

Industrial Organisation

Lecture 2: Basic oligopoly

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Outline

- ▶ Game theory refresher 1
- ▶ Bertrand competition
- ▶ Cournot competition

Game theory refresher 1 plan

- ▶ Game 1: The prisoner's dilemma.
 - What are the elements of a game?
 - What does it mean for a strategy to be dominant?
- ▶ Game 2: Stag hunt.
 - What is a Nash equilibrium?
- ▶ See OZ 2.1

The prisoner's dilemma

- ▶ Two prisoners are separately questioned about the same crime.
- ▶ Each is offered a deal:
 - Give evidence that the other was involved and *walk free*, providing the other does not offer evidence on you.
 - In this case the other goes to jail for *three years*.
 - If both offer evidence, then both end up in jail for *two years*.
 - If neither offers evidence, then both go to jail for *one year*.

Representing this game

Prisoner B →		Cooperate	Defect
		(Stay quiet)	(Give evidence)
↓ Prisoner A	Cooperate (Stay quiet)	2, 2	3, 0
	Defect (Give evidence)	0, 3	1, 1

- ▶ Numbers now represent utility.
 - Calculated as 3 minus number of years in prison.
- ▶ Games always have three elements:
 - A set of players.
 - A set of actions for the players to take.
 - The utilities the players obtain from taking those actions.

How should the prisoners act?

- ▶ Suppose that player A knew that B would cooperate.

Prisoner B →		Cooperate	Defect
		(Stay quiet)	(Give evidence)
↓ Prisoner A	Cooperate	2	3
	(Stay quiet)	2	0
Defect	(Give evidence)	3	1
	0	1	

How should the prisoners act?

- ▶ Suppose that player A knew that B would defect.

Prisoner B →		Cooperate	Defect
		(Stay quiet)	(Give evidence)
↓ Prisoner A	Cooperate (Stay quiet)	2, 2	0, 3
	Defect (Give evidence)	3, 0	1, 1

How should the prisoners act?

- ▶ No matter what B does, A wishes to defect.
- ▶ Defecting is a strictly dominant strategy for A.

Prisoner B →		Cooperate	Defect
		(Stay quiet)	(Give evidence)
↓ Prisoner A	Cooperate (Stay quiet)	2, 2	3, 0
	Defect (Give evidence)	0, 3	1, 1

- ▶ By symmetry, both players will defect.
- ▶ This is an equilibrium in strictly dominant strategies.

Stag hunt: description

- ▶ Two hunters are lying in wait for a stag.
- ▶ A pair of hares run past.
- ▶ Either hunter can jump to catch a hare, but if they do so the stag will be frightened off for good.
- ▶ If they both remain patient they will eventually catch the stag.

Stag hunt: payoffs

		Hunter B →	
		Stag (Wait patiently)	Hare (Catch the hare)
↓ Hunter A	Stag (Wait patiently)	2, 2	1, 0
	Hare (Catch the hare)	0, 1	1, 1

- ▶ The “3s” in the prisoner’s dilemma have become “1s”.
- ▶ Is there a dominant strategy?

Stag hunt: Pure Nash equilibrium

		Hunter B →	
		Stag (Wait patiently)	Hare (Catch the hare)
↓ Hunter A	Stag (Wait patiently)	2, 2	1, 0
	Hare (Catch the hare)	0, 1	1, 1

- ▶ Suppose both players believe the other will play “Stag”, does either want to play Hare?
- ▶ Suppose both players believe the other will play “Hare”, does either want to play Stag?
- ▶ An outcome is called a “Nash equilibrium” if given how everyone else is behaving, each player is behaving optimally.

Stag hunt: Mixed Nash equilibrium

		Hunter B →	
		Stag (Wait patiently)	Hare (Catch the hare)
↓ Hunter A	Stag (Wait patiently)	2	1
	Hare (Catch the hare)	0	1

- ▶ Suppose both players believe the other will toss a (hidden) coin, and play stag if it's heads, and tails otherwise.
- ▶ Can either player do better than following this strategy?
- ▶ What would the mixed Nash equilibrium look like if we replaced the "2"s by "3"s above?

Bertrand (price) competition (OZ 6.3)

- ▶ n firms produce an identical product.
- ▶ Firm 1 has constant marginal cost c_1 , firm 2 has constant marginal cost c_2 , etc.
- ▶ Firm 1 sets a price p_1 , firm 2 sets a price p_2 , etc.
 - For convenience, we assume price is “discrete”, with all prices and costs specified as a multiple of some small amount ϵ (e.g. one penny).
- ▶ Demand curve for the product is $Q(p)$.
 - Consumers always buy from the cheapest firm.
 - If several firms set the same price, consumers are split evenly between them.
- ▶ Consumers will pay a price: $p^* := \min_{i=1,\dots,n} p_i$.

