

Industrial Organisation

Lecture 6: Profit reducing price discrimination,
and product differentiation

Tom Holden

<http://io.tholden.org/>

Plan for test feedback

- ▶ Tests are still being marked I'm afraid.
- ▶ Performance seems good overall so far.
- ▶ It is important that you are able to see what you did wrong, and how to improve.
- ▶ So, there will be a special class to go through the test next week, once you have your tests back.
 - Time will be circulated in advance, by e-mail.

Outline

- ▶ Left from last week:
 - Durable goods.
 - Price discrimination in time.
- ▶ Horizontal product differentiation
 - Models in which different consumers prefer different products.
 - The [Dixit–Stiglitz \(1977\)](#) model of exogenous differentiation.
 - The [Hotelling \(1929\)](#) linear–city model with location choice.
 - Social welfare.
 - Price discrimination in the Hotelling set–up.
 - The [Salop \(1979\)](#) circular–city model.
 - Product proliferation.
- ▶ Next week, vertical product differentiation
 - Models in which all consumers prefer the same products if they have the same price

Durable goods (1 / 2) (OZ 5.5.1)

- ▶ E.g. cars/washing machines etc.
- ▶ If a firm charges a high price for a durable good today and so sells to all of the high value customers, then tomorrow, it will be tempted to cut its price to sell to the low value ones.
- ▶ Knowing this, the high value consumers will delay purchasing.
 - This hurts profits!
- ▶ The firm would prefer not to be able to discriminate (i.e. not to be able to set different prices in different periods).

Durable goods (2 / 2)

- ▶ Maths on slides from last week gave a linear example in which, given the choice, firms would prefer to commit to set the same price in both periods.
 - Maths is “icing”.
- ▶ Such commitment is generally difficult.
- ▶ A few examples of this in practice:
 - Chrysler offered a “lowest price guarantee” on their cars. If the price is lower in future, people who buy now will be reimbursed the difference.
 - Xerox only leased their copiers in the ‘60s and ‘70s.
 - If you lease you have to pay every period, so there’s no point delaying.

Product differentiation:

Main questions

- ▶ How do firms choose which products to produce?
- ▶ How does product differentiation affect price competition?
- ▶ Are there too many products, or too few?

The Dixit–Stiglitz (1977) model (OZ 7.2)

- ▶ A model of consumers' preference for variety within a market.
 - E.g. breakfast cereals.
- ▶ Rather than assuming different consumers want different products, assumes the existence of a representative consumer who wants a little of everything.

Consumer preferences in the Dixit–Stiglitz (1977) model

- ▶ The representative consumer's utility is given by:

$$U = q_0 + \left[\sum_{i=1}^n q_i^{\frac{1}{1+\lambda}} \right]^{1+\lambda} = q_0 + \left[q_1^{\frac{1}{1+\lambda}} + q_2^{\frac{1}{1+\lambda}} + \dots + q_n^{\frac{1}{1+\lambda}} \right]^{1+\lambda}$$

- ▶ Good zero represents e.g. money (a good that is useful for other things).
- ▶ Good $i > 0$ is produced by the i^{th} firm.
- ▶ Adding another good (increasing n) makes consumers better off.
 - Consumers value variety.
- ▶ With this utility function, all products are equally close substitutes for all other products.
 - When $\lambda = 0$, this is linear utility, so goods are perfect substitutes.
 - When λ is large, goods are poor substitutes.
 - OZ 7.2.1 covers the $\lambda = 1$ case.

