Industrial Organisation

Topic 5: Price discrimination and nonlinear pricing

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Motivation

- Last week:
  - We showed how competing firms following “trigger” strategies may sustain monopoly profits in an industry.
- This week:
  - We see if there are any times in which a monopolist can obtain profits higher than the monopoly level.
  - Only possible if the monopolist can charge different consumers different prices.
  - A break from oligopoly models.

Outline

- What is price discrimination?
- Types of price discrimination.
  - Welfare effects.
- Tying and bundling.
- Durable goods.
  - Price discrimination in time.
- Alternative reading this week:
  - Church and Ware Chapter 5 (online at http://is.gd/XHBLz4)
  - The maths is covered in the OZ refs to be given.

What is price discrimination?

- Price discrimination means selling the same good at different prices. E.g.:
  - Buy two get one free offers.
  - Student discounts.
  - Off-peak rail fares.

- Slightly more generally:
  - Price discrimination is selling two *similar* goods at different ratios to marginal costs.
  - So, e.g. the fact that posting a letter to Scotland costs the same as posting one to someone in Guildford is discriminatory.

When is discrimination possible?

- Need all of the following conditions:
  - The good cannot be resold.
    - Else e.g. a student would buy a load of discounted copies of MS Office then sell them on EBay at full price.
    - Services (e.g. haircuts) usually pass this test.
  - The firm has some market power (so we can have $P > MC$).
  - Consumers are not all identical.
  - Firms can somehow charge different prices to the different types of consumers.

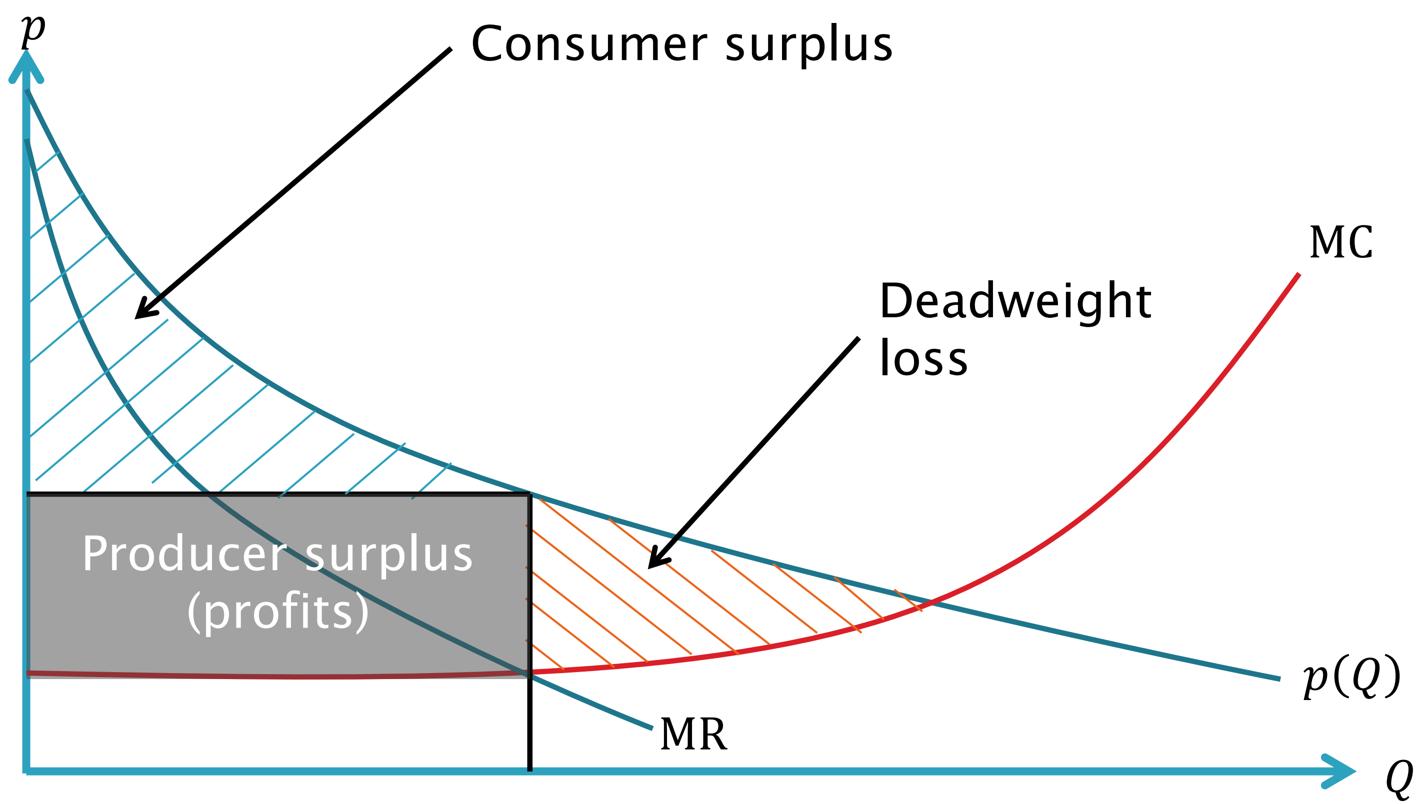
Why would firms want to discriminate?

