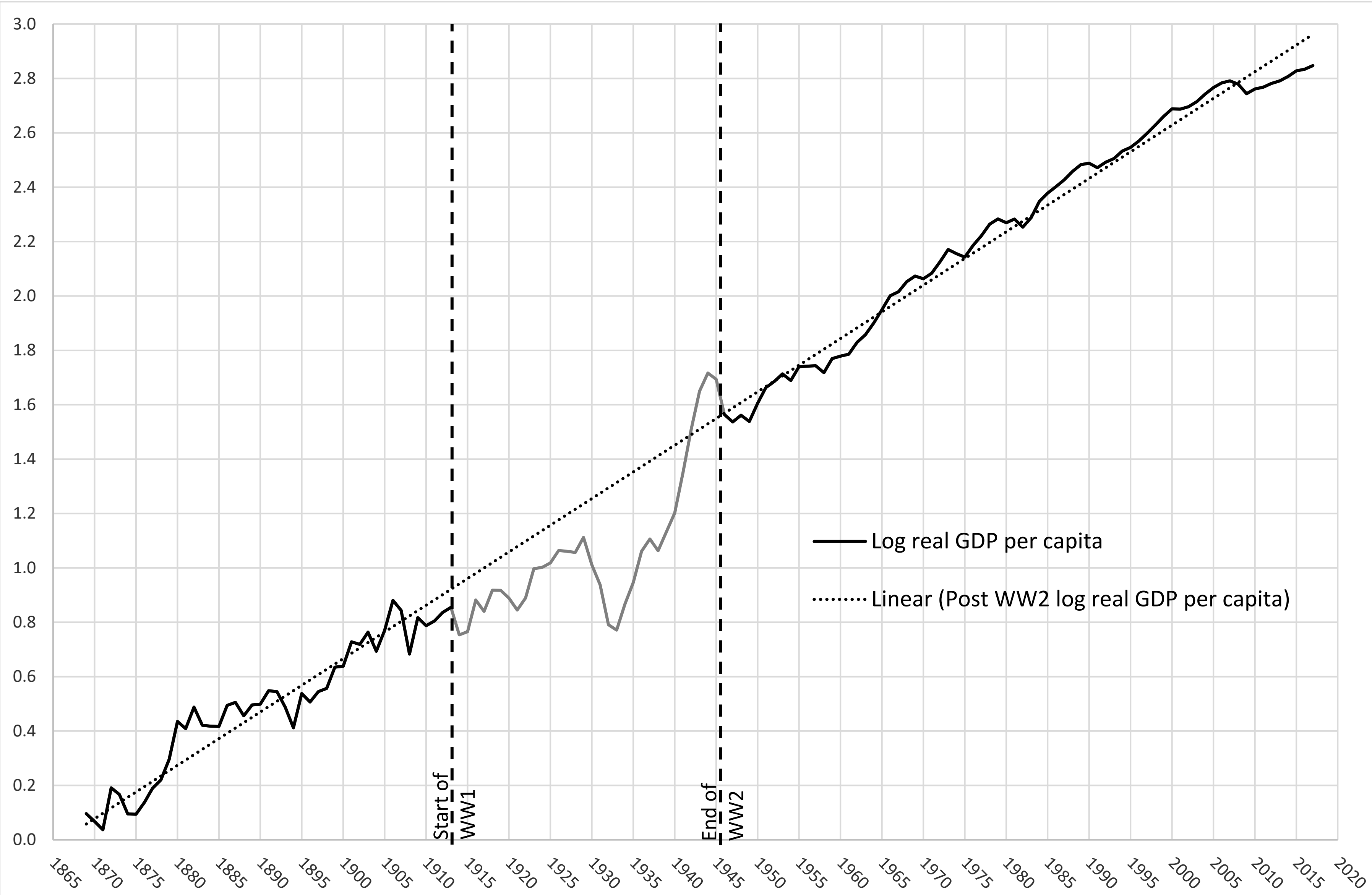
Medium frequency cycles in a stationary world