Demand functions

- ▶ The representative consumer maximises utility subject to the budget constraint $q_0 + \sum_{i=1}^n p_i q_i = y$, where y is their income.
- ▶ Using the BC we can substitute q_0 out of utility giving: $U = y - [p_1 q_1 + p_2 q_2 + \dots + p_n q_n] + \left[q_1^{\frac{1}{1+\lambda}} + q_2^{\frac{1}{1+\lambda}} + \dots + q_n^{\frac{1}{1+\lambda}} \right]^{1+\lambda}$
- ▶ FOC q_1 gives:
 - $0 = -p_1 + (1 + \lambda) \left[q_1^{\frac{1}{1+\lambda}} + q_2^{\frac{1}{1+\lambda}} + \dots + q_n^{\frac{1}{1+\lambda}} \right]^{\lambda} \left(\frac{1}{1+\lambda} \right) q_1^{\frac{1}{1+\lambda}-1}$
 - i.e.: $p_1 = A q_1^{\frac{1}{1+\lambda}-1}$ where $A = \left[q_1^{\frac{1}{1+\lambda}} + q_2^{\frac{1}{1+\lambda}} + \dots + q_n^{\frac{1}{1+\lambda}} \right]^{\lambda}$
 - Key simplification: when n is large the effect of p_1 on A is negligible.

Firm choices

- ▶ So from the last slide, we know firm i faces the demand curve $p_i = Aq_i^{\frac{1}{1+\lambda}-1}$ for their good.
 - Each firm then sets their quantity as a monopolist would, when facing this (iso-elastic!) inverse demand curve.
 - We call this “monopolistic competition”.
- ▶ Firm profits (assuming constant MC of c_i): $q_i(p_i - c_i) = q_i \left(Aq_i^{\frac{1}{1+\lambda}-1} - c_i \right) = Aq_i^{\frac{1}{1+\lambda}} - c_i q_i$
- ▶ FOC: $0 \approx \frac{1}{1+\lambda} Aq_i^{\frac{1}{1+\lambda}-1} - c_i = \frac{1}{1+\lambda} p_i - c_i$.
- ▶ So $p_i \approx (1 + \lambda)c_i$. I.e. each firm charges the same mark-up over its marginal cost.
 - When $\lambda = 0$ we get $p_i \approx c_i$ i.e. perfect competition.

Free entry

- ▶ Suppose firms have to pay a fixed cost F to enter, and suppose all firms have the same MC, c .

- ▶ Then the zero-profit condition says:

$$F = (p_i - c)q_i = \lambda c q_i$$

- ▶ So, in equilibrium, each firm produces $\frac{F}{\lambda c}$ units.
Thus, firms are larger when:
 - the entry cost is high, and when
 - goods are close substitutes.

Too much, or too little entry?

- ▶ Dixit–Stiglitz (1977) show that the market equilibrium with free entry is a constrained optimum.
 - It is the value for n, p_i, q_i a social planner would choose if they were maximising total utility subject to:
 - There being no lump sum transfers/subsidies.
 - Firms not making negative profits.
- ▶ Recall with Bertrand competition there was too little entry, and with Cournot there was too much.
 - Under Dixit–Stiglitz (1977) competition we have the “Goldilocks” level—the optimal balance between variety and scale.

So, do we never need to worry about inefficient entry?

- ▶ No. The “Goldilocks” property is a consequence of special properties of this particular utility function.
- ▶ More general utility functions lead to variable P.E.D. and there are two opposing effects.
 1. “Non-appropriability of social surplus.”
 - A new firm entering benefits consumers because of their preference for variety. Firms cannot capture this surplus, and so there will tend to be too little entry.
 2. “Business stealing.”
 - Just as we saw with Cournot, when a new firm enters all other firms have to cut their quantity, since the new firm will sell to some of their old customers. This negative externality of entry means there will tend to be too much entry.

Endogenous differentiation models

- ▶ In the Dixit–Stiglitz (1977) model, firms do not really choose which product to produce.
 - They enter, and then they are magically producing a differentiated product.
 - All products are equally close substitutes.
- ▶ In models of endogenous product differentiation, firms will choose how different to make their product from those of their rivals.
 - How close a substitute a product is becomes a choice variable.

The Hotelling (1929) linear-city model (OZ 7.3.1)

- ▶ One way firms can differentiate themselves is by location choice.
 - E.g. imagine a long beach, with sunbathers spread along it.
 - Two competing ice cream sellers want to serve the sunbathers.
 - Where should they locate?
 - The Hotelling model has exactly this structure.
 - We'll start by analysing the problem with location fixed.
- ▶ Location is also a metaphor for any difference in preference:
 - E.g. spicy versus non-spicy food.
 - Alcoholic versus non-alcoholic drinks.