- Even under monopoly, if there are consumers who value a good highly (above the price they pay), consumer surplus will be high.

- Price discrimination enables firms to “steal” that CS and turn it into profits.
  - While not putting off consumers who value the good less.

- Usually price discrimination increases profits, though there are two notable exceptions to this:
  - When selling durable goods.
    - Price discrimination in time.
  - When firms are discriminating under oligopoly.
    - To be looked at in a future lecture.

The lost (producer) surpluses



First degree price discrimination (1/2)

- The firm sells each unit at a different, take-it-or-leave-it price.

- Goods sold at consumer's reservation prices.

- Example: DeBeers' sales of uncut diamonds:
  - "The diamonds are sold on a take-it-or-leave-it basis. A sightholder is given a small box of uncut diamonds priced between $1 and $25 million. De Beers set the price – there is no haggling and no re-selling of diamonds in uncut form. It is rare for sightholders to refuse a diamond package offered to them, for fear of not being invited back. And those who dare to purchase diamonds from other sources than De Beers will have their sightholder privilege revoked."
  - http://www.neatorama.com/2008/12/01/10-facts-about-diamonds-you-should-know/

First degree price discrimination (2/2)

- Maximises social welfare
  - Though it all goes to the firm…

- Very hard for most firms to do, since they do not know each consumer's reservation price.

- Not "incentive compatible".
  - High-value customers have an incentive to pretend to be low-value ones.

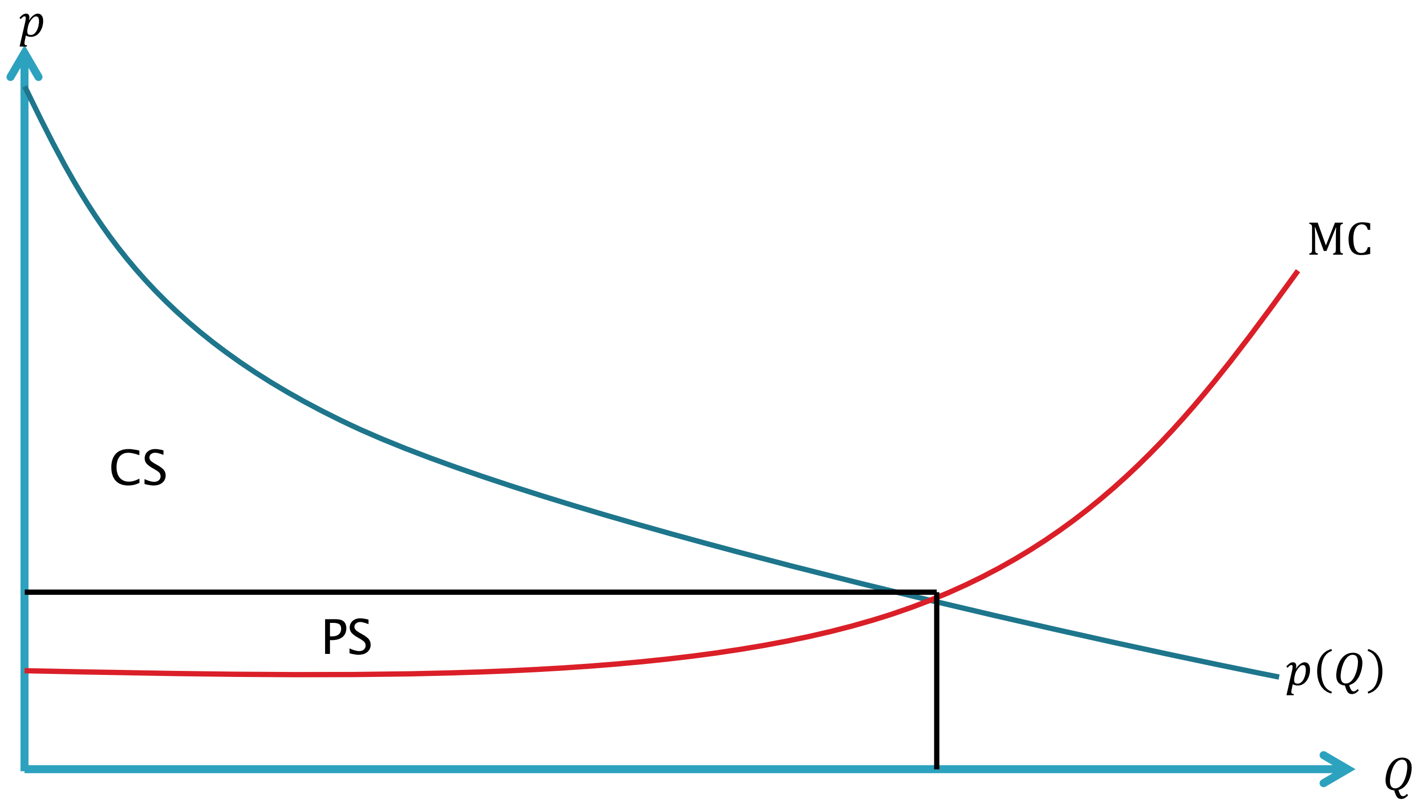
Two part tariffs (1/3) (OZ 13.1)