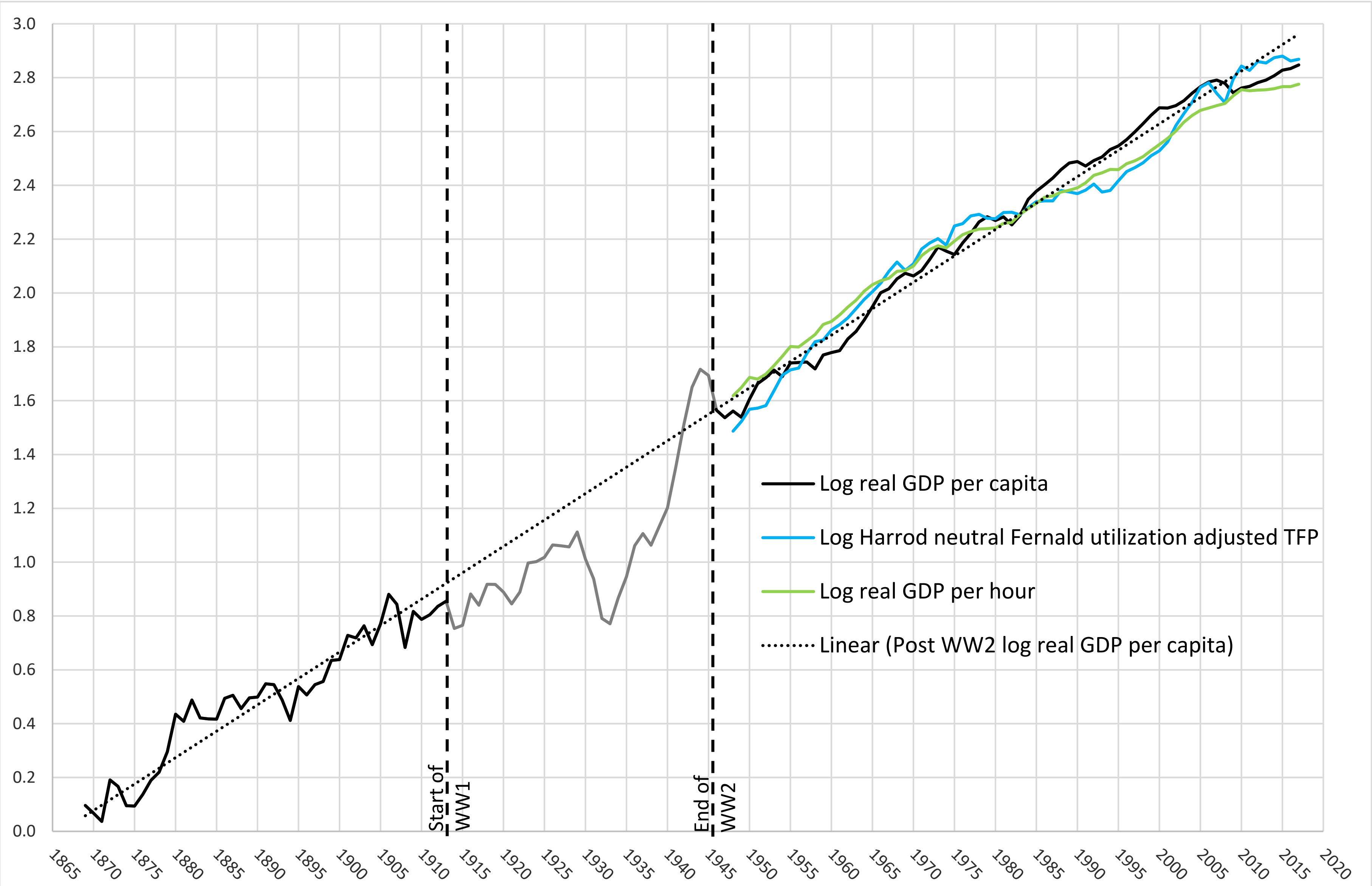
Tom Holden

Deutsche Bundesbank

Log US real GDP/capita, Post-WW2 trend



As before + TFP and GDP per hour



The challenge

- Existing endogenous growth models only generate log-linear growth in the presence of knife-edge assumptions.
  - In 2nd generation models, the required assumption is the symmetry of spill-overs from process to product innovation. (Li 2000)

- Existing endogenous growth models generate trend breaks in productivity following standard business cycle shocks.
  - Shocks knock the model away from cancelling out the strong scale effect.
  - If the measure of industries adjusted immediately, there would be no movement in productivity.

- We want a model capable of simultaneously generating:
  - the medium frequency cycles noted by Comin and Gertler (2006),
  - trend-stationary, or near trend-stationary GDP (i.e. low variance at frequency 0 of differenced GDP).

Our model: Stable growth

- 2nd generation endogenous growth models feature two margins of adjustment:
  - Process innovation, driving productivity growth.
  - Product invention, leading to the spreading out of process innovation efforts over an increasing measure of industries.
- We add a third margin, the number of firms per industry.
  - Requires a non-quality ladder approach to modelling process improvement.
- Although the measure of industries cannot react immediately to demand shocks, the number of firms per industry can.
  - Means firms stay roughly the same size over the cycle.
  - Thus, firms have near constant process improvement incentives.

Our model: Medium frequency cycles

- The invention of a new product, creates a new industry.
  - The inventor is granted a time limited patent on the invention allowing them to control entry to the new industry.
  - Inventors charge a cost to firms wanting to produce the new product.
  - Thus, patent protected industries are more expensive to enter.

- Higher entry costs imply less entry in patent protected industries.
  - Less entry means higher mark-ups, and higher returns to process improvement.
  - Endogenously then, patent protected industries have higher productivity and mark-ups.

- Demand up $\Rightarrow$ invention up $\Rightarrow$ patent protected share up $\Rightarrow$ productivity and mark-ups up.
- Return to trend happens when products go out of patent protection.
  - Naturally a medium-frequency phenomenon.

Empirical evidence

The stationarity of US real GDP per capita

- Run a bootstrap version (Chang, Sickles, and Song 2016) of Hansen's covariate augmented unit root test (Hansen 1995).
  - Test $\alpha = 0$ in $\Delta y_t = \alpha y_{t-1} + \sum_{k=1}^{l} \rho_k \Delta y_{t-k} + \sum_{k=1}^{m} \phi_k \Delta z_{t-k} + \varepsilon_t$.

- y_t is linearly detrended log US real GDP per capita, z_t is linearly detrended log real GDP per capita in another country.
  - (Only require that Δz_t is stationary.)

