t) = 0$:

$$\lim_{\zeta \to 0} \left[\frac{1}{\zeta} \log \frac{P(t)}{P(t-\zeta)} \right] = \frac{\int_{-\infty}^{t} P_s(t) Y_s(t) \frac{\dot{P}_s(t)}{P_s(t)}\, ds}{\int_{-\infty}^{t} P_s(t) Y_s(t)\, ds}.$$

- I.e. if applied to capital quantities and rental rates, this gives the theoretically correct aggregate price.
  - This is not what the BEA does as rental rates are hard to infer.
- In general, the biases we find in this paper do not occur for non-durable goods.
  - Capital services are a non-durable good with price equal to the rental rate of capital.

Further model primitives (1/2)

- The law of motion for capital good vintage s is:

$$\dot{K}_s(t) = I_s(t) - \delta K_s(t).$$

- Capital and labour are combined to produce an intermediate good X_t, the numeraire. (Details later.)

- One unit of the intermediate good may be converted into $e^{\gamma t}$ units of the consumption good in period t.

  - Produced under perfect competition.

  - I.e. the price of a unit of consumption is $P_C(t) \coloneqq e^{-\gamma t}$.

  - Effectively: γ is the TFP growth of the consumption good producing sector.

Further model primitives (2/2)

- One unit of the intermediate good becomes:

$$\psi(t-s)^{\phi}e^{\kappa s}$$

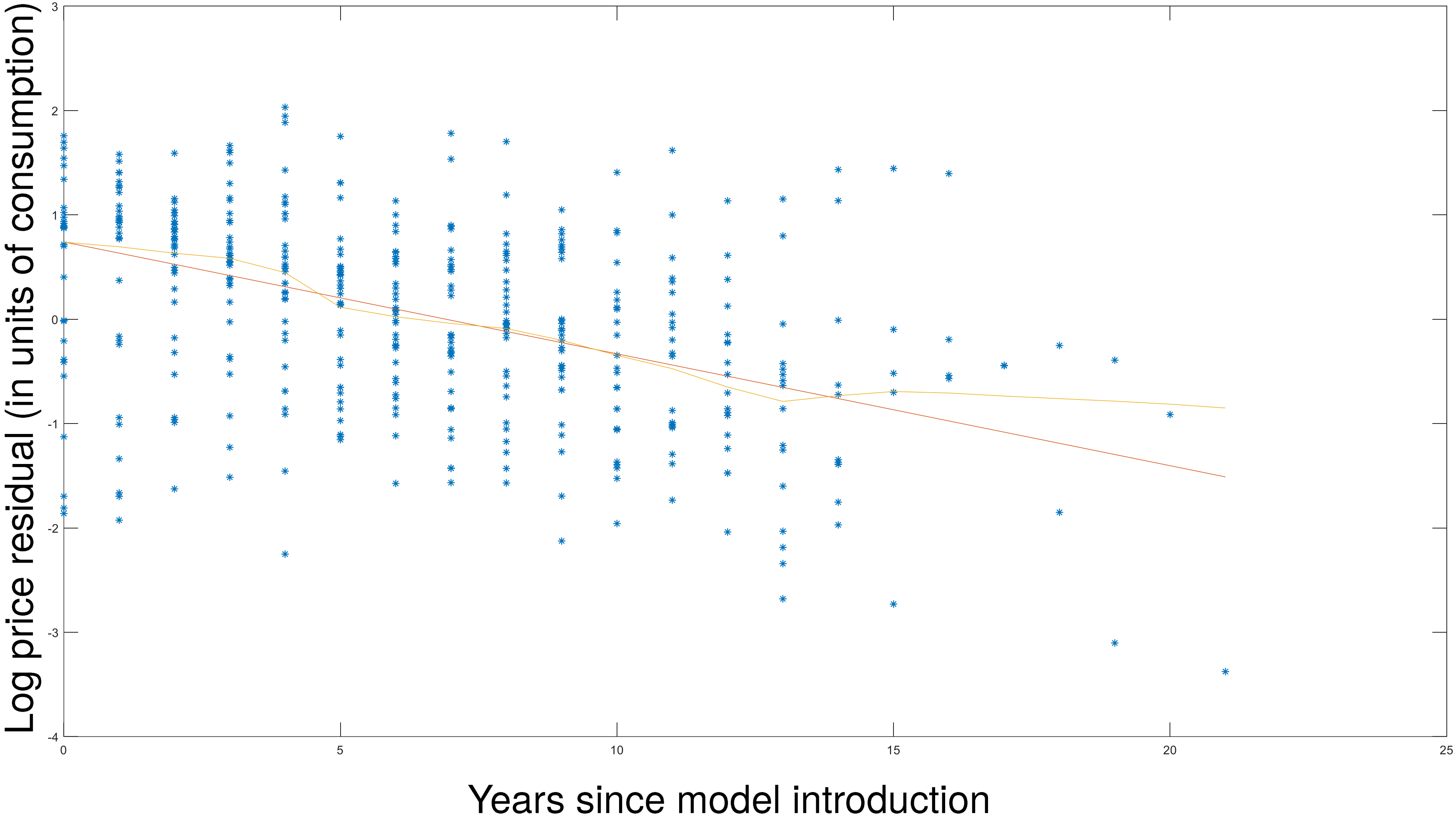
units of capital variety s in period t. Assume $\kappa > 0$.

