The One Equation New Keynesian Model

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What drives inflation in New Keynesian models? How does monetary policy work in them? Answers to these questions usually begin with a simple three equation model. However, the mechanisms can still be difficult to explain to students and policymakers. This note shows that even large New Keynesian (NK) models collapse to a single independent equation for inflation if the monetary rule is slightly simplified. This special case provides a laboratory for understanding the behaviour of inflation in NK models. In this special case, inflation's dynamics are unrelated to the slope of the Phillips curve.

Monetary rules in NK models often take the form:

$$i_t = \tilde{r}_t + \pi^* + \phi_\pi(\pi_t - \pi^*),$$

where:

$$\tilde{r}_t := r^* + \phi_i i_{t-1} + \phi_y y_t + \phi_{\Delta y}(y_t - y_{t-1}) + \varepsilon_t,$$

and where i_t is the nominal interest rate, r^* is the long-run real interest rate, π_t is inflation, π^* is the inflation target, y_t is output relative to a trend or efficient level, and ε_t is a monetary policy shock.

In estimated medium-scale models, $\tilde{r}_t$ is usually highly correlated with r_t, the real interest rate. This is due to the persistence of expansions in the data, and the Euler equation's link between expected consumption growth and current real interest rates. For example, in the Smets & Wouters (2007) model, the correlation between $\tilde{r}_t$ and r_t is 83%,[2] and they have similar standard deviations (0.46% for r_t, 0.50% for $\tilde{r}_t$). Not much is lost then by using the simpler rule:

$$i_t = r_t + \pi^* + \phi_\pi(\pi_t - \pi^*) + \nu_t, \tag{1}$$

in which the central bank responds to the current real interest rate and inflation. (Such rules have appeared in Adão, Correia & Teles (2011) amongst other places.) We allow for an extra monetary policy shock, ν_t, to capture any difficulties in observing r_t.

Any linearized model will also include the Fisher equation:

$$i_t = r_t + \mathbb{E}_t \pi_{t+1}. \tag{2}$$

Combining (1) and (2) gives:

$$\boxed{\mathbb{E}_t \pi_{t+1} - \pi^* = \phi_\pi(\pi_t - \pi^*) + \nu_t} \tag{3}$$

Equation (3) is the one equation New Keynesian model. It completely characterises the evolution of inflation, independently of all of the model's other endogenous variables, and all of the model's other equations. For example, the slope of the Phillips curve cannot matter for the dynamics of inflation in this model.

If $\phi_\pi > 1$ and ν_t follows the AR(1) process $\nu_t = \rho\nu_{t-1} + \eta_t$, then the unique non-explosive solution to equation (3) takes the form:

$$\pi_t - \pi^* = -\frac{\nu_t}{\phi_\pi - \rho}.$$

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[1] The views expressed in this paper are those of the author and do not represent the views of the Deutsche Bundesbank, the Eurosystem or its staff.

[2] The correlation is 63% if i_t and $\tilde{r}_t$ are redefined so that ε_t enters i_t but not $\tilde{r}_t$.

If the central bank is more aggressive, so ϕ_π is larger, then inflation is less volatile. Contractionary monetary policy shocks (i.e. increases in ν_t) lead to falls in inflation, as expected. Inflation inherits persistence from the persistence of the monetary policy shock. No other shocks effect inflation. Of course, if there is a nominal rigidity in the model, such as sticky prices or wages, monetary shocks may have an impact on real variables. But as long as the central bank follows the rule in equation (1), these real disruptions have no feedback to inflation. We can understand inflation without worrying about the rest of the economy.

It is natural to wonder if this model is so weird as to be useless. No central bank justifies movements in nominal interest rates via observed movements in real rates, though many appeal to output gaps or unemployment. The key point is that responding to output growth can look a lot like responding to real rates. Current output growth is correlated with future consumption growth, given appropriate rigidities. Future consumptions growth is correlated with real rates via the Euler equation. Even if in actual economies central banks are not responding one-for-one to real interest rates, if they come close to doing this, outcomes should be similar to those in this simple model. The observed impact of cost-push shocks may relate to the fact that cost push shocks create a wedge between r_t and $\tilde{r}_t$, so cost-push shocks act like measurement error in r_t. For example, in the Smets & Wouters (2007) model, the correlation between r_t and $\tilde{r}_t$ is lowest conditional on a price or wage mark-up shock, with correlations of 73% and 19% respectively.

If this model does provide a rough description of central bank behaviour, then we should not be surprised that inflation apparently follows an exogenous stochastic process, independent of the output gap. This point was made in an optimal monetary policy context by McLeay & Tenreyro (2019), who also survey the empirical literature showing the seeming independence of inflation. Additionally, we should not expect the dynamics of inflation to change by much if there is a change in the slope of the Phillips curve. This is contrary to the traditional story in which the Phillips curve is the mechanism by which monetary policy impacts inflation. With the central bank following equation (1) the Phillips curve determines the output gap, given inflation. Causation runs in one direction only: from monetary policy to inflation to output.

Would it be good for central banks to follow equation (1) more closely? They could do so by incorporating information from inflation indexed bonds into their decision making. According to the model, with no measurement error, there would be no monetary policy shock, and thus no variance in inflation. Whether this is desirable depends on the breadth of the central bank's mandate and the weight it places on output stabilisation.

References

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