- Double bootstrap with 10000×10000 samples to remedy any biases from (AICc) lag selection.
- Maddison project data.

The stationarity of US real GDP per capita

Covariate is log real GDP per capita in:	First observation	Last observation	Single bootstrapped p-value	Double bootstrapped p-value
Canada	1873	2016	0.0109*	0.0084**
Australia	1870	2016	0.0363*	0.0288*
Western Europe	1873	2016	0.0485*	0.0454*
Western Offshoots	1873	2016	0.0466*	0.0440*

- Also significant at 5% with Belgium, Brazil, Chile, Columbia, Finland, France, Greece, Italy, Netherlands, New Zealand, Norway, Peru, Portugal, Spain, Sri Lanka, Sweden, Switzerland, Venezuela. (22 countries/regions in total.)
- Not significant at 5% with Austria, Denmark, Germany, Japan, UK, Uruguay. (6 countries/regions in total.)

Patenting facts

- Firms with patents are (17%) more productive; firms gaining a patent get (7.4%) more productive (Balasubramanian and Sivadasan 2010).
- Patent transfer is strongly correlated with aggregate TFP (Serrano 2007).
- R&D leads to invention which leads to increased productivity, with the productivity increase more persistent than the invention increase (Raymond et al. 2015).
- Countries with longer patent protection have larger medium frequency cycles (next slide).

Patents and medium frequency cycles

Source	Variable	Spec. 1	Spec. 2	Spec. 3	Spec. 4	Spec. 5	Spec. 6	Spec. 7
	Constant	-1.6115***	-2.4216***	-3.2014***	-4.8790***	-5.2405***	-7.1666***	-8.4497***
Penn World Tables 9.0 (Feenstra, Inklaar, and Timmer (2015)	Mean log real GDP (expenditure side)							1.3607***
	Slope log real GDP (expenditure side)							-24.0884**
	Initial log real GDP (output side)						0.1907	-1.0528***
	Mean consumption share				0.7515	0.8946	1.5112	2.2762**
	Mean government spending share				3.1945**	3.4249**	4.1499**	4.1747***
	Mean export share				0.8562	0.9537	1.0415	2.3900**
	Mean import share				-0.1229	0.0436	0.0182	1.3484*
International Country Risk Guide	Mean internal conflict		-0.0974	-0.0866	-0.1225*	-0.1258*	-0.1185*	-0.1459**
	Mean external conflict		0.1160**	0.1151**	0.1381**	0.1310**	0.1244**	0.1309***
	Mean socioeconomic conditions		0.0360	0.1021	0.1378*	0.1968**	0.1914**	0.1946**
	Mean military in politics		-0.0756	-0.0365	-0.0595	-0.0737	-0.0889	-0.1597**
	Mean religion in politics		0.0998	0.1163	0.1577**	0.1957**	0.1910**	0.1862**
La Porta, Lopez-de-Silanes, and Shleifer (2008)	German legal origin					-0.5228	-0.3913	-0.7089*
	Scandinavian legal origins					0.0114	0.0592	0.1229
	Socialist legal origins					-0.6746	-0.4313	-0.7187
	British legal origins					-0.0387	-0.0211	0.0249
	Corruption			-0.2303	-0.2876*	-0.3182*	-0.3856**	-0.4366***
Park (2008)	**Slope of patent duration index**	**25.4341**	**33.9635***	**38.5254***	**48.1122****	**45.1931***	**44.9586***	**49.9016****
	Initial patent duration index	**0.6849***	**0.8356***	**1.0083****	**1.2423*****	**1.1498****	**1.1288****	**0.9650****
	Observations	98	98	98	98	98	98	98
	Minimum normality test p-value	0.0672	0.2139	0.4542	>0.5	>0.5	0.4300	0.4851

The impact of patent duration on the strength of medium frequency cycles.

Coefficients from assorted regression specifications. (Stars denote significance at 5%, 1% and 0.1%.)