  - Again produced under perfect competition.
  - I.e. the price of investment goods of vintage s is:

$$Q_s(t) := \frac{1}{\psi(t-s)^{\phi}e^{\kappa s}} \Rightarrow \frac{\dot{Q}_s(t)}{Q_s(t)} = -\frac{\phi}{t-s}.$$

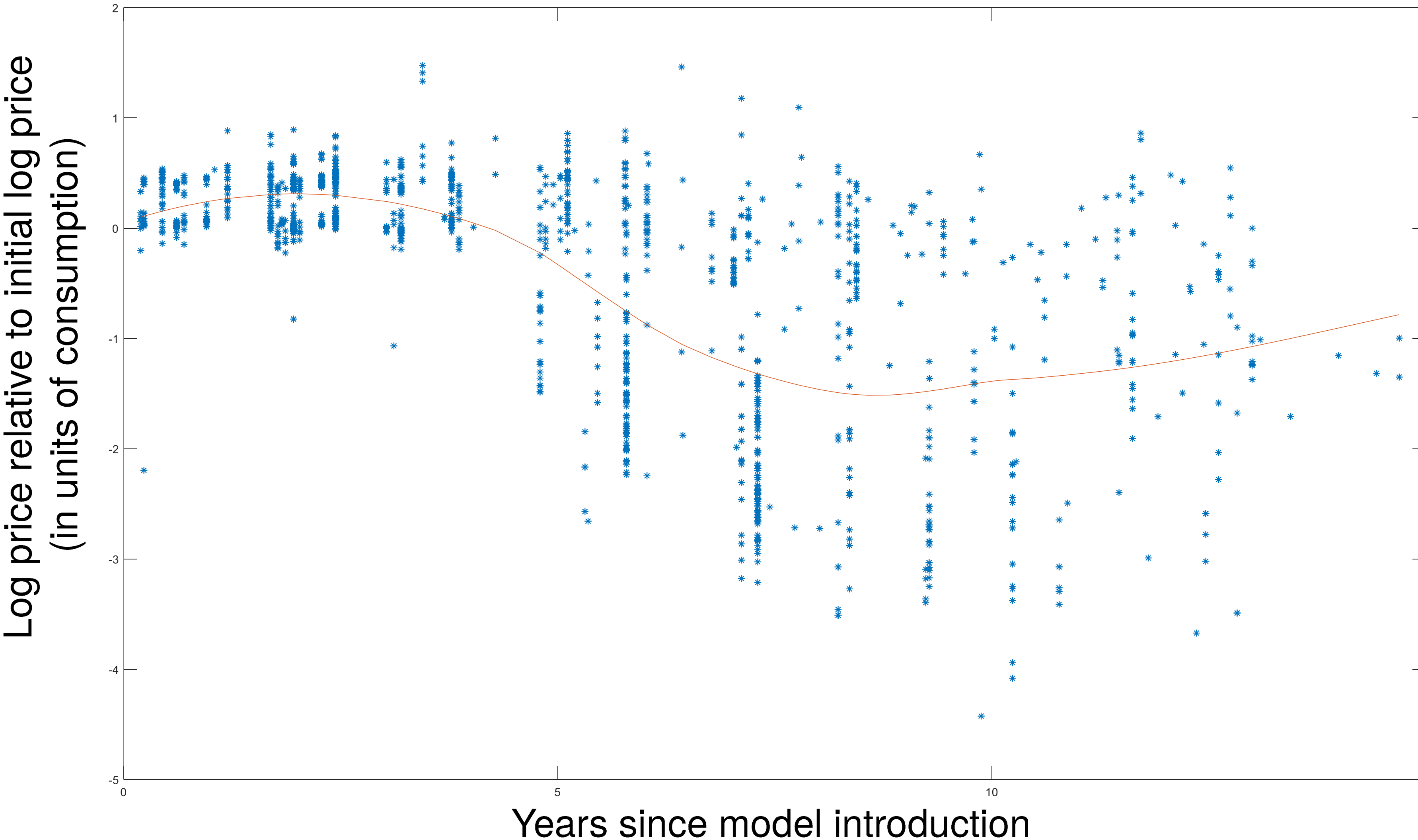
  - $\psi > 0$ just scales productivity.
  - $\phi > 0$ determines the growth rate of young products. (All products start with zero productivity, but then experience rapid growth.)
  - $\kappa > 0$ determines how productivity grows over time, holding fixed variety age $(t-s)$.