- Suppose that all consumers have the same demand function $p(Q)$.

- And suppose the monopolist chose a pricing structure under which to buy $q > 0$ units a consumer had to pay a fixed fee of f plus an additional price p per unit.
  - I.e. total payment for quantity q, $T(q) = f + pq$.
  - Examples:
    - Mobile phone contracts.
    - Gym membership.
    - Etc.

Two part tariffs (2/3)

- What are the optimal choices of f and p for the firm?



Two part tariffs (3/3)

- Is this price discrimination?
  - Everyone has the same preferences…
  - Everyone pays the same price…

- Re-sale?

- Variations in demand across consumers?

Second degree price discrimination

- Firms cannot see consumers characteristics.

- Try to get consumers to self-select into the different price bands.

- Works by e.g.:
  - Offering menus of tariffs (such as different telephone contracts).
  - Offering nonlinear tariffs:
    - As a function of quantity (such as 3 for 2 offers).
    - As a function of quality (such as hardback/paperback books or deliberately "damaged" computer processors/graphics cards).

Second degree discrimination with two part tariffs (1/2)

- Suppose there are equal numbers of two types of consumers, those with low demand (L) and those with high demand (H).
  - High types demand more at any price.
  - For phones, you might think of the low demand types as being households, and the high demand types as being businesses.