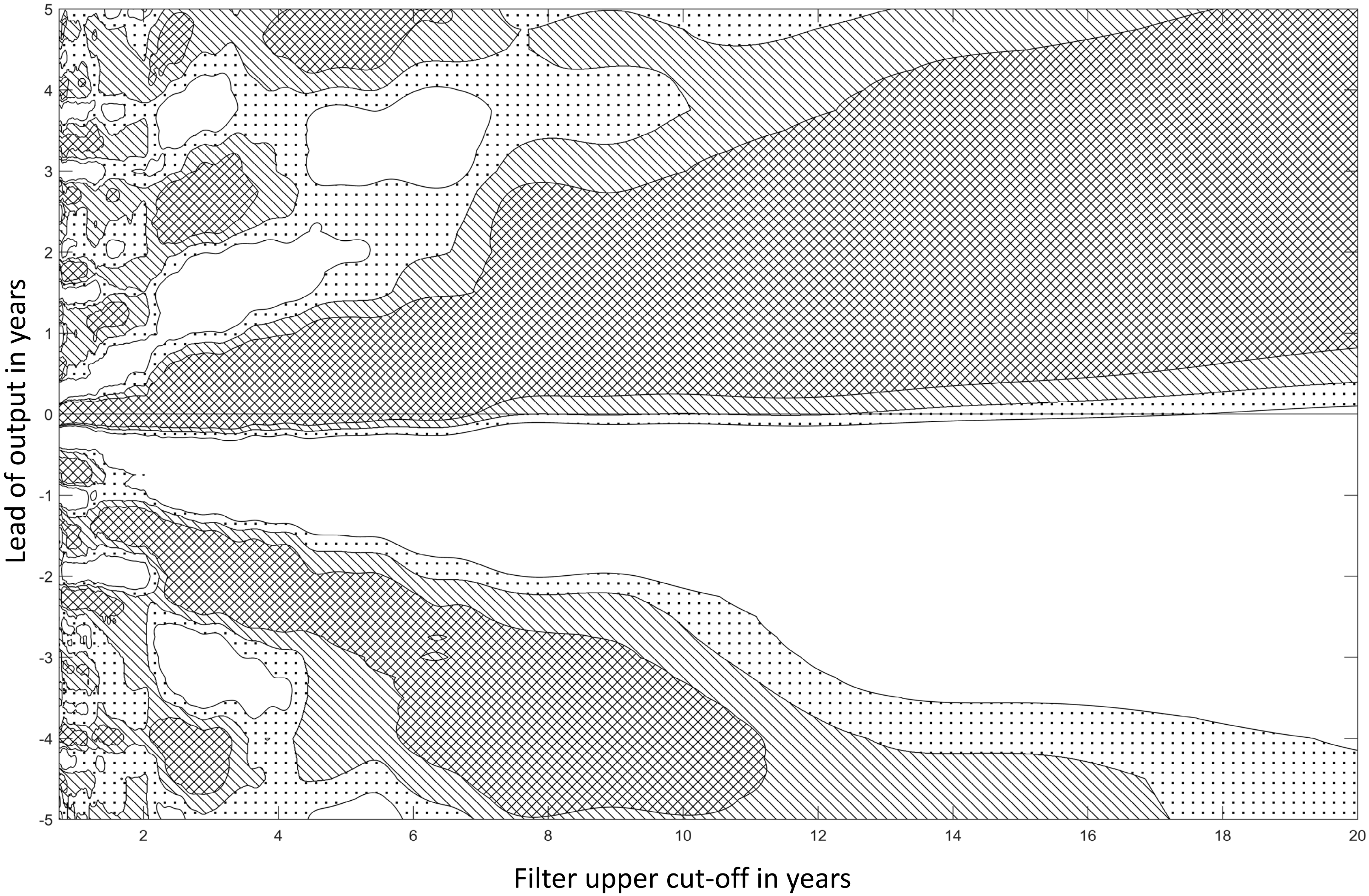
In each, the dependent variable is a logit transform of the proportion of real GDP per capita growth variance that is at frequencies with periods greater than eight years. We use a model for the variance of the form $\sigma_i^2 = s_0 + \frac{s_1}{\mathrm{POP}_i} + \frac{s_2}{\mathrm{NUMOBS}_i} + \nu_i$, where POP_i is mean population from the Penn World Tables, NUMOBS_i is the number of observations used in constructing the dependent variable, and where all coefficients are constrained to be positive. Estimation is by iterated FGLS. Medium frequency variance shares are constructed by taking the ratio of the variance of real GDP per capita when filtered with the Christiano and Fitzgerald (2003) band-pass filter set to accept cycles from 9 to 50 years, to the variance of real GDP per capita when filtered with the Christiano and Fitzgerald (2003) band-pass filter set to accept cycles from 2 to 50 years. For all countries, we use the maximum possible time span from the Penn World Tables data.

Other facts matched by the model

- Product creation is pro-cyclical (Broda and Weinstein 2010).
- TFP is pro-cyclical (Bils 1998; Campbell 1998; Comin and Gertler 2006).
- Productivity dispersion is counter-cyclical (Kehrig 2011).
- Productivity growth dispersion is counter-cyclical (Eisfeldt and Rampini, 2006; Bachmann and Bayer 2009).

- Mark-ups lead output at business cycle frequencies (Nekarda and Ramey 2010), and at medium ones (next slide).
- Aggregate mark-ups are weakly pro-cyclical, but mark-ups in any particular industry are counter-cyclical (Boulhol 2007).
- The cross-correlation between output and mark-ups is much clearer than the contemporaneous correlation (next slide).