New aircraft prices



Residuals after standardizing at the good level. Data kindly provided by A. Lanteri.

Intel CPU Prices



Current price data web scraped from NewEgg.com. Initial price data web scraped from Wikipedia.

Optimal capital variety holdings (1/2)

- Suppose a firm holds capital vintage s.
- If $r(t)$ is the deterministic interest rate at t, the firm's value is:

$$\int_0^\infty e^{-\int_0^\xi r(t+\tau)\,d\tau}[R_s(t+\xi)K_s(t+\xi) - Q_s(t+\xi)I_s(t+\xi)]\,d\xi\,.$$

- Optimisation implies:

$$\frac{R_s(t)}{Q_s(t)} = \delta + r(t) + \frac{\phi}{t-s}.$$

- (Unsurprisingly) Price of capital good s is:

$$\int_0^\infty e^{-\int_0^\xi r(t+\tau)\,d\tau - \delta\xi} R_s(t+\xi)\,d\xi = Q_s(t).$$

Optimal capital variety holdings (2/2)

- If we define:

$$R^*(\tau) := \frac{\delta + r^* + \frac{\phi}{\tau}}{\psi \tau^{\phi} e^{-\kappa\tau}}, \qquad R^* := \left[\int_0^\infty R^*(\tau)^{-\frac{1}{\lambda}}\, d\tau\right]^{-\lambda},$$

- Then if $r(t) \equiv r^*$ (i.e. we are on the BGP):

$$R(t) = e^{-\kappa t} R^*, \qquad R_s(t) = e^{-\kappa t} R^*(t-s).$$

- Both individual and aggregate rental rates are declining at rate κ.

- Note:

$$\frac{R^{*\prime}(\tau)}{R^*(\tau)} = \kappa - \frac{\phi}{(\delta + r^*)\tau^2 + \phi\tau} - \frac{\phi}{\tau}.$$

  - So $R^*(\tau)$ is U-shaped.

Some notation

- Suppose throughout that the random variable T has p.d.f.:

$$\tilde{\tau} \mapsto \frac{Q_{t-\tilde{\tau}}(t) K_{t-\tilde{\tau}}(t)}{\int_0^\infty Q_{t-\tau}(t) K_{t-\tau}(t)\, d\tau}.$$

- $\mathbb{E}\mathrm{T}$ is the average years since variety introduction of the capital stock with weights given by value shares of that age.
    - Not the same as average age from the national accounts.
    - Chiefly convenient for stating results.