The model (simple version)

- ▶ Consumers are uniformly distributed along $[0,1]$ (“the beach”).
- ▶ There are two firms A and B , both with constant MC of 0.
 - Firm A locates at point 0 and charges a price p_A for ice cream.
 - Firm B locates at point 1 and charges a price p_B for ice cream.
- ▶ Consumers really want ice cream.
 - They are prepared to buy it at any price.
- ▶ Consumers are lazy:
 - The cost to a consumer located at x to buy from firm A is: $p_A + tx$
 - The cost to a consumer located at x to buy from firm B is: $p_B + t(1 - x)$
 - t measures just how lazy consumers are.
- ▶ Two stages: firms choose price, then consumers choose which firm to buy from.

Where is the indifferent consumer?

- ▶ There must be a point x^* such that the consumer at x^* is just indifferent between buying from firm A (and walking left) or firm B (and walking right).

- ▶ At x^* we must have:

$$p_A + tx^* = p_B + t(1 - x^*)$$

- ▶ So: $x^* = \frac{1}{2} + \frac{p_B - p_A}{2t}$

- So if B is expensive, the indifferent consumer is further along, meaning more buy from A .

What price should firms set?

- ▶ Consumers located left of x^* will buy from A and consumers located right of x^* will buy from B .
- ▶ Firm A 's profits are thus $p_A x^* = p_A \left(\frac{1}{2} + \frac{p_B - p_A}{2t} \right)$ and firm B 's are $p_B (1 - x^*) = p_B \left(\frac{1}{2} + \frac{p_A - p_B}{2t} \right)$.
- ▶ FOC for firm A : $\frac{1}{2} + \frac{p_B - p_A}{2t} - \frac{p_A}{2t} = 0$. I.e. $p_A = \frac{t}{2} + \frac{p_B}{2}$
- ▶ Similarly from firm B 's FOC we get: $p_B = \frac{t}{2} + \frac{p_A}{2}$
- ▶ Solving gives: $p_A = p_B = t$.
 - Both firms make a profit of $\frac{t}{2}$.
 - Despite competing in prices.

Interim conclusions on product differentiation

- ▶ In the [Dixit–Stiglitz \(1977\)](#) model firms do not choose what product to produce.
 - Entry automatically creates a product equally different to all other products in the market.
 - With the model's special preferences, there is just the right amount of entry.
- ▶ In the [Hotelling \(1929\)](#) model, firms can choose what product to produce on a taste continuum.
 - We have assumed their choice is fixed for the time being.
 - Equilibrium is solved for by first finding the indifferent consumer.

Location choice in the Hotelling (1929) linear-city model

- ▶ By choosing which product to produce, firms can decide how intense will be the competition they face.
- ▶ Strictly, the model we will present here is that of d'Aspremont, Gabszewicz and Thisse (1979).
 - Hotelling's original conclusions about location choice were incorrect. (See proposition 7.7 of OZ 7.3.1, and OZ 7.5 if you're interested.)