Cross-correlation: output and mark-ups



Prior literature

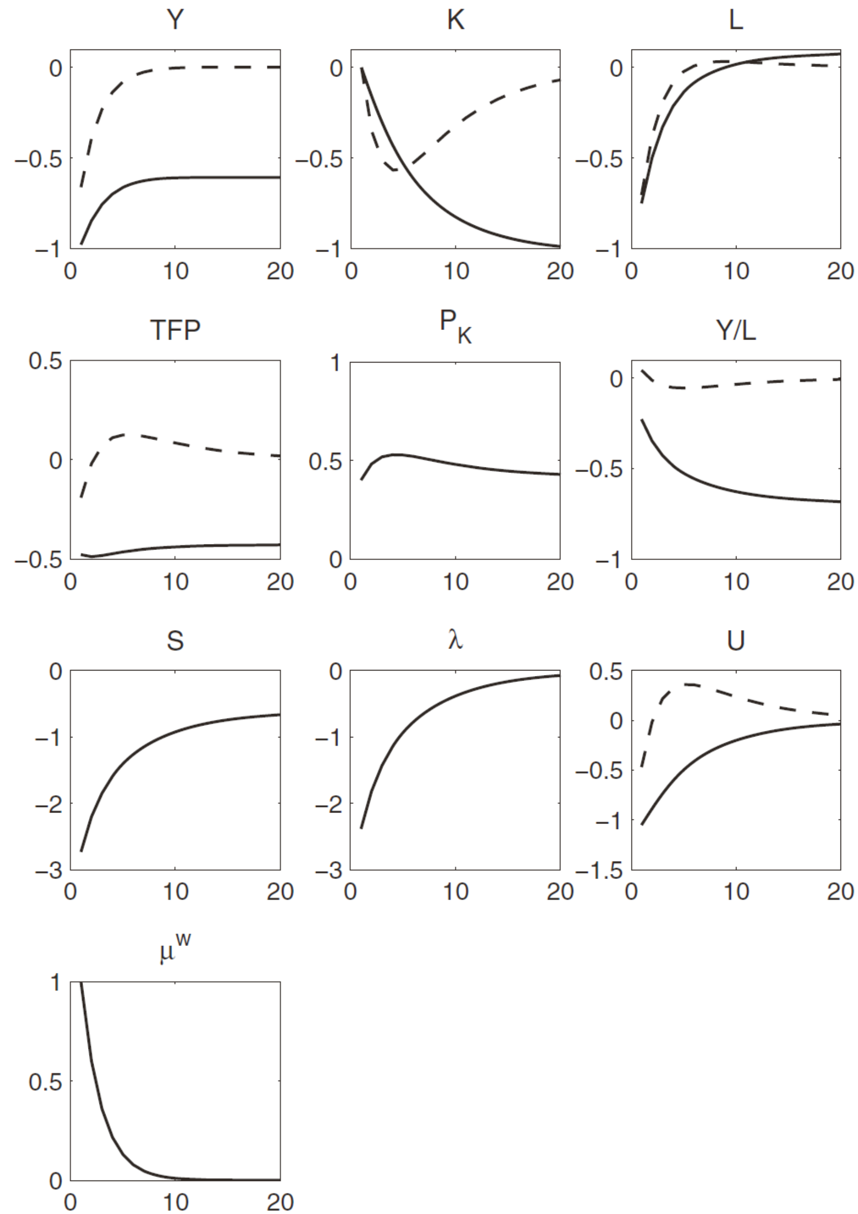
Prior literature

- Business cycle models with endogenous growth include:
  - Comin and Gertler (2006), Comin (2009), Comin, Gertler, and Santacreu (2009), Phillips and Wrase (2006), Nuño (2008; 2009; 2011).
- Endogenous growth models that generate cycles include:
  - Bental and Peled (1996), Matsuyama (1999), Wälde (2005), Francois and Lloyd-Ellis (2008; 2009), Comin and Mulani (2009).
- All have problems with scale effects in either the long or the short run.
  - Fixing these problems would destroy the central mechanism of the papers.
- Business cycle models with endogenous competition include:
  - Bilbiie, Ghironi, and Melitz (2007), Jaimovich (2007).

Impulse response to a wage mark-up shock in the Comin and Gertler (2006) model.

In their model: large unit root in output, increases in mark-ups lead to drops in TFP and output.

In the data: small unit root in output, increases in mark-ups are followed by increases in output (Nekarda and Ramey 2010)



The model

Model outline: Structure

- To illustrate the mechanism, we present a model without capital.
- Output is produced from differentiated inputs.
- The measure of differentiated inputs is increased by invention.
  - Inventors lack the skills to produce their own product at scale.
  - Each product is produced by firms in a dedicated industry.
- Firms within an industry are slightly differentiated (following Jaimovich 2007).
  - Paying the fixed entry cost is sufficient to attain this differentiation.
- Preference for variety across and within industries is turned off (for simplicity).

Model outline: Patenting

- New products are patentable.
  - Patenting is also a stand-in for e.g. copyright, NDAs, obfuscation etc.
  - Patent holders bargain with the firms producing their product to set the rents they must pay in period t in order to produce the protected product in period $t + 1$.
  - Patents have finite lives.

- Process improvements are not patentable.
  - Broadly in line with the law. (“Machine or transformation” test suggests e.g. business processes are not patentable.)
  - Technology transfer across industries is the result of a costly process of “appropriation”.
  - Technology transfer within an industry is free.

Model outline: Firms

- Firms pay a fixed cost in period t to produce in period $t + 1$.
- If firms perform research/appropriation in period t, they benefit from higher productivity in period $t + 1$.
  - Process improvement is incremental, with regular small changes.
  - Research and appropriation are getting harder over time.
- Assumptions imply no firm specific state variables, only industry specific ones.
- WLOG then, firms exist for two periods each with overlapping generations.
- In the first, they:
  - pay the fixed cost,
  - pay any required rents to the patent holder,
  - possibly perform appropriation,
  - possibly perform appropriation.
- In the second, they produce.