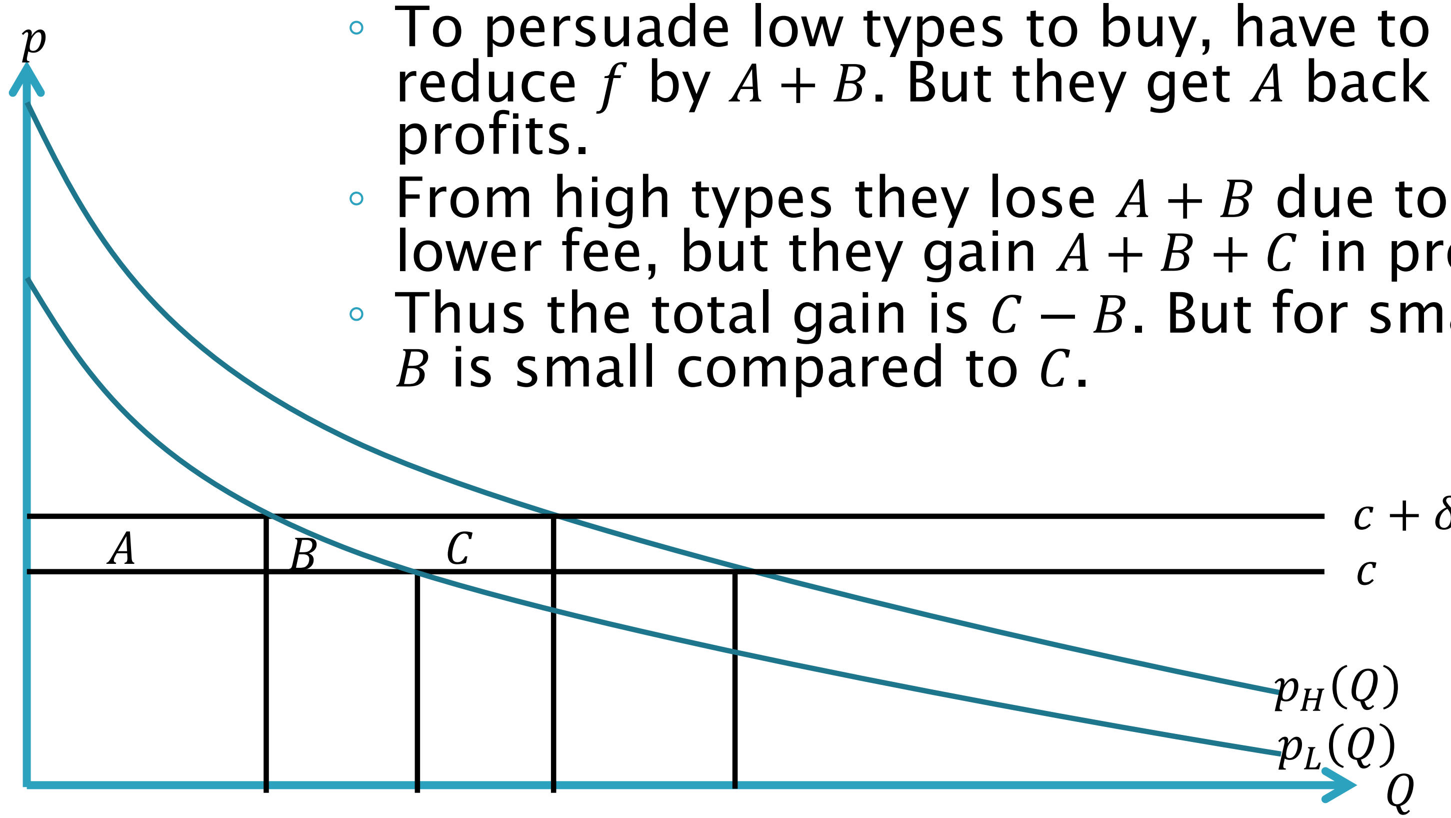
- The monopolist would like to offer the low type $T_L(q) = \mathrm{CS}_L + cq$ where c is MC, and CS_L is the consumer surplus the low types would get from perfect competition.

- Likewise they would like to offer $T_H(q) = \mathrm{CS}_H + cq$ to the high types.

- But firms cannot tell low from high types, and, given this, high types would always pretend to be low types to pay the lower fee.

2^{nd} degree discrimination with two part tariffs (2/2)

- Can the firm do better than offering $T(q) = \mathrm{CS}_L + cq$ to everyone?
- Yes. Suppose they increase price by δ.
  - To persuade low types to buy, have to reduce f by $A + B$. But they get A back in profits.
  - From high types they lose $A + B$ due to the lower fee, but they gain $A + B + C$ in profits.
  - Thus the total gain is $C - B$. But for small δ, B is small compared to C.



Linear example

- Suppose there are two types of consumers of equal number (normalized to one for both), with demand curves:
  - $q_H = \max\{0, 1 - p\}$
  - $q_L = \max\{0, a - p\}$ where $a \leq 1$.
- Constant marginal cost of $c < a$.
- Tariff of $T(q) = f + pq$ offered.

- If the firm sells in both markets (requires $p < a$), profits are:
$$2f + (p - c)[(1 + a) - 2p]$$
  - Maximised when f is as large as possible, i.e. when the low type makes zero surplus.
  - Exercise: draw a diagram to show this means $f = \frac{(a-p)^2}{2}$.
  - Exercise: show that with this level of f, firms will set $p = c + \frac{1-a}{2}$.
  - Hence: the bigger the difference between types, the higher is p and the lower is f.
  - Exercise: Does the firm always want to sell in both markets?
    - Hint: suppose $c = 0$ and compare the cases when $a = 1/2$ and $a = 3/4$.