- Lemma 1 (exercise in integration by parts):

$$\kappa = \mathbb{E}\left[(1+\lambda)\frac{\phi}{(\delta + r^*)\mathrm{T}^2 + \phi\mathrm{T}} + \frac{\phi}{\mathrm{T}}\right].$$

The true price of capital

- The true price of the aggregate capital stock satisfies:

$$P_K(t) = \frac{\int_{-\infty}^{t} Q_s(t) K_s(t)\, ds}{K(t)} = \frac{e^{-\kappa t} R^*}{\delta + r^* + \phi \mathbb{E}\mathrm{T}^{-1}},$$

- **Shrinking at rate $\boldsymbol{\kappa}$.**

- By Lemma 1:

$$P_K(t) > \frac{e^{-\kappa t} R^*}{\delta + r^* + \kappa} = \int_0^{\infty} e^{-\int_0^{\xi} r(t+\tau)\, d\tau - \delta \xi} R(t+\xi)\, d\xi\,,$$

- With equality in limit $\kappa \to 0$.

- Buying a unit of the aggregate capital stock at t gets you period $t + \xi$ returns of:

$$\int_{-\infty}^{t} e^{-\delta \xi} R_s(t+\xi) \frac{K_s(t)}{K(t)} ds \neq R(t+\xi)!$$

Investment and effective depreciation

- Define $\Delta(t)$ ($= \Delta^*$ on BGP) to solve:

$$\frac{e^{-\kappa t} R^*}{\delta + r^* + \phi \mathbb{E}\mathrm{T}^{-1}} = P_K(t) = \int_0^\infty e^{-\int_0^\xi (r(t+\tau)+\Delta(t+\tau))\, d\tau} R(t+\xi)\, d\xi$$

$$= e^{-\kappa t} R^* \int_0^\infty e^{-(\Delta^* + r^* + \kappa)\xi}\, d\xi = \frac{e^{-\kappa t} R^*}{\Delta^* + r^* + \kappa}$$

- I.e.:

$$\Delta^* = \delta - (\kappa - \phi \mathbb{E}\mathrm{T}^{-1}) < \delta.$$

- We then **define** aggregate investment by:

$$I(t) \coloneqq \dot{K}(t) + \Delta(t) K(t).$$

- These definitions will ensure:
    1. The aggregate capital good is equivalent to a homogeneous capital good with depreciation $\Delta(t)$ and rental rate $R(t)$.
    2. The aggregate price of investment equals that of capital.

The true price of investment

- Let $z(t) \coloneqq \frac{I(t)}{K(t)}$ ($= z^*$ on the BGP), so $\frac{\dot{K}(t)}{K(t)} = z^* - \Delta$ on the BGP.

- Lemma 2: $\mathbb{E}\left[z^* + \delta - \Delta^* - \frac{1+\lambda}{\lambda}\frac{R^{*\prime}(\mathrm{T})}{R^*(\mathrm{T})}\right] = z^*$.

- True price of aggregate investment is given by:

$$P_I(t) \coloneqq \frac{\int_{-\infty}^{t} Q_s(t) I_s(t)\, ds}{I(t)} = \frac{P_K(t)}{z^*}\mathbb{E}\left[z^* + \delta - \Delta^* - \frac{1+\lambda}{\lambda}\frac{R^{*\prime}(\mathrm{T})}{R^*(\mathrm{T})}\right] = P_K(t).$$

- The true investment price is **equal** to the capital price.

The observed price of capital

- The observed price of capital grows at rate:

$$\frac{\dot{P}_K^{\mathrm{OBS}}(t)}{P_K^{\mathrm{OBS}}(t)} := \frac{\int_{-\infty}^{t} Q_s(t) K_s(t) \frac{\dot{Q}_s(t)}{Q_s(t)}\, ds}{\int_{-\infty}^{t} Q_s(t) K_s(t)\, ds} = -\mathbb{E}\, \frac{\phi}{\mathrm{T}}$$
$$= -\kappa + \mathbb{E}\left[(1+\lambda)\, \frac{\phi}{(\delta + r^*)\mathrm{T}^2 + \phi \mathrm{T}}\right]$$
$$\geq -\kappa.$$

- Using Lemma 1.

- The observed price of capital is growing more quickly than the true price!