Aggregators

- Final good is produced from industry aggregates by a perfectly competitive industry with the production function:

$$X_t = |\mathbb{I}_{t-1}| \left[\frac{1}{|\mathbb{I}_{t-1}|} \int_{i \in \mathbb{I}_{t-1}} X_t(i)^{\frac{1}{1+\lambda}} di \right]^{1+\lambda}$$

- Industry aggregates are produced from firm output by a perfectly competitive industry with production function:

$$X_t(i) = J_{t-1}(i) \left[\frac{1}{J_{t-1}(i)} \sum_{j=1}^{J_{t-1}(i)} X_t(i,j)^{\frac{1}{1+\eta\lambda}} \right]^{1+\eta\lambda}$$

  - $\eta \in (0,1)$ controls the degree of differentiation between firms, relative to that between industries.

Firm production

- In period t, firm j in industry i has access to the linear production technology:

$$X_t(i,j) = A_t(i,j)L_t^P(i,j).$$

- Symmetry across firms in industry i and profit maximization implies:

$$P_t(i,j) = \left(1+\mu_{t-1}(i)\right)\frac{W_t^W}{A_t(i,j)} = \left(1+\mu_{t-1}(i)\right)\frac{W_t^W}{A_t(i)} = P_t(i)$$

  - where $\mu_t(i) \coloneqq \lambda\frac{\eta J_t(i)}{J_t(i)-(1-\eta)} \in (\eta\lambda, \lambda]$. ($t-1$ subscripts since entry occurs the period before production.)

- Consequently $W_t^W = \frac{A_t}{1+\mu_{t-1}}$ where:

$$\frac{1}{1+\mu_t} = \left[\frac{1}{|\mathbb{I}_t|}\int_{i\in\mathbb{I}_t}\left[\frac{1}{1+\mu_t(i)}\right]^{\frac{1}{\lambda}} di\right]^{\lambda}, \qquad A_t \coloneqq \frac{\left[\frac{1}{|\mathbb{I}_{t-1}|}\int_{i\in\mathbb{I}_{t-1}}\left[\frac{A_t(i)}{1+\mu_{t-1}(i)}\right]^{\frac{1}{\lambda}} di\right]^{\lambda}}{\left[\frac{1}{|\mathbb{I}_{t-1}|}\int_{i\in\mathbb{I}_{t-1}}\left[\frac{1}{1+\mu_{t-1}(i)}\right]^{\frac{1}{\lambda}} di\right]^{\lambda}}$$

Appropriation

- Notation:

$$A_t^*(i) := \max_{j \in \{1,\ldots,J_{t-1}(i)\}} A_t(i,j)\,, \qquad A_t^* := \sup_{i \in \mathbb{I}_{t-1}} A_t^*(i)$$

- Effective appropriation input: $\mathcal{L}_t^A(i,j) := E_t^A(i) L_t^A(i,j)$
- Appropriation productivity: $E_t^A(i) := A_t^*(i)^{-\zeta^{A1}} {A_t^*}^{\zeta^{A2}} |\mathbb{I}_t|^{\phi^A} \Psi^A$
  - where $\zeta^A = \zeta^{A1} - \zeta^{A2} > 0$ and $\phi^A \geq 0$ is small enough that ${A_t^*}^{-\zeta^A} |\mathbb{I}_t|^{\phi^A} \to 0$ as $t \to \infty$.
  - R&D getting harder over time in line with evidence of Bloom et al. (2017).
- Output of appropriation is given by:

$$A_t^{**}(i,j) = \left[A_t^*(i)^\tau + \left({A_t^*}^\tau - A_t^*(i)^\tau \right) \frac{\mathcal{L}_t^A(i,j)}{1 + \mathcal{L}_t^A(i,j)} \right]^{\frac{1}{\tau}}$$

Research

- Effective research input: $\mathcal{L}_t^R(i,j) \coloneqq E_t^R(i) L_t^R(i,j)$
- Research productivity: $E_t^R(i) := A_t^{**}(i)^{-\zeta^{R1}} A_t^{*\zeta^{R2}} |\mathbb{I}_t|^{\phi^R} \Psi^R$
  - where $\zeta^R = \zeta^{R1} - \zeta^{R2} > 0$ and $\phi^R \geq 0$ is small enough that $A_t^{*-\zeta^R} |\mathbb{I}_t|^{\phi^R} \to 0$ as $t \to \infty$,
  - and $\zeta^{R1} > \zeta^{A1}$, $\zeta^{R2} \leq \zeta^{A2}$ and $\phi^R \leq \phi^A$ implying that the difficulty of research is increasing over time faster than the difficulty of appropriation.

- Output of research is given by:

$$A_{t+1}(i,j) = A_t^{**}(i,j)\left(1 + \gamma Z_{t+1}(i,j) \mathcal{L}_t^R(i,j)\right)^{\frac{1}{\gamma}}$$

- For simplicity, assume $Z_t(i,j) \coloneqq Z_t$ in the following.

Firm profits

- Not assuming symmetry, firm expected discounted profits are:

$$\beta \mathbb{E}_t \left[\Xi_{t+1} \left(P_t(i,j) - \frac{W_{t+1}^W}{A_{t+1}(i,j)} \right) X_t(i,j) \right]$$
$$- \left[L_t^R(i,j) + L_t^A(i,j) + L_t^{\mathcal{R}}(i) + L^F \right] W_t^W$$

- $\beta \Xi_{t+1}$ is the household's stochastic discount factor.

- $\mathcal{R}_t(i) \coloneqq L_t^{\mathcal{R}}(i) W_t^W$ is the required rent payment in industry i.
  - Paid in goods, not in labour.
- L^F is the fixed entry cost.
  - Paid in labour.