General second degree price discrimination (1/2)

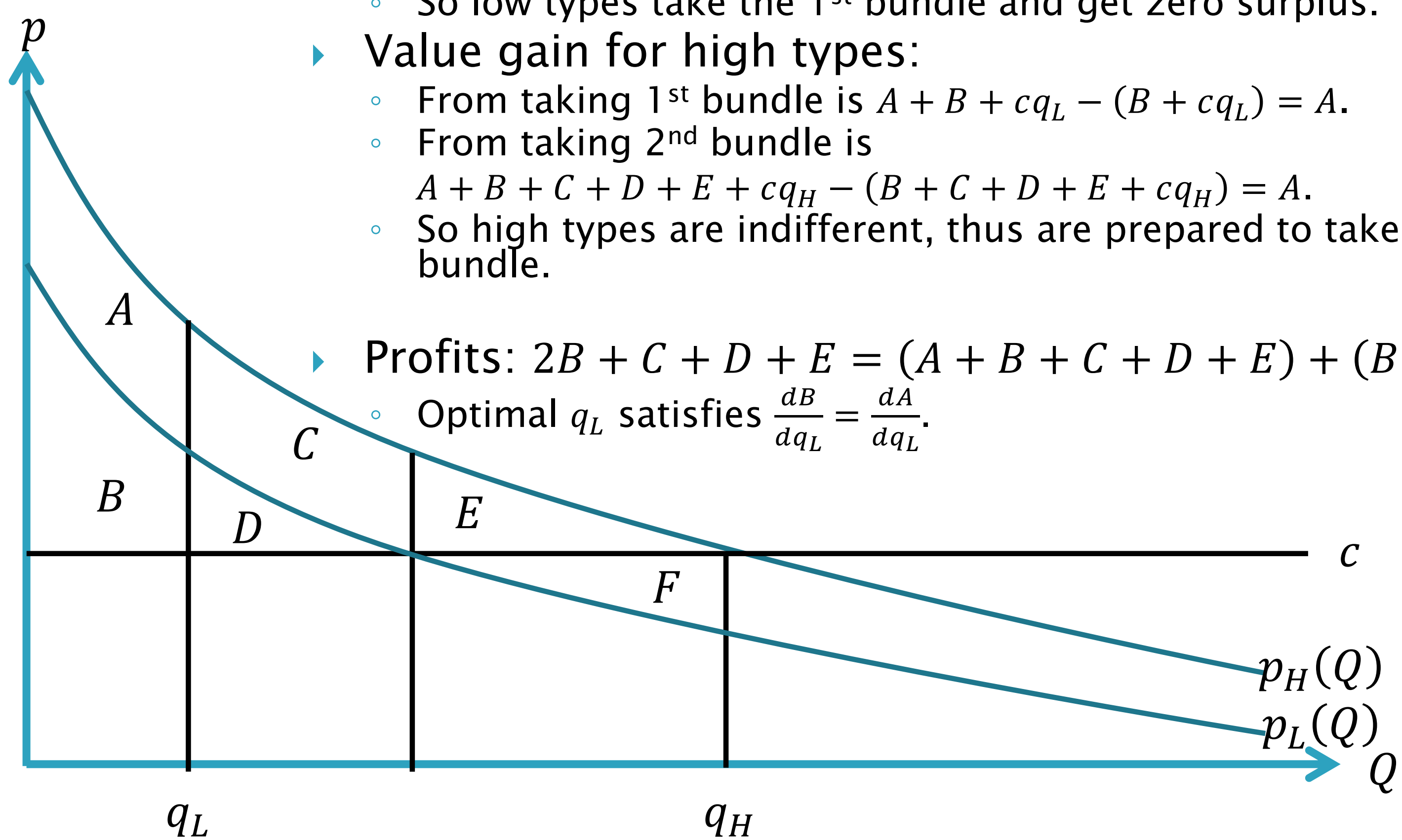
- Suppose that rather than offering a two-part tariff, the firm offers a choice between two (quantity, total-payment) bundles.
- Can trivially implement the solution to the optimal two-part tariff using these bundles.
- Can they do better? Yes.
  - Exercise: Show that under the two part tariff considered in our linear example, the high type strictly prefers their bundle to the low type's one.
- Firm profits maximised when high types are just indifferent between the two tariffs, so optimal bundles have higher total costs for the high type.
  - But: always optimal to have high type consuming the efficient quantity.
  - If you're interested, Church and Ware p.166-176 has one proof of this.
    - Not necessary for the exam.