Bertrand example

- ▶ Suppose two car firms compete in price, and must price in whole pounds.
- ▶ Firm 1 has marginal costs of £4000.
- ▶ Firm 2 has marginal costs of £6000.
- ▶ Suppose demand is inelastic.
 - What is the maximum difference between the prices set by both firms in any Nash equilibrium?
 - In the Nash equilibrium in which firm 1 makes the highest possible profits, what prices does each firm set? Why?
- ▶ Now suppose the demand curve for cars is given by $Q(p) = 5000 - p$.
 - What price would firm 1 set if firm 2 wasn't around?
 - In the Nash equilibrium in which firm 1 makes the highest possible profits, what prices does each firm set? Why?
- ▶ Are there any other equilibria in either case?

Bertrand: Pure Nash equilibrium, first result.

- ▶ Suppose for a firm i it was true that:
 $p_i > p^* \geq c_i + \epsilon$.
 - Then firm i is not currently selling anything. (Why?)
 - If it instead set p_i to $c_i + \epsilon$, it would make a strict profit.
- ▶ Thus, in any (pure-Nash) equilibrium, and for any firm i , if $p^* \geq c_i + \epsilon$, then $p_i = p^*$.

Bertrand: Pure Nash equilibrium, second result.

- ▶ Now suppose there were two firms i and j , for which $p_i = p_j = p^*$.
 - If it was the case that $p^* > c_i + \epsilon$, then firm i could steal the whole market by undercutting its rival with a price of $p_i - \epsilon$.
 - Providing ϵ is small, the gain in profits from increased demand will outweigh the cost from slightly reduced price.
- ▶ Thus, (for small enough ϵ) in any (pure-Nash) equilibrium, there can be at most one firm i for which $p^* > c_i + \epsilon$.
 - In the limit as ϵ goes to 0, this means that at most one firm can make a profit.

Bertrand: Pure Nash equilibrium with symmetric firms

- ▶ If $c_i = c$ for all firms i , then by our second result there can be at most one firm i with $c_i + \epsilon = c + \epsilon < p^*$.
- ▶ By symmetry, this is only possible if either:
 - there are no such firms (i.e. $c \leq p^* \leq c + \epsilon$), OR
 - there is only one firm (i.e. $n = 1$), in which case we get the monopoly solution.
- ▶ As long as there are at least two firms then, $c \leq p^* \leq c + \epsilon$ when costs are symmetric.
 - Firms price at (or very near) marginal cost.
 - The competitive solution.

Bertrand: General Pure Nash equilibrium

- ▶ If there are at least two firms with marginal cost equal to $\min_{i=1\dots n} c_i$ (the lowest marginal cost), then:
 - By our second result, $\min_{i=1\dots n} c_i \leq p^* \leq \min_{i=1\dots n} c_i + \epsilon$ and we again get the competitive solution.
- ▶ Otherwise:
 - There is a unique firm k with $c_k = \min_{i=1\dots n} c_i$.
 - That firm sets a price in the interval $[c_k + \epsilon, \min\{c_j, p_M\}]$, where:
 - j is the firm with the next smallest marginal cost, and
 - p_M is the price a monopolist with marginal cost c_k would set.
 - With small enough ϵ , only firm k sells anything.

Further Bertrand examples

- ▶ Each with pricing in whole pounds and 3 firms.
 - $c_1 = £100, c_2 = £100, c_3 = £100$
 - $c_1 = £100, c_2 = £100, c_3 = £200$
 - $c_1 = £100, c_2 = £200, c_3 = £200$
 - $c_1 = £100, c_2 = £102, c_3 = £104$
 - $c_1 = £100, c_2 = £101, c_3 = £102$
- ▶ In each case, which firm(s) sell? And at what price(s)?
- ▶ Demand curve left unspecified.
 - Say how the demand curve might affect your answers.

Symmetric Bertrand: the twist

- ▶ The above analysis was for pure Nash equilibria.
- ▶ Suppose that:
 - There are two firms (i.e. $n = 2$).
 - Firms have zero marginal cost (i.e. $c_1 = c_2 = 0$).
 - Consumers demand one unit of the good at any price.
- ▶ Monopoly profits are infinite, pure-Nash Bertrand profits are zero, but...

Mixed strategy Nash equilibria

- ▶ Suppose for some fixed constant z :
 - Each firm never chooses a price less than z .
 - For any price p , with $p > z$, each firm chooses a price greater than p with probability $\frac{z}{p}$.
- ▶ Why is this an equilibrium?
 - Firm 1 knows firm 2 is picking their price at random like this. So given this, their expected profits from choosing a price p_1 is:

$$p_1 \Pr(p_2 > p_1) = p_1 \frac{z}{p_1} = z$$

- So firm 1's profits do not depend on price!
 - Thus, they are happy to pick at random.