Research solution

- Interior research FOC:

$$\beta \frac{1}{|\mathbb{I}_t| J_t(i)} \frac{\mu_t(i)}{1+\mu_t(i)} \left(\frac{1+\mu_t}{1+\mu_t(i)}\right)^{\frac{1}{\lambda}} \mathbb{E}_t \Xi_{t+1} X_{t+1} \left(\frac{A_{t+1}(i)}{A_{t+1}}\right)^{\frac{1}{\lambda}} \frac{d_t(i)}{\mu_t(i)} \frac{Z_{t+1} E_t^R(i)}{1+\gamma Z_{t+1} E_t^R(i) L_t^R(i)}$$
$$= W_t^W$$

  - (For $J_t(i) \geq 2$, $d_t(i) \in \left(\frac{2}{3}, 1\right)$. $d_t(i) \to 1$ as $J_t(i) \to \infty$.)

- Free entry:

$$\beta \frac{1}{|\mathbb{I}_t| J_t(i)} \frac{\mu_t(i)}{1+\mu_t(i)} \left(\frac{1+\mu_t}{1+\mu_t(i)}\right)^{\frac{1}{\lambda}} \mathbb{E}_t \Xi_{t+1} X_{t+1} \left(\frac{A_{t+1}(i)}{A_{t+1}}\right)^{\frac{1}{\lambda}}$$
$$= \left[L_t^R(i,j) + L_t^A(i,j) + L_t^{\mathcal{R}}(i) + L^F\right] W_t^W$$

- Combination:

$$\mathcal{L}_t^R(i) = \max\left\{0, \frac{d_t(i) E_t^R(i)\left(L_t^A(i,j) + L_t^{\mathcal{R}}(i) + L_t^F\right) - \mu_t(i)}{\gamma \mu_t(i) - d_t(i)}\right\}$$

Inventors

- Patents always last at least one period.
- Patent decay at the stochastic rate q.
- Products depreciate at the rate $\delta_{\mathbb{I},t}$.
- Products invented with productivity $A_t^*(i) = S_t A_t^*$, $S_t \in (0,1)$.

- Value of a patent is:

$$V_t^{\mathbb{I}}(i) \coloneqq L_{t+s}^{\mathcal{R}}(i) W_{t+s}^{W} J_{t+s}(i) + \mathbb{E}_t \beta \Xi_{t+1} (1-q) \big(1 - \delta_{\mathbb{I},t+1}\big) V_{t+1}^{\mathbb{I}}(i)$$

- Free entry of inventors implies $\frac{\mathcal{L}_t^I}{E_t^I} W_t^W = V_t^{\mathbb{I}}(\sup \mathbb{I}_t)$,
  - where $\mathcal{L}_t^I$ is a stationary stochastic process giving the amount of effective labour to invent a new product,
  - and $E_t^I \coloneqq (S_t A_t^*)^{-\zeta^{I1}} A_t^{*\zeta^{I2}} |\mathbb{I}_{t-1}|^{-\phi^I}$, with $\phi^I \geq 0$ and $\zeta^{I1} > \zeta^{I2}$.

Bargaining

- If firms did not pay the patent license fee, U.S. law states that:
  - “the court shall award the claimant damages adequate to compensate for the infringement but in no event less than a reasonable royalty for the use made of the invention by the infringer, together with interest and costs as fixed by the court”.
- The established legal definition of a “reasonable royalty” is the outcome of a hypothetical bargaining process that took place immediately before production.
  - Patent holders may as well bargain in this way then.
  - (Some subtleties, discussed at length in the paper w.r.t. the U.S. law.)

- Post entry and R&D decision, the size of the “pie” is:

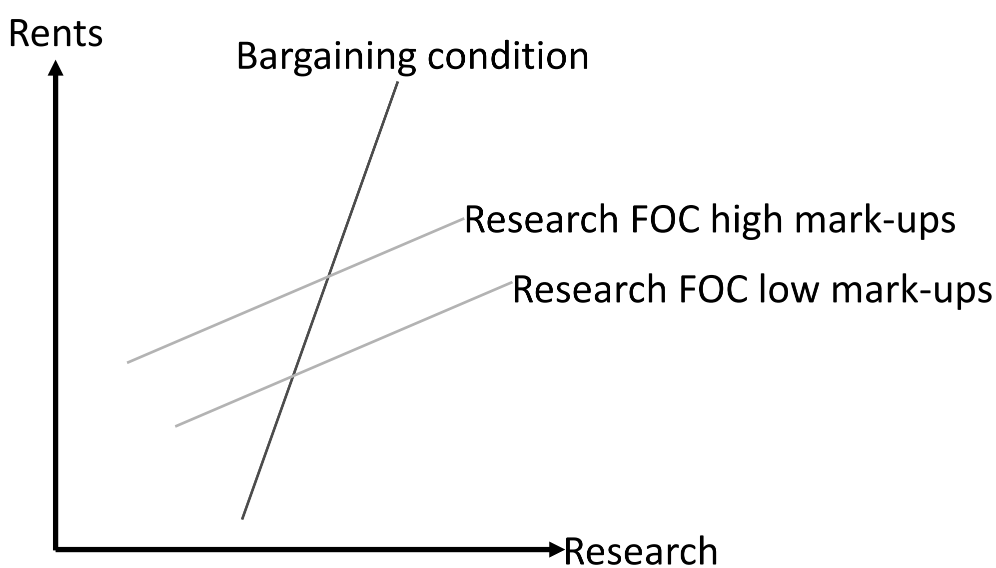
$$\left[L_t^R(i,j) + L_t^A(i,j) + L_t^{\mathcal{R}}(i) + L^F\right] W_t^W$$

- Result: $L_t^{\mathcal{R}}(i) = \frac{1-\wp}{\wp}\left[L_t^R(i) + L_t^A(i) + L^F\right]$
  - $\wp \in (0,1)$ is the (generalized Nash) bargaining power of the firm.