General 2nd degree discrimination (2/2)

- Suppose the firm offers a choice of $(q_L, B + cq_L)$ and $(q_H, B + C + D + E + cq_H)$.

- Value gain for low types:
  - From taking 1st bundle is $B + cq_L - (B + cq_L) = 0$.
  - From taking 2nd bundle is $B + D - F + cq_H - (B + C + D + E + cq_H) = -C - E - F < 0$.
  - So low types take the 1st bundle and get zero surplus.
- Value gain for high types:
  - From taking 1st bundle is $A + B + cq_L - (B + cq_L) = A$.
  - From taking 2nd bundle is $A + B + C + D + E + cq_H - (B + C + D + E + cq_H) = A$.
  - So high types are indifferent, thus are prepared to take the 2nd bundle.

- Profits: $2B + C + D + E = (A + B + C + D + E) + (B - A)$.
  - Optimal q_L satisfies $\frac{dB}{dq_L} = \frac{dA}{dq_L}$.



Third degree price discrimination

- Firms base price on consumers' observable characteristics. E.g.:
  - OAP discounts for museums.
  - Student discounts on software.
  - Academic discounts for conferences.
  - Magazine price varying by country.
    - The New Statesman is €5.80 throughout the EU, except in Greece, where it is €5.40.
  - AEA membership price varying by income.

- Most common form of price discrimination.

- The firm sets the monopoly price in each market (i.e. MR=MC).

Maths of third degree price discrimination (OZ 5.3)

- Market is equally split between type 1 and type 2 consumers:
  - Type 1 consumers have demand: $p_1(q_1)$
  - Type 2 consumers have demand: $p_2(q_2)$
- Firm has costs $C(Q)$ to produce $Q = q_1 + q_2$.

- Profits: $p_1(q_1)q_1 + p_2(q_2)q_2 - C(q_1 + q_2)$
- FOC q_1: $0 = p_1'(q_1)q_1 + p_1(q_1) - C'(q_1 + q_2)$
- I.e.: $MR_1 = MC$.
- Likewise: $MR_2 = MC$.

- Exercise: Show that this condition is still valid when there are n type 1 consumers and m type 2s.

Which market has the higher price? (OZ 3.2.2 and 5.3)

- Recall the FOC for q_1 says: $p_1'(q_1)q_1 + p_1(q_1) = C'(q_1 + q_2)$.
  - So: $\frac{p_1'(q_1)q_1}{p_1(q_1)} + 1 = \frac{C'(q_1+q_2)}{p_1(q_1)}$

- Note: $\frac{p_1'(q_1)q_1}{p_1(q_1)} = \frac{q_1}{p_1}\frac{dp_1}{dq_1} = \frac{1}{\frac{p_1 dq_1}{q_1 dp_1}} = \frac{1}{\varepsilon}$ where ε is the price elasticity of demand.
- Remember:
  - ε will almost always be negative. $-\varepsilon$ large means elastic demand.
  - In general ε is a function of the price/quantity.

- So: $p_1(q_1) = \frac{C'(q_1+q_2)}{1+\frac{1}{\varepsilon}}$.
  - Elastic demand means $-\frac{1}{\varepsilon}$ is small, so $p_1(q_1) \approx C'(q_1 + q_2)$.
  - I.e. the market with the more elastic demand will have the lower price.
  - Students are more put-off by high prices, so you should charge them less.

Is third degree price discrimination welfare improving?

- Ambiguous:
  - The firm gains.
    - It can always get the same as before by setting the same price in both markets.
  - Consumers offered the high price lose out.
  - Consumers offered the low price gain.
    - Before they might not have been buying the good even.

- A necessary condition for a welfare improvement is that output increases.
  - Varian (1985) or Varian (1989)
    - No need to understand the proof.

Linear demand example (1/3)

- Suppose:
  - $q_1 = \max\{0{,}1 - p_1\}$
  - $q_2 = \max\{0, a - p_2\}$ where $a \leq 1$.
  - $C(Q) = 0$.

- Decisions under discrimination:
  - Profit in first market is $(1 - p_1)p_1$.
    - Maximised when $p_1 = \frac{1}{2}$ so $q_1 = \frac{1}{2}$.
  - Profit in second market is $(a - p_2)p_2$.
    - Maximised when $p_2 = \frac{a}{2}$ so $q_2 = \frac{a}{2}$.
  - Total output is $\frac{1+a}{2}$.