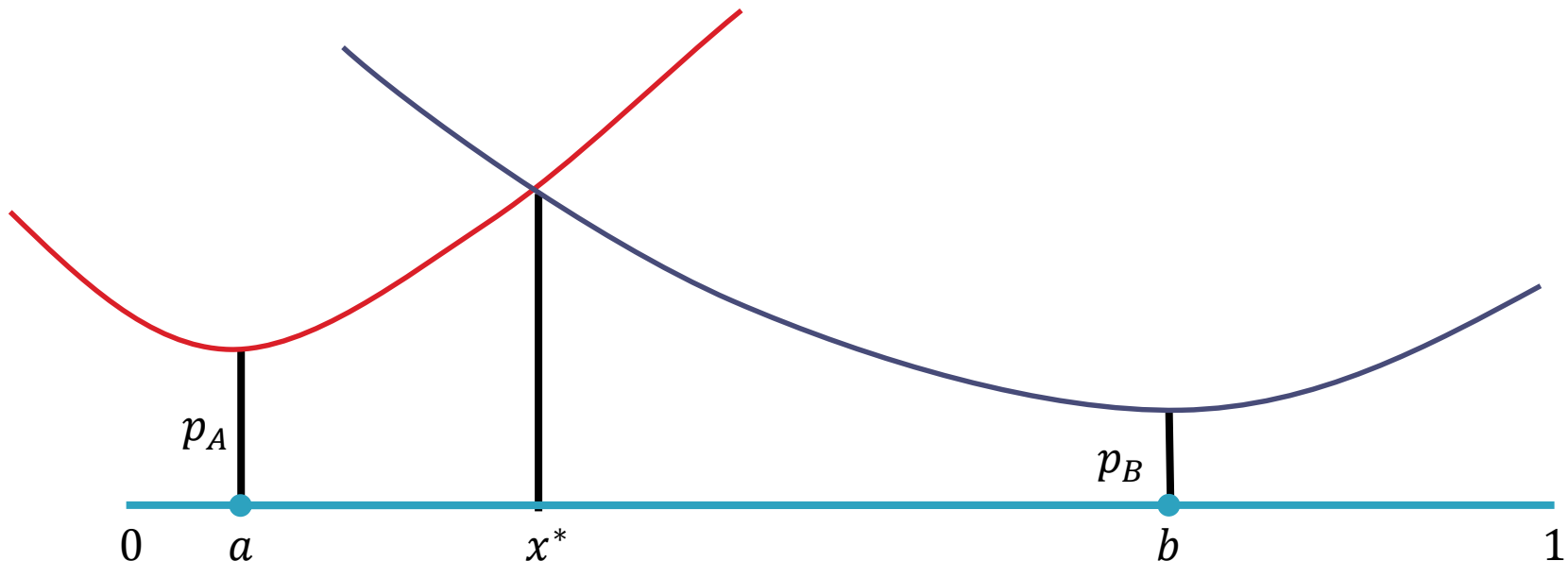
The model

- ▶ Consumers are uniformly distributed along $[0,1]$ (“the beach”).
- ▶ There are two firms A and B , both with MC of c .
 - Firm A locates at point $a \in [0,1]$ along the beach and charges a price p_A for ice cream.
 - Firm B locates at point $b \in [0,1]$ along the beach and charges a price p_B for ice cream.
- ▶ Consumers really want ice cream.
 - They are prepared to buy it at any price.
- ▶ Consumers are lazy:
 - The cost to a consumer located at x to buy from firm A is: $p_A + t(x - a)^2$
 - The cost to a consumer located at x to buy from firm B is: $p_B + t(x - b)^2$
 - t measures just how lazy consumers are.
 - Note the quadratic costs!
- ▶ Three stages: firms choose location, then they choose price, then consumers choose which firm to buy from.



The third stage: which firm will consumers buy from?

- ▶ Let us assume, (without loss of generality) that $a < b$.
- ▶ Then there must be a consumer located at some point $x^* \in (a, b)$ who is totally indifferent between buying from A or B .



Where is the indifferent consumer?

- ▶ At x^* we must have:

$$p_A + t(x^* - a)^2 = p_B + t(x^* - b)^2$$

- ▶ So:
$$\begin{aligned} \frac{p_B - p_A}{t} &= (x^* - a)^2 - (x^* - b)^2 \\ &= x^{*2} - 2ax^* + a^2 - x^{*2} + 2bx^* - b^2 \\ &= 2(b - a)x^* + a^2 - b^2 \end{aligned}$$

- ▶ Thus:
$$x^* = \frac{\frac{p_B - p_A}{t} + b^2 - a^2}{2(b - a)} = \frac{p_B - p_A + t(b^2 - a^2)}{2t(b - a)}$$

- So if B is expensive, the indifferent consumer is further along, meaning more buy from A .

