Industrial Organisation

Lecture 2: Basic oligopoly

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Outline

- Game theory refresher 1
- Bertrand competition
- Cournot competition

Game theory refresher 1 plan

- Game 1: The prisoner’s dilemma.
  - What are the elements of a game?
  - What does it mean for a strategy to be dominant?

- Game 2: Stag hunt.
  - What is a Nash equilibrium?

- See OZ 2.1

The prisoner's dilemma

- Two prisoner's are separately questioned about the same crime.
- Each is offered a deal:
  - Give evidence that the other was involved and ***walk free***, providing the other does not offer evidence on you.
    - In this case the other goes to jail for ***three years***.
  - If both offer evidence, then both end up in jail for ***two years***.
  - If neither offers evidence, then both go to jail for ***one year.***

Representing this game

Prisoner B → ↓ Prisoner A	Cooperate (Stay quiet)	Defect (Give evidence)
Cooperate (Stay quiet)	2 2	3 0
Defect (Give evidence)	0 3	1 1

- Numbers now represent utility.
  - Calculated as 3 minus number of years in prison.
- Games always have three elements:
  - A set of players.
  - A set of actions for the players to take.
  - The utilities the players obtain from taking those actions.

How should the prisoners act?

- Suppose that player A knew that B would cooperate.

Prisoner B → ↓ Prisoner A	Cooperate (Stay quiet)	Defect (Give evidence)
Cooperate (Stay quiet)	2 2	3 0
Defect (Give evidence)	0 3	1 1

How should the prisoners act?

- Suppose that player A knew that B would defect.

Prisoner B → ↓ Prisoner A	Cooperate (Stay quiet)	Defect (Give evidence)
Cooperate (Stay quiet)	2 2	3 0
Defect (Give evidence)	0 3	1 1

How should the prisoners act?

- No matter what B does, A wishes to defect.
- Defecting is a strictly dominant strategy for A.

Prisoner B → ↓ Prisoner A	Cooperate (Stay quiet)	Defect (Give evidence)
Cooperate (Stay quiet)	2 2	3 0
Defect (Give evidence)	0 3	1 1

- By symmetry, both players will defect.
- This is an equilibrium in strictly dominant strategies.

Stag hunt: description

- Two hunters are lying in wait for a stag.
- A pair of hares run past.
- Either hunter can jump to catch a hare, but if they do so the stag will be frightened off for good.
- If they both remain patient they will eventually catch the stag.

Stag hunt: payoffs

Hunter B → ↓ Hunter A	Stag (Wait patiently)	Hare (Catch the hare)
Stag (Wait patiently)	2 2	1 0
Hare (Catch the hare)	0 1	1 1

- The "3s" in the prisoner's dilemma have become "1s".
- Is there a dominant strategy?

Stag hunt: Pure Nash equilibrium

Hunter B → ↓ Hunter A	Stag (Wait patiently)	Hare (Catch the hare)
Stag (Wait patiently)	2 2	1 0
Hare (Catch the hare)	0 1	1 1

- Suppose both players believe the other will play "Stag", does either want to play Hare?
- Suppose both players believe the other will play "Hare", does either want to play Stag?

- An outcome is called a "Nash equilibrium" if given how everyone else is behaving, each player is behaving optimally.

Stag hunt: Mixed Nash equilibrium

Hunter B → ↓ Hunter A	Stag (Wait patiently)	Hare (Catch the hare)
Stag (Wait patiently)	2 2	1 0
Hare (Catch the hare)	0 1	1 1

- Suppose both players believe the other will toss a (hidden) coin, and play stag if it's heads, and tails otherwise.
- Can either player do better than following this strategy?
- What would the mixed Nash equilibrium look like if we replaced the "2"s by "3"s above?

Bertrand (price) competition (OZ 6.3)

- n firms produce an identical product.
- Firm 1 has constant marginal cost c_1, firm 2 has constant marginal cost c_2, etc.
- Firm 1 sets a price p_1, firm 2 sets a price p_2, etc.
  - For convenience, we assume price is "discrete", with all prices and costs specified as a multiple of some small amount ϵ (e.g. one penny).
- Demand curve for the product is $Q(p)$.
  - Consumers always buy from the cheapest firm.
  - If several firms set the same price, consumers are split evenly between them.
- Consumers will pay a price: $p^* \coloneqq \min_{i=1,\ldots,n} p_i$.