Linear demand example (2/3)

- Decisions without discrimination:
  - Firm can decide to sell in one or both markets.
  - Total market demand when a price P is set in both markets is: $Q = q_1 + q_2 = \max\{0,1-P\} + \max\{0,a-P\}$.
    - So profits are: $P[(1+a)-2P] = 2P\left[\frac{1+a}{2} - P\right]$ providing $1-P \geq 0$ and $a-P \geq 0$ (i.e. if $P \leq a$ since $a \leq 1$).
    - Maximised when $P = \frac{1+a}{4}$ (valid providing $a \geq \frac{1}{3}$), so $Q = (1+a) - 2\frac{1+a}{4} = \frac{1+a}{2}$, giving profits of $\frac{(1+a)^2}{8}$.
    - Hence, total output does not increase under discrimination, so welfare cannot increase. (It will have fallen as long as $a < 1$.)

Linear demand example (3/3)

- Decisions without discrimination continued:
  - The firm always has the option to just sell in the first market, in which case profits are $P(1-P)$.
    - Maximised when $P=\frac{1}{2}$, so $Q=\frac{1}{2}$ and profits are $\frac{1}{4}$.
    - Thus if the firm only sells in one market without discrimination, discrimination increases output, and increases welfare. (Example: AIDS drugs.)
  - Exercise: show the firm will sell in both markets when discrimination is not possible if $a \geq \sqrt{2}-1$. (Hint: first assume $a=\sqrt{2}-1$ and compare profits.)
  - Exercise: show that welfare with price discrimination is $\frac{3}{8}(1+a^2)$ and welfare without price discrimination is $\frac{3}{8}$ if $a<\sqrt{2}-1$ and $\frac{14a^2-4a+14}{32}$ otherwise.

Tying

- Printers and cartridges are complements, but not in fixed proportions.
- Given resale is possible, only one price can be charged for printers.
- If this price is low, high demand consumers get a large surplus.
- Tying enables firms to extract some of this.
  - E.g. "If you buy a printer from me, you have to buy cartridges from me too."
  - Enables $P > MC$ for cartridges.
  - A kind of two part tariff.
- OZ 14.1 gives a slightly strange definition of tying.
  - More usual one is that the purchase of one good requires the future purchase of another. See https://en.wikipedia.org/wiki/Tying_%28commerce%29

Welfare effects of tying

- As in the cases above, depends on whether by tying the firm can open up a new market.
  - E.g. suppose that without tying printers would be priced so high that only businesses could buy them.
  - In this situation tying (if performed) will generally increase welfare.
  - But if all consumers would buy even without tying, welfare will generally fall.

Pure Bundling (OZ 14.1.2)

- =Selling two goods in fixed proportions.
- Imagine you are Rupert Murdoch. What channels do you want to bundle into Sky?

Valuations	Sky Sports	Discovery	Total
Jock	15	10	25
Geek	8	12	20

- If you sell both channels separately (and there are as many Geeks as Jocks) the optimal prices are 8 and 10 for Sky Sports and Discovery respectively, giving a total profit of $2 * 8 + 2 * 10 = 36$.
  - Exercise: How would this change if Jocks valued Sky Sports at 17.
- If you sell a bundle, then the optimum price is 20, giving a profit of 40.
- Key requirement for profitability of bundling: valuations must be negatively correlated across types.

Mixed Bundling (OZ 14.1.3)

- =Selling both a fixed proportion bundle, and the components separately.
- Strategy one: Word and Excel are both 30.
  - Revenue: $120 * 30 = 3600$.
- Strategy two: Word and Excel are both 50.
  - Revenue: $80 * 50 = 4000$.
- Strategy three: Word and Excel are sold in a bundle at price 50.
  - Revenue: $100 * 50 = 5000$.
- Strategy four: Word and Excel are sold in a bundle at price 60.
  - Revenue: $20 * 60 = 1200$.
- Strategy five: Word and Excel are sold individually at price 50, or in a bundle at price 60.
  - Revenue: $80 * 50 + 20 * 60 = 5200$.