The second stage: what price should firms set? (1 / 2)

- ▶ With $a < b$, consumers located below x^* will buy from a and consumers located above x^* will buy from b .
- ▶ Firm A 's profits are thus $(p_A - c)x^*$ and firm B 's are $(p_B - c)(1 - x^*)$.
- ▶ Note that $\frac{dx^*}{dp_A} = -\frac{1}{2t(b-a)}$ and $\frac{dx^*}{dp_B} = \frac{1}{2t(b-a)}$.
- ▶ So from firm A 's FOC: $0 = \frac{p_B - p_A + t(b^2 - a^2)}{2t(b-a)} - \frac{p_A - c}{2t(b-a)}$, i.e.
$$p_A = \frac{c + p_B + t(b^2 - a^2)}{2}.$$
- ▶ Likewise from firm B 's FOC: $p_B = \frac{c + 2t(b-a) + p_A - t(b^2 - a^2)}{2}$.
 - Exercise: verify.

The second stage: what price should firms set? (2 / 2)

▶ $p_A = \frac{c + p_B + t(b^2 - a^2)}{2}$, $p_B = \frac{c + 2t(b - a) + p_A - t(b^2 - a^2)}{2}$.

▶ So:

$$p_A = \frac{c + \frac{c + 2t(b - a) + p_A - t(b^2 - a^2)}{2} + t(b^2 - a^2)}{2}$$
$$= \frac{3}{4}c + \frac{1}{2}t(b - a) + \frac{1}{4}t(b^2 - a^2) + \frac{1}{4}p_A$$

▶ I.e. $p_A = c + \frac{2}{3}t(b - a) + \frac{1}{3}t(b^2 - a^2)$.

▶ Similarly: $p_B = c + \frac{4}{3}t(b - a) - \frac{1}{3}t(b^2 - a^2)$

◦ Exercise: Verify.

▶ Note:

- Firms price above marginal costs unless both firms are in the same location.
- The further apart a and b are the higher prices are.
- The lazier consumers are, the higher prices are.

The first stage: Location choice

- ▶ Claim: firm A locates at 0 and firm B locates at 1.
 - Implies that $p_A = p_B = c + t$.
 - What are equilibrium profits in this case?
- ▶ To prove we would look at B 's optimum location choice given $a = 0$, and A 's optimum choice given $b = 1$.
 - Lots of tedious algebra though.
- ▶ Intuition: There are two effects of A moving closer to B :
 - The closer A gets, the greater share of demand it will get, meaning profits will tend to increase.
 - The closer A gets, the more intense price competition will be, meaning profits will tend to fall.
- ▶ Which effect dominates depends on how steeply consumer's transport costs increase.
 - With the quadratic specification we have here, the second "strategic" effect will always dominate.
 - With linear transportation costs, initially the first effect dominates, but when the two firms are too close together, the price setting equilibrium we described no longer exists. (And when the two firms are in the same location, they both price at MC.)

Practical considerations

- ▶ Although we do see restaurants tending to position themselves on the extremes of the spiciness spectrum, we do not see firms locating at opposite ends of towns. Why?
 - Transportation costs may be closer to linear.
 - Consumers may be concentrated in the centre.
 - Shops may have their prices set centrally.
 - Exercise: verify that if firms take prices as fixed, they will choose to locate in the centre.
 - Interesting fact: Most US shops (inc. supermarkets) have discretion to set their own prices locally. Most UK ones do not. Does this explain why US towns are more spread out?
 - Being close together may facilitate collusion (deviations may be observed and punished easier).
 - There are positive externalities associated with concentration.
 - Labour's easier to recruit as labour transport costs are lower.
 - Consumer's save on search costs, increasing the market size.

Social welfare

- ▶ Is $a = 0$, $b = 1$ optimal, in terms of social welfare?
 - Total surplus is CS+PS, which (because the price is just a transfer from CS to PS) equals the consumers' total valuation for the ice-cream, minus transport costs, minus production costs.
 - Valuations and production costs are fixed, so the optimum would minimise transportation cost.
 - Minimising transport costs means $a = \frac{1}{4}$ and $b = \frac{3}{4}$, so no consumer has to walk further than $\frac{1}{4}$.
 - So the ice cream firms locate too far apart. The government should (?!) pay them to come closer.
 - And should pay Indian restaurants to make their food less spicy, etc. etc.

