PhD Macro 2 Topic 8 Questions

1) Suppose households choose $C(\cdot)$ and $L(\cdot)$ to maximise $\int_0^\infty e^{-\rho t}\left[\log C(t) - \frac{1}{1+\nu}L(t)^{1+\nu}\right]dt$ subject to $Y(t) = K(t)L(t)^{1-\alpha} = C(t) + I(t)$ and $\dot{K} = I(t) - \delta K(t)$. Solve for the optimum. *Hint: You may find it helpful to conjecture that* $\frac{C(t)}{K(t)}$ *is constant.*

2) Suppose households choose $s(\cdot)$ to maximise $\int_0^\infty N(t)e^{-\varrho t}\log c(t)\,dt$ subject to $c(t) = \frac{C(t)}{N(t)}$, $C(t) = Y(t) = A(t)L_Y(t)$, $\quad \dot{A}(t) = \kappa A(t)^\phi L_A(t)$, $\quad L_A(t) = s(t)N(t)$, $\quad L_Y(t) = \big(1 - s(t)\big)N(t)$, $N(t) = N_0 e^{nt}$, $A(0) = (\psi N_0)^{\frac{1}{1-\phi}}$. Solve for the optimum. *Hint: You may find it helpful to conjecture that* $\frac{\dot{A}(t)}{A(t)}$ *and* $s(t)$ *are constant.* Assuming that $0 < n$, $0 < \psi$, $0 < \kappa$, $0 < \phi < 1$ show that your solution is only valid if $\phi\kappa < \psi\left(\frac{\phi}{1-\phi}n + \varrho\right) < \kappa$.

3) Suppose the unit cost final good and the "machine" good are produced by perfectly competitive industries with (respectively) the production function $Y(t) = M(t)^\alpha L_Y(t)^{1-\alpha}$ and the production function $M(t) = \left[\int_0^{\mathcal{J}(t)} M_i(t)^{\frac{1}{1+\mu}}\,di\right]^{1+\mu}$, where $M_i(t)$ is the quantity of the ith intermediate good used at t, which is purchased at a price $P_i(t)$ and where the labour $L_Y(t)$ is hired at a wage $W(t)$. Suppose that the ith intermediate good is produced by a monopolist using the production function $M_i(t) = AX_i(t)^\beta L_i(t)^{1-\beta}$, where the labour input $L_i(t)$ is again hired at a wage $W(t)$ and where $X_i(t)$ is an input of the final good. Further suppose that a new firm can enter, producing a new product, if they pay a one off invention cost of ω units of the final good. New inventors then become a perpetual monopolist for the product they have just invented (e.g. because they have been granted a patent). Apart from the invention cost, there is free entry of new firms. Firms are owned by households, and so discount the future at the real interest rate $r(t)$. Finally suppose that households choose consumption $C(t)$, labour $L(t)$ and bonds $B(t)$ to maximise $\int_0^\infty e^{-\rho t}[\log C(t) - L(t)]\,dt$. Given all this, the market clearing conditions are $Y(t) = C(t) + \int_0^{\mathcal{J}(t)} X_i(t)\,di + \omega\dot{\mathcal{J}}(t)$ and $L(t) = L_Y(t) + \int_0^{\mathcal{J}(t)} L_i(t)\,di$. Solve for the path of this economy.

4) Repeat question 3 assuming that rather than entry being governed by a free entry condition, it is instead chosen by a social planner (i.e. the social planner chooses entry $\dot{\mathcal{J}}(t)$, consumption $C(t)$, labour $L(t)$ and bonds $B(t)$ to maximise $\int_0^\infty e^{-\rho t}[\log C(t) - L(t)]\,dt$). Does the growth rate you found agree with that you found in 3?

5) Repeat question 3 assuming that there are taxes or subsidies on the cost of inventing a new product, so rather than costing ω, inventing a new product instead costs $\omega(1+\tau_\omega)$, where the additional $\omega\tau_\omega$ units of the final good paid are rebated lump sum to the household. Are there taxes/subsidies that replicate the social planner solution from 4?

6) Repeat questions 3, 4 and 5 assuming that monopolists have a constant probability of $\phi\, dt$ of losing their "patent" (i.e. their ability to exclude competitors) in the interval $[t, t + dt)$. Assume that once the ith monopolist has lost their patent, the ith intermediate good is produced by a perfectly competitive industry.