Bertrand example

- Suppose two car firms compete in price, and must price in whole pounds.
- Firm 1 has marginal costs of £4000.
- Firm 2 has marginal costs of £6000.
- Suppose demand is inelastic.
  - What is the maximum difference between the prices set by both firms in any Nash equilibrium?
  - In the Nash equilibrium in which firm 1 makes the highest possible profits, what prices does each firm set? Why?
- Now suppose the demand curve for cars is given by $Q(p) = 5000 - p$.
  - What price would firm 1 set if firm 2 wasn't around?
  - In the Nash equilibrium in which firm 1 makes the highest possible profits, what prices does each firm set? Why?
- Are there any other equilibria in either case?

Bertrand: Pure Nash equilibrium, first result.

- Suppose for a firm i it was true that:
  $p_i > p^* \geq c_i + \epsilon$.
  - Then firm i is not currently selling anything. (Why?)
  - If it instead set p_i to $c_i + \epsilon$, it would make a strict profit.

- Thus, in any (pure–Nash) equilibrium, and for any firm i, if $p^* \geq c_i + \epsilon$, then $p_i = p^*$.

Bertrand: Pure Nash equilibrium, second result.

- Now suppose there were two firms i and j, for which $p_i = p_j = p^*$.
  - If it was the case that $p^* > c_i + \epsilon$, then firm i could steal the whole market by undercutting its rival with a price of $p_i - \epsilon$.
  - Providing ϵ is small, the gain in profits from increased demand will outweigh the cost from slightly reduced price.

- Thus, (for small enough ϵ) in any (pure-Nash) equilibrium, there can be at most one firm i for which $p^* > c_i + \epsilon$.
  - In the limit as ϵ goes to 0, this means that at most one firm can make a profit.

Bertrand: Pure Nash equilibrium with symmetric firms

- If $c_i = c$ for all firms i, then by our second result there can be at most one firm i with $p^* > c_i + \epsilon = c + \epsilon$.
- Thanks to the first result, this is only possible if either:
  - there are no such firms (i.e. $c \leq p^* \leq c + \epsilon$), OR
  - there is only one firm (i.e. $n = 1$), in which case we get the monopoly solution.
- As long as there are at least two firms then, $c \leq p^* \leq c + \epsilon$ when costs are symmetric.
  - Firms price at (or very near) marginal cost.
  - The competitive solution.

Bertrand: General Pure Nash equilibrium

- If there are at least two firms with marginal cost equal to $\min_{i=1\ldots n} c_i$ (the lowest marginal cost), then:
  - By our second result, $\min_{i=1\ldots n} c_i \le p^* \le \min_{i=1\ldots n} c_i + \epsilon$ and we again get the competitive solution.

- Otherwise:
  - There is a unique firm k with $c_k = \min_{i=1\ldots n} c_i$.
  - That firm sets a price in the interval $\left[c_k + \epsilon, \min\{c_j, p_M\}\right]$, where:
    - j is the firm with the next smallest marginal cost, and
    - p_M is the price a monopolist with marginal cost c_k would set.
  - With small enough ϵ, only firm k sells anything.

Further Bertrand examples

- Each with pricing in whole pounds and 3 firms.
  - $c_1 = £100,\ c_2 = £100,\ c_3 = £100$
  - $c_1 = £100,\ c_2 = £100,\ c_3 = £200$
  - $c_1 = £100,\ c_2 = £200,\ c_3 = £200$
  - $c_1 = £100,\ c_2 = £102,\ c_3 = £104$
  - $c_1 = £100,\ c_2 = £101,\ c_3 = £102$
- In each case, which firm(s) sell? And at what price(s)?
- Demand curve left unspecified.
  - Say how the demand curve might affect your answers.

Symmetric Bertrand: the twist

- The above analysis was for pure Nash equilibria.

- Suppose that:
  - There are two firms (i.e. $n = 2$).
  - Firms have zero marginal cost (i.e. $c_1 = c_2 = 0$).
  - Consumers demand one unit of the good at any price.

- Monopoly profits are infinite, pure-Nash Bertrand profits are zero, but…

Mixed strategy Nash equilibria

- Suppose for some fixed constant z:
  - Each firm never chooses a price less than z.
  - For any price p, with $p > z$, each firm chooses a price greater than p with probability $\frac{z}{p}$.

- Why is this an equilibrium?
  - Firm 1 knows firm 2 is picking their price at random like this. So given this, their expected profits from choosing a price p_1 is:

$$p_1 \Pr(p_2 > p_1) = p_1 \frac{z}{p_1} = z$$

  - So firm 1's profits do not depend on price!
    - Thus, they are happy to pick at random.