The observed price of investment

- The observed price of investment grows at rate:

$$\frac{\dot{P}_I^{\mathrm{OBS}}(t)}{P_I^{\mathrm{OBS}}(t)} := \frac{\int_{-\infty}^{t} Q_s(t) I_s(t) \frac{\dot{Q}_s(t)}{Q_s(t)}\, ds}{\int_{-\infty}^{t} Q_s(t) I_s(t)\, ds} = -\frac{\mathbb{E}\frac{\phi}{\mathrm{T}}\left[z^* + \delta - \Delta^* - \frac{1+\lambda}{\lambda}\frac{R^{*\prime}(\mathrm{T})}{R^*(\mathrm{T})}\right]}{\mathbb{E}\left[z^* + \delta - \Delta^* - \frac{1+\lambda}{\lambda}\frac{R^{*\prime}(\mathrm{T})}{R^*(\mathrm{T})}\right]}$$

$$\leq -\mathbb{E}\frac{\phi}{\mathrm{T}} = \frac{\dot{P}_K^{\mathrm{OBS}}(t)}{P_K^{\mathrm{OBS}}(t)}.$$

- Thus observed investment growth is below observed capital growth!

- Providing $\Delta^* + r^* + \kappa - z^* > 0$ (which will hold automatically in our GE setup), we can prove that for sufficiently large λ:

$$\frac{\dot{P}_I^{\mathrm{OBS}}(t)}{P_I^{\mathrm{OBS}}(t)} \leq -\kappa \leq \frac{\dot{P}_K^{\mathrm{OBS}}(t)}{P_K^{\mathrm{OBS}}(t)}$$

- The observed price of investment is growing more slowly than the true price!

“ϕ bias”

- “ϕ bias” disappears when $\phi = 0$. If $\phi = 0$ then:

$$\frac{\dot{P}_I^{\mathrm{OBS}}(t)}{P_I^{\mathrm{OBS}}(t)} = \frac{\dot{P}_I(t)}{P_I(t)} = -\kappa = \frac{\dot{P}_K(t)}{P_K(t)} = \frac{\dot{P}_K^{\mathrm{OBS}}(t)}{P_K^{\mathrm{OBS}}(t)}.$$

- With ϕ positive, a wedge is driven between observed and true price growth, pushing down observed investment price growth, and pushing up observed capital price growth.

- ϕ bias comes from the facts that:

  1. The investment bundle is skewed towards newer goods than the capital bundle.

  2. New goods experience rapid productivity growth.

Closing the model

- Let $C(t)$ be aggregate consumption, $N(t)$ population, $w(t)$ labour income per capita in units of consumption goods, $A(t)$ household total asset holdings.

- Assume on the BGP $w(t) = w^* e^{gt}$, where we will solve for g.

- At t, households maximise:

$$\int_0^\infty e^{-\rho\tau} N(t+\tau) \frac{\left(\frac{C(t+\tau)}{N(t+\tau)}\right)^{1-\sigma} - 1}{1-\sigma} d\tau \,,$$

- Subject to:

$$\dot{A}(t) = r(t)A(t) + P_C(t)w(t)N(t) - P_C(t)C(t).$$

- Assume the intermediate good is produced under perfect competition with the technology:

$$\begin{aligned} K(t)^\alpha N(t)^{1-\alpha} = X(t) &= P_C(t)C(t) + P_I(t)I(t) + E(t) \\ &= R(t)K(t) + P_C(t)w(t)N(t). \end{aligned}$$

In equilibrium

- Consumption per capita grows at rate:

$$g \coloneqq \gamma + \frac{\alpha}{1-\alpha}\kappa.$$

- Capital and investment per capita grow at rate:

$$h \coloneqq \frac{\kappa}{1-\alpha}.$$

- We have:

$$r^* = \rho + \sigma g - \gamma = \rho + \frac{\alpha}{1-\alpha}\sigma\kappa - (1-\sigma)\gamma,$$
$$z^* = h + n + \delta.$$

- Investment bias condition holds:

$$\Delta^* + r^* + \kappa - z^* = \rho - n - g(1-\sigma) > 0,$$

  - As the RHS of the equality is the effective discount rate.

Calibration (1/3)

- Fix:
  - $\lambda \leftarrow \frac{1}{2.2-1} \approx 0.83$. Broda & Weinstein (2006), 2.2 is median e.o.s. at SITC-3 level, 1990-2001, also e.o.s. for "automatic data process machines", quite similar to e.o.s. for vehicles, 3.0.
  - $\sigma \leftarrow \frac{1}{0.594} \approx 1.68$. Havranek et al. (2015), unimportant for results.