The Hotelling model with price discrimination (1 / 2)

- ▶ Suppose now that both firms can observe consumers' locations, so offer a consumer at x a price of $p_A(x)$ or $p_B(x)$.
 - And suppose locations are fixed at 0 and 1.
- ▶ If the consumer at $x < \frac{1}{2}$ was just indifferent between the two firms we would have: $p_A(x) + tx^2 = p_B(x) + t(1-x)^2$. Given that $x < \frac{1}{2}$, this means $p_A(x) > p_B(x)$.
 - If $p_B(x) > 0$, firm B can increase their profits by cutting their price by some small amount, and stealing the whole market at x .
 - If $p_B(x) = 0$, firm A can increase their profit by cutting their price by some small amount, and stealing the whole market at x .
 - Thus no consumer is indifferent between the two firms, and for $x < \frac{1}{2}$, $p_B(x) = 0$.
 - Given this, for $x < \frac{1}{2}$, $p_A(x) = t(1-x)^2 - tx^2 = t(1-2x)$ (minus one penny).

The Hotelling model with price discrimination (2/2)

- ▶ By symmetry then, $p_A(x) = \max\{0, t(1 - 2x)\}$ and $p_B(x) = \max\{0, t(2x - 1)\}$.
- ▶ The consumer at x buys from A if $x < \frac{1}{2}$ and from B if $x > \frac{1}{2}$.
- ▶ Firm A 's profits are $\int_0^{\frac{1}{2}} t(1 - 2x) dx = \frac{t}{4} < \frac{t}{2}$!
 - (Draw a graph to see this integral.)
- ▶ So price discrimination lowers profits under oligopoly.

The Salop (1979) circular-city model (OZ 7.3.2)

- ▶ Consumers are uniformly distributed around a circle of circumference 1.
- ▶ Transport costs are linear (t per unit distance).
- ▶ Three stages:
 1. Firms decide whether or not enter. Those that enter must pay a fixed cost F .
 2. The n firms that enter are placed evenly around the circle.
 - This means the distance between two firms is $\frac{1}{n}$.
 - With quadratic transportation costs we could allow firms to choose location, but this is an extra complication.
 3. Firms choose prices and produce (with zero MC).

Pricing in the Salop model

- ▶ Suppose firm $i \in \{1, \dots, n\}$ sets price p , but all other firms set a price p^* .
- ▶ Firm i has two neighbours, so there is an indifferent consumer both clockwise from it and anti-clockwise from it.
 - Let x^* be the distance to these indifferent consumers (symmetry means they are both the same distance).
Then $p + tx^* = p^* + t\left(\frac{1}{n} - x^*\right)$, so $x^* = \frac{1}{2n} + \frac{p^* - p}{2t}$.
 - Firm i 's profits are then: $2x^*p = p\left(\frac{1}{n} + \frac{p^* - p}{t}\right)$.
 - FOC: $\frac{1}{n} + \frac{p^* - p}{t} - \frac{p}{t} = 0$, so since by symmetry $p = p^*$: $p^* = \frac{t}{n}$.

Entry in the Salop model

- ▶ Given $p^* = \frac{t}{n}$, each firm make profits of $p^* \left(\frac{1}{n} + \frac{p^* - p^*}{t} \right) = \frac{t}{n^2}$.
- ▶ Free entry then means that $F = \frac{t}{n^2}$, so $n = \sqrt{\frac{t}{F}}$.
 - Meaning $p^* = \sqrt{tF}$.
- ▶ Too much or too little?
 - Total surplus is $CS + PS$. Since the price is a transfer, surplus is maximised when transport costs plus entry costs are minimised.
 - Total transport cost is: $2n \int_0^{\frac{1}{2n}} tx \, dx = nt \left(\frac{1}{2n} \right)^2 = \frac{t}{4n}$.
 - So the social planner wants to minimise $\frac{t}{4n} + nF$.
 - FOC: $-\frac{t}{4n^2} + F = 0$, so $4Fn^2 = t$, i.e. $n = \frac{1}{2} \sqrt{\frac{t}{F}}$.
 - Thus there is two times too much entry under free entry.
 - Business stealing effect (like Cournot) dominates non-appropriability of social surplus effect (love of variety).