Does this generalise?

- We showed that with completely inelastic demand the Bertrand model has equilibria in which profits are arbitrarily high.
  - Completely inelastic demand is rather implausible.

- Baye and Morgan (1999) show there are mixed equilibria like this whenever, either:
  - a monopolist's profits would be infinite, or
  - there is uncertainty about the location of a choke point in demand (and up to that point demand is sufficiently inelastic).

- We will see similar things hold when the firms compete in multiple periods.

General Cournot (quantity) competition (OZ 6.1+6.7)

- Inverse demand curve: $p(Q)$
  - Q is now total quantity

- Firms: $i = 1, \ldots, n$

- Firm i:
  - Produces q_i
  - Total cost function: $c_i(q_i)$
  - Profits: $p(Q)q_i - c_i(q_i)$

- Total quantity is given by:

$$Q = \sum_{i=1}^{n} q_i = q_1 + q_2 + \cdots + q_n$$

Cournot (quantity) duopoly

- Easy case, $n = 2$. Total quantity is given by: $Q = q_1 + q_2$

- Firm 1:
  - Profits assuming firm 2 is playing their optimum, q_2^*: $p(q_1 + q_2^*)q_1 - c_1(q_1)$
  - FOC q_1: $p'(q_1^* + q_2^*)q_1 + p(q_1^* + q_2^*) - c_1'(q_1^*) = 0$

- Firm 2:
  - Profits assuming firm 1 is playing their optimum, q_1^*: $p(q_1^* + q_2)q_2 - c_2(q_2)$
  - FOC q_2: $p'(q_1^* + q_2^*)q_2 + p(q_1^* + q_2^*) - c_2'(q_2^*) = 0$

- Add up the two first order conditions, then divide by two:

$$\frac{1}{2}p'(Q^*)Q^* + p(Q^*) = \frac{1}{2}\left(c_1'(q_1^*) + c_2'(q_2^*)\right)$$

  - Suppose the two firms merged, how would this equation change?

General Cournot (quantity) competition

- Firm i:
  - Profits assuming other firms are playing their optimum: $p(q_1^* + \cdots + q_{i-1}^* + q_i + q_{i+1}^* + \cdots + q_n^*)q_i - c_i(q_i)$
  - FOC: $p'(Q^*)q_i^* + p(Q^*) - c_i'(q_i^*) = 0$
  - Should also check profits are positive at the optimum.

- Add up all of the first order conditions, and divide by n:

$$\frac{1}{n}p'(Q^*)Q^* + p(Q^*) = \frac{1}{n}\sum_{i=1}^{n} c_i'(q_i^*)$$

what happens as $n \to \infty$?

Cournot problems

- Suppose $p(Q) = p_0 - p_1 Q$ and $C_i(q_i) = c_{i,0} + c_{i,1} q_i$ (for all i)
  - Show that under Cournot competition: $Q^* = \frac{p_0 - \bar{c}_1}{\left(1+\frac{1}{n}\right)p_1}$
    - where $\bar{c}_1$ is average marginal cost.
  - Recall that under perfect competition with symmetric marginal costs c_1, and no fixed costs: $Q^* = \frac{p_0 - c_1}{p_1}$.
- Suppose $p(Q) = kQ^{-\beta}$ and $C_i(q_i) = c_{i,0} + c_{i,1} q_i$.
  - Show that under Cournot competition: $p(Q^*) = \frac{\bar{c}_1}{1-\frac{\beta}{n}}$
    - Mark-up pricing (still!)
- What happens in each case as $n \to \infty$?
  - Harder: Is it efficient?

Further problems

- OZ Ex 2.6
  - Questions 1 to 4.
- OZ Ex 6.8
  - Questions 1(parts c and d are optional), 2 and 4)a)+c).
- OZ Extra exercises:
  - http://ozshy.50webs.com/io-exercises.pdf
  - Set #2 and set #6

Conclusion

- Bertrand competition with symmetric marginal costs attains efficiency in pure strategies.
  - Non-efficient mixed strategy equilibria may also exist.
  - If marginal costs are not symmetric, one firm may still make profits.

- Cournot competition leads to similar expressions for quantity and prices as under monopoly, except:
  - Average marginal costs replace marginal costs.
  - The distortion away from perfect competition is smaller, the more firms there are.