- Calibrate (LHS are observed geometric means 1965 to now):
  - $n \leftarrow \frac{d}{dt} \log N(t) \approx 0.0098$.
  - $\alpha \leftarrow \frac{Y^{\mathrm{GDP}}(t)-Y^{\mathrm{PROP}}(t)-Y^{\mathrm{LAB}}(t)}{Y^{\mathrm{GDP}}(t)-Y^{\mathrm{PROP}}(t)} \approx 0.36$.
    - $Y^{\mathrm{PROP}}(t)$ is proprietor's income.
    - $Y^{\mathrm{LAB}}(t)$ is compensation of employees, plus government social benefits to persons, less contributions for government social insurance.
  - $g \leftarrow \frac{d}{dt} \log \left(\frac{Y^{\mathrm{GDP}}(t)}{N(t) P_C(t)} \right) \approx 0.017$.
  - $z^* \leftarrow \frac{P_I(t) I(t)}{P_K(t) K(t)} \approx 0.073$.
  - $\mathcal{R} \leftarrow \frac{R(t) K(t)}{P_K(t) K(t)} \approx 0.12$.

Calibration (2/3)

- Calibrate (LHS are observed geometric means 1965 to now):
  - $g_{PK} \leftarrow \frac{d}{dt}\log\left(\frac{P_K^{\text{OBS}}(t)}{P_C(t)}\right) \approx 0.0049.$
  - $g_{PI} \leftarrow \frac{d}{dt}\log\left(\frac{P_I^{\text{OBS}}(t)}{P_C(t)}\right) \approx -0.0067.$
  - $\delta^{DATA} \leftarrow \frac{CFC(t)}{P_K(t)K(t)} \approx 0.050.$
    - Only used to check the over-identifying restriction: $\delta^{DATA} = z^* - n - g + g_{PK}$.
    - We use $\delta \approx 0.052$ (the RHS of the previous restriction).
    - Error: 0.15 percentage points, 2.9 percent.

Calibration (3/3)

- Guess ϕ, Δ^*, then evaluate:
  - $h \leftarrow z^* - n - \Delta^* \approx 0.019.$
  - $\kappa \leftarrow h(1-\alpha) \approx 0.012.$
  - $\gamma \leftarrow g - \alpha h \approx 0.010.$
  - $\phi \mathbb{E}\mathrm{T}^{-1} \leftarrow -\mathscr{g}_{PK} + \gamma \approx 0.0050.$
  - $\frac{\mathbb{E}\frac{\phi}{\mathrm{T}}\left[z^*+\delta-\Delta^*-\frac{1+\lambda R^{*\prime}(\mathrm{T})}{\lambda \quad R^*(\mathrm{T})}\right]}{\mathbb{E}\left[z^*+\delta-\Delta^*-\frac{1+\lambda R^{*\prime}(\mathrm{T})}{\lambda \quad R^*(\mathrm{T})}\right]} \leftarrow -\mathscr{g}_{PI} + \gamma \approx 0.017.$
  - $\mathscr{r}^* \leftarrow \mathcal{R} - \Delta^* - \kappa \approx 0.066.$
  - $\rho \leftarrow \mathscr{r}^* + \gamma - \sigma g \approx 0.048.$

- Calculate residuals from directly evaluating the expectations above.
  - We evaluate the required integrals numerically, by treating them as expectations of appropriate gamma distributed random variables, mapping these into uniforms, and then using a Fejer Type 1 rule.
  - If they are not zero, adjust our guesses of ϕ, Δ^*.

Calibration consequences

- Annual growth of capital price in units of consumption goods:
  - Observed: 0.49%.
  - True: -0.21%.
  - Bias: -0.70 percentage points
- Annual growth of investment price in units of consumption goods:
  - Observed: -0.67%
  - True: -0.21%
  - Bias: 0.47 percentage points
- Annual growth of output price in units of consumption goods:
  - Observed: -0.15%
  - True: -0.05%
  - Bias: 0.10 percentage points

- Annual real output growth is overstated by 0.10 percentage points in the NIPA!
- Annual real investment growth is overstated by 0.47 percentage points!

