Inverse demand curve: $p = 1 - Q$

Zero costs of production. Entry cost of F.

Profits of firm i: $\pi_i = (1 - Q)q_i$

FOC of firm i: $0 = \frac{\partial \pi_i}{\partial q_i} = -q_i + 1 - Q$

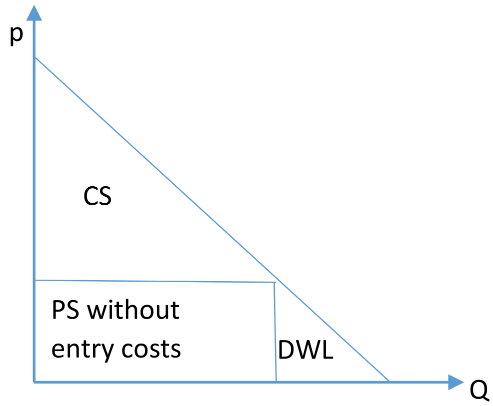
Adding FOCs across the i firms gives: $0 = -Q + n(1 - Q)$

So: $(1 + n)Q = n$

I.e. $Q = \frac{n}{1+n}$

Hence $\pi_i = \left(1 - \frac{n}{1+n}\right)\left(\frac{1}{1+n}\right) = \left(\frac{1}{1+n}\right)^2$.

Firms enter until $F \approx \pi_i = \left(\frac{1}{1+n}\right)^2$. Hence: $n \approx -1 + \sqrt{\frac{1}{F}} = -1 + F^{-\frac{1}{2}}$



Producer surplus is given by: $PS = \sum_{i=1}^{n} \pi_i - nF = (1 - Q)Q - nF$

Consumer surplus is given by: $CS = \frac{1}{2}Q^2$

Thus welfare is given by: $W = CS + PS = \frac{1}{2}Q^2 + (1 - Q)Q - nF = \frac{1}{2}\left(\frac{n}{1+n}\right)^2 + \left(\frac{1}{1+n}\right)\left(\frac{n}{1+n}\right) - nF$

The social planner would choose the number of firms to maximise welfare.

Social planner FOC:

$$0 = \frac{dW}{dn} = \left(\frac{n}{1+n}\right)\frac{1+n-n}{(1+n)^2} + \frac{(1+n)^2 - 2n(1+n)}{(1+n)^4} - F = \frac{n}{(1+n)^3} + \frac{1+n-2n}{(1+n)^3} - F$$
$$= \frac{1}{(1+n)^3} - F$$

Hence: $n = -1 + \sqrt[3]{\frac{1}{F}} = -1 + F^{-\frac{1}{3}}$

Now, if $F > 1$, then $n < 0$ with or without the social planner. So suppose $F < 1$. Then $F^{\frac{1}{3}} > F^{\frac{1}{2}}$, so $F^{-\frac{1}{3}} < F^{-\frac{1}{2}}$. Thus the social planner would prefer fewer firms to enter than will enter under laissez-faire.