Product proliferation in the Salop model (1 / 2)

- ▶ Suppose that there were n (even) brands around the circle, but they were owned by only 2 firms (e.g. Kellogg's and General Mills).
 - And suppose that their products alternated around the circle.
 - Then both firms will set a price equal to $\frac{t}{n}$, since as before each product faces competition from a rival on both sides.
 - So both firms will make a profit (not including any entry or brand creation costs) of $\frac{n}{2} \frac{t}{n^2} = \frac{t}{2n}$.
 - If creating each brand costs a firm F , this means net profits are $\frac{t}{2n} - \frac{n}{2} F$. This is positive if $\sqrt{\frac{t}{F}} > n$.

Product proliferation in the Salop model (2/2)

- ▶ Suppose now that Kellogg's created its brands first.
 - If it creates more than $b^* = \frac{1}{2} \sqrt{\frac{t}{F}}$ brands, then General Mills will never be able to make a profit from creating any brands afterwards.
 - So by filling up the product space, Kellogg's can prevent entry.
 - This will be profitable for them if consumers' valuations for the product is high enough.
- ▶ See [Schmalensee \(1978\)](#) for an alternative analysis + an example of this happening in the cereal industry.

Empirical work on product differentiation

- ▶ There is a lot of empirical work estimating demand functions in differentiated product markets. See e.g. Carlton and Perloff p.231–233 for a summary.
 - Often take a characteristics approach, running regressions like $\text{valuation} = \alpha \text{characteristics} - \beta \text{price} + \text{other factors}$. Useful in antitrust investigations to work out consequences of e.g. a merger.
- ▶ Another line of research tries to quantify the gains from variety.
 - [Hausman \(1997\)](#) is an early example, that finds a very large value of consumer surplus from the introduction of Apple–Cinnamon Cheerios.
 - Gain in value of cereal consumption is around 25% under perfect competition, this falls to around 20% under imperfect competition, since introducing Apple–Cinnamon Cheerios means that the price of other Cheerios brands can be increased.
 - [Broda and Weinstein \(2010\)](#) use scanner data about every good purchased by a sample of 55000 households.
 - Conclude that true inflation is overstated by 0.9% because of the extra value consumers are getting from variety. So “...consumers are willing to pay around seven percent of their income to access the set of goods available in 2003 relative to those available in 1994.”

Market power without product differentiation

- ▶ Must be careful to distinguish product differentiation from situations in which we get the results of product differentiation (market power etc.) while firms are selling identical goods.
 - Consumer search (OZ 16):
 - Suppose consumers must pay a cost to find out each firm's price. Then there are equilibria in which all firm's charge the monopoly price (so there is no point visiting more than one firm), and equilibria in which firms choose a price at random, above MC. ([Burdett and Judd 1983](#)) If some consumers have higher search costs than others then we can get partial sorting by search cost. (Consider e.g. tourist shops.)
 - Switching costs/habits:
 - Many switching costs to changing products (time to change bank accounts, lost airline status points, time to learn how to use new operating system/keyboard). We also become attached to products we are familiar with (=habits). Firms have an incentive to offer low prices early on and increase them later. But even these introductory prices may be higher than MC. ([Klemperer 1987](#))

Product differentiation exercises

- ▶ OZ Ex. 7.6
 - Question 2, 3, 4

Conclusions on product differentiation

- ▶ When the firms choose location, products will be too different, relative to the social optimum.
 - ▶ At least with quadratic costs.
- ▶ Price discrimination does not necessarily increase profits when products are differentiated.
- ▶ The Salop model is one in which the business stealing effect dominates, leading to excess entry.
- ▶ Product proliferation may be used to deter entry.
- ▶ Under vertical differentiation firms want to produce as different qualities as possible, and even the low quality firm will make profits.
- ▶ Empirical work suggests the returns to variety are large, and that product differentiation is pervasive.
- ▶ But $P > MC$ does not always mean products are differentiated.