User Type	Number	Valuation of Word	Valuation of Excel	Total Valuation
Writer	40	50	0	50
Accountant	40	0	50	50
Generalist	20	30	30	60

Durable goods (1/6)

- E.g. cars/washing machines etc.
- If a firm charges a high price for a durable good today and sells to all of the high value customers, then tomorrow, it will be tempted to cut its price to sell to the low value ones.
- Knowing this, the high value consumers will delay purchasing.
  - This hurts profits!
- The firm would prefer not to be able to discriminate (i.e. not to be able to set different prices in different periods).

Durable goods (2/6) (OZ 5.5.1)

- Two periods.
- Customers have per-period valuations uniformly distributed on $[0,1]$.
  - I.e. half of all consumers have a valuation below $\frac{1}{2}$.
  - $\frac{2}{3}$ of all consumers have a valuation above $\frac{1}{3}$, etc.
- A customer with a per-period valuation of v:
  - gets a surplus of $2v - p$ if they buy in period 1.
  - gets a surplus of $v - p$ if they buy in period 2.
- MC is zero.
- Firm sets p_1 in the first period and p_2 in the second.

Durable goods (3/6)

- Suppose the firm can commit to $p_1 = p_2 = p$.
- Then there is no point consumers delaying purchasing.
- Consumers with a valuation v such that $2v - p \geq 0$ (i.e. $v \geq \frac{p}{2}$) will buy.
- Firm profit is then $p\left(1 - \frac{p}{2}\right) = \frac{1}{2}p(2-p)$, which is maximized at $p = 1$.
- So demand is $\frac{1}{2}$ and profits are $\frac{1}{2}$.

Durable goods (4/6)

- Without commitment (i.e. with discrimination):
- Suppose in period 1, the q_1 consumers with the highest valuation purchased the good.
- Then the remaining consumers are uniformly distributed on $[0,1-q_1]$ and will buy if $v \geq p_2$.
- Firm second period profit is then

$$p_2(1-q_1-p_2),$$

which is maximised at $p_2^* = \frac{1-q_1}{2}$.

- At this point, second period profits are $\frac{(1-q_1)^2}{4}$.

Durable goods (5/6)
Don't worry about the maths!

- Consumers will then buy in period 1 if: $2v - p_1 \geq v - p_2^*$ and $2v - p_1 \geq 0$, i.e. if $v \geq \max\left\{\frac{p_1}{2}, p_1 - p_2^*\right\}$.
- Guess (to be verified): $\frac{p_1}{2} < p_1 - p_2^*$, so $q_1 = 1 - (p_1 - p_2^*) = 1 - p_1 + \frac{1-q_1}{2}$.
- Thus $q_1 = 1 - \frac{2}{3}p_1$.
- Then, total (both period) profits are then:

$$p_1\left(1 - \frac{2}{3}p_1\right) + \frac{\left(1 - \left(1 - \frac{2}{3}p_1\right)\right)^2}{4}$$

- Maximised at $p_1 = \frac{9}{10}$. So $q_1 = \frac{2}{5}$, $p_2 = \frac{3}{10}$ and $q_2 = \frac{3}{10}$.
- Check guess: $\frac{p_1}{2} = \frac{9}{20} < \frac{1}{2} < \frac{6}{10} = p_1 - p_2$.
- From subbing p_1 into total profits, total profits are $\frac{9}{20} < \frac{1}{2}$!

Durable goods (6/6)

- So, given the choice, firms would prefer to commit to set the same price in both periods.
  - Such commitment is generally difficult.

- A few examples of this in practice:
  - Chrysler offered a "lowest price guarantee" on their cars. If the price is lower in future, people who buy now will be reimbursed the difference.
  - Xerox only leased their copiers in the '60s and '70s.
    - If you lease you have to pay every period, so there's no point delaying.

Exercises

- OZ Ex. 5.7
  - Question 3, 4, 5, 6 (tricky)

- OZ Ex. 13.5
  - Question 1

- OZ Extra exercises:
  - http://ozshy.50webs.com/io-exercises.pdf
  - Set #5, 20

Conclusion

- Price discrimination (generally) enables firms to extract additional profit.
- If consumer characteristics are observable, firms perform 1^{st} or (more likely) 3^{rd} degree price discrimination.
- If they are unobservable, firms perform 2^{nd} degree price discrimination, subject to the incentive compatibility constraints.
- Versioning, tying, bundling and time (i.e. durables) provide other avenues for discrimination.
- Welfare is usually improved by discrimination when it opens new markets, but is not otherwise.