Does this generalise?

- ▶ We showed that with completely inelastic demand the Bertrand model has equilibria in which profits are arbitrarily high.
 - Completely inelastic demand is rather implausible.
- ▶ [Baye and Morgan \(1999\)](#) show there are mixed equilibria like this whenever, either:
 - a monopolist's profits would be infinite, or
 - there is uncertainty about the location of a choke point in demand (and up to that point demand is sufficiently inelastic).
- ▶ We will see similar things hold when the firms compete in multiple periods.

General Cournot (quantity) competition (OZ 6.1 + 6.7)

- ▶ Inverse demand curve: $p(Q)$
 - Q is now total quantity
- ▶ Firms: $i = 1, \dots, n$
- ▶ Firm i :
 - Produces q_i
 - Total cost function: $c_i(q_i)$
 - Profits: $p(Q)q_i - c_i(q_i)$
- ▶ Total quantity is given by:

$$Q = \sum_{i=1}^n q_i = q_1 + q_2 + \dots + q_n$$

Cournot (quantity) duopoly

- ▶ Easy case, $n = 2$. Total quantity is given by: $Q = q_1 + q_2$
- ▶ Firm 1:
 - Profits assuming firm 2 is playing their optimum, q_2^* : $p(q_1 + q_2^*)q_1 - c_1(q_1)$
 - FOC q_1 : $p'(q_1^* + q_2^*)q_1 + p(q_1^* + q_2^*) - c_1'(q_1^*) = 0$
- ▶ Firm 2:
 - Profits assuming firm 1 is playing their optimum, q_1^* : $p(q_1^* + q_2)q_2 - c_2(q_2)$
 - FOC q_2 : $p'(q_1^* + q_2^*)q_2 + p(q_1^* + q_2^*) - c_2'(q_2^*) = 0$
- ▶ Add up the two first order conditions, then divide by two:
$$\frac{1}{2}p'(Q^*)Q^* + p(Q^*) = \frac{1}{2}(c_1'(q_1^*) + c_2'(q_2^*))$$
 - Suppose the two firms merged, how would this equation change?

General Cournot (quantity) competition

- ▶ Firm i :
 - Profits assuming other firms are playing their optimum:
 $p(q_1^* + \dots + q_{i-1}^* + q_i + q_{i+1}^* + \dots + q_n^*)q_i - c_i(q_i)$
 - FOC: $p'(Q^*)q_i^* + p(Q^*) - c'_i(q_i^*) = 0$
 - Should also check profits are positive at the optimum.
- ▶ Add up all of the first order conditions, and divide by n :

$$\frac{1}{n}p'(Q^*)Q^* + p(Q^*) = \frac{1}{n}\sum_{i=1}^n c'_i(q_i^*)$$

what happens as $n \rightarrow \infty$?

Cournot problems

- ▶ Suppose $p(Q) = p_0 - p_1 Q$ and $C_i(q_i) = c_{i,0} + c_{i,1} q_i$ (for all i)
 - Show that under Cournot competition: $Q^* = \frac{p_0 - \bar{c}_1}{(1 + \frac{1}{n})p_1}$
 - where \bar{c}_1 is average marginal cost.
 - Recall that under perfect competition with symmetric marginal costs c_1 , and no fixed costs: $Q^* = \frac{p_0 - c_1}{p_1}$.
- ▶ Suppose $p(Q) = kQ^{-\beta}$ and $C_i(q_i) = c_{i,0} + c_{i,1} q_i$.
 - Show that under Cournot competition: $p(Q^*) = \frac{\bar{c}_1}{1 - \frac{\beta}{n}}$
 - Mark-up pricing (still!)
- ▶ What happens in each case as $n \rightarrow \infty$?
 - Harder: Is it efficient?

Further problems

- ▶ OZ Ex 2.6
 - Questions 1 to 4.
- ▶ OZ Ex 6.8
 - Questions 1 (parts c and d are optional), 2 and 4)a)+c).
- ▶ OZ Extra exercises:
 - <http://ozshy.50webs.com/io-exercises.pdf>
 - Set #2 and set #6

Conclusion

- ▶ Bertrand competition with symmetric marginal costs attains efficiency in pure strategies.
 - Non-efficient mixed strategy equilibria may also exist.
 - If marginal costs are not symmetric, one firm may still make profits.
- ▶ Cournot competition leads to similar expressions for quantity and prices as under monopoly, except:
 - Average marginal costs replace marginal costs.
 - The distortion away from perfect competition is smaller, the more firms there are.