Further consequences

- Lower role for investment specific technological change as an engine of growth.
  - IST ($\kappa > \gamma$) explains 6.9% of growth in our calibration.
  - Compared to 60% in Greenwood, Hercowitz & Krusell (1997).
  - IST may still drive growth in particular sectors (e.g. equipment, IPP, durable cons.).

- Unlikely can explain the decline in the labour share with falling capital prices and capital/labour substitution (Neiman and Karabarbounis 2014).
  - In fact, capital and labour appear to be gross compliments:
    - Best practice estimates: León-Ledesma, McAdam & Willman (2010).
    - Surveys: Chirinko (2008), Klump, McAdam & Willman (2012).
    - Meta-study: Knoblach, Rößler & Zwerschke (2016)
  - Were capital prices falling rapidly, a rise in the labour share would be expected.
  - We at least reduce this puzzle.

- Piketty (2014) / Piketty & Zucman (2014) via Rognlie (2014).
  - With lower real investment growth, we should expect $\frac{P_K K}{P_Y Y} \to \frac{s}{g_I + \delta}$ to be larger.

Conclusions

- We built a simple model to explain the apparent divergence of capital and investment prices.
  - All of this divergence can be explained by incorrect aggregation of the prices of fixed assets.
  - Traditional price aggregates are inappropriate for durable goods.
  - Aggregation should be on implied rental rates, not prices.
- Model implies a greatly reduced role for IST.
- Model predicts output growth in units of consumption goods is overstated by about 0.10 percentage points per year.
- Currently working on reconstructing US national accounts from lowest level up.

Appendix: IST Literature

- Nominal capital LOM:

$$P_{K,t}K_t = (1-\delta)P_{K,t}K_{t-1} + P_{I,t}I_t$$
$$\Rightarrow K_t = (1-\delta)K_{t-1} + \frac{P_{I,t}}{P_{K,t}}I_t$$

- Greenwood, Hercowitz & Krusell (1997) call $\frac{P_{I,t}}{P_{K,t}}$ "investment specific technological change" (IST).
  - GHK identify it with $\frac{P_{C,t}}{P_{I,t}}$, the price of consumption in units of investment.
  - This ratio is rising in the US, so could be an engine of growth.
  - $\frac{P_{I,t}}{P_{K,t}}$ is roughly inversely proportional to Tobin's Q $= \frac{\text{market}}{\text{replacement}}$.

- BUT $\frac{P_{I,t}}{P_{K,t}}$ is the price of investment in units of capital goods, not $\frac{P_{C,t}}{P_{I,t}}$!
  - Even if $\frac{P_{I,t}}{P_{K,t}}$ and $\frac{P_{I,t}}{P_{C,t}}$ are falling, $\frac{P_{K,t}}{P_{C,t}}$, the price of capital in units of consumption goods, may still be rising, in which case IST would not be an engine of growth.

Appendix: What should the BEA do?

- To (roughly) correctly measure the price of capital and investment, the BEA could proceed as follows:

1. Guess an interest rate $r(t)$.
2. Measure individual good prices and depreciation rates as at present.
3. From the good prices $Q_s(t)$, depreciation rates $\delta_s(t)$ and guessed interest rate $r(t)$, construct implied rental rates $R_s(t)$ using:
$$\frac{R_s(t)}{Q_s(t)} = \delta_s(t) + r(t) - \frac{\dot{Q}_s(t)}{Q_s(t)}.$$
3. Aggregate $R_s(t)$ and $K_s(t)$ to obtain $R(t)$ and $K(t)$ using the Fisher index formula.
   - Note that this is implicitly using second derivatives of $Q_s(t)$!
4. Evaluate $P_K(t) = \frac{\int_{-\infty}^{t} Q_S(t) K_S(t)\, ds}{K(t)}$ and $I(t) = \frac{\int_{-\infty}^{t} Q_S(t) I_S(t)\, ds}{P_K(t)}$ (so $P_I(t) = P_K(t)$).
5. Evaluate $\Delta(t) = \frac{I(t) - \dot{K}(t)}{K(t)}$, and check $\frac{R(t)}{P_K(t)} - \Delta(t) = r(t) - \frac{\dot{P}_K(t)}{P_K(t)}$, if not, adjust $r(t)$.