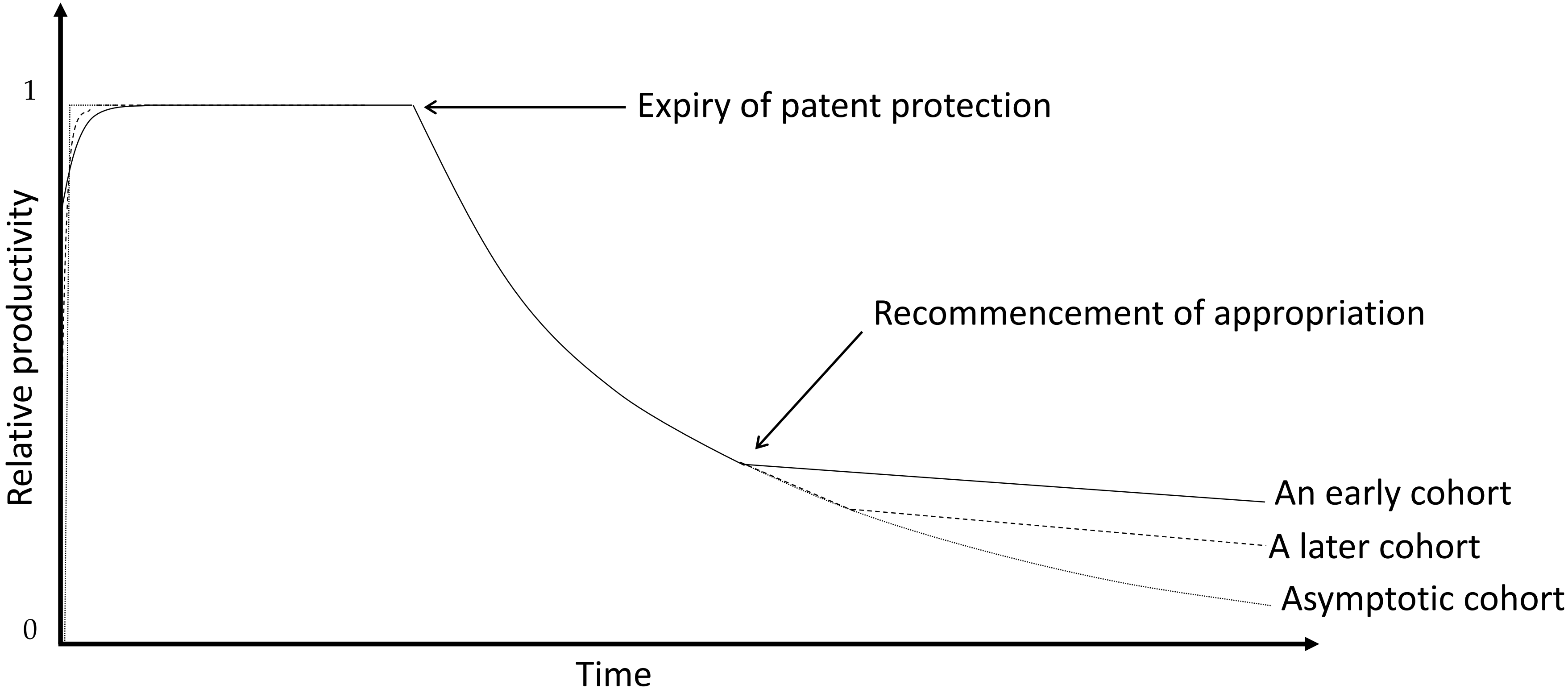
Research in patent protected industries

- Rents are such a steeply sloped function of equilibrium research that the signs of other terms in the research solution are flipped.
- With $Z_{t+1} = 1$:

$$\mathcal{L}_t^R(i) = \frac{\wp\mu_t(i) - d_t(i)E_t^R(i)(L_t^A(i) + L^F)}{d_t(i) - \gamma\wp\mu_t(i)}$$



Life cycle of an industry



Broad decline in R&D activity with age is in line with the results of Graham et al. (2018).

Closing the model

- Households choose C_t and L_t^S to maximise:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s N_{t+s} \left[\log \frac{C_{t+s}}{N_{t+s}} - \frac{1}{1+\nu} \left(\frac{L_{t+s}^S}{N_{t+s}} \right)^{1+\nu} \right]$$

- subject to the budget constraint:

$$C_t + B_t = L_t^S W_t^W + B_{t-1} R_{t-1} + \Pi_t$$

- Market clearing conditions: $C_t = X_t$, $B_t = 0$,

$$L_t^S = \frac{\mathcal{L}_t^I}{E_t^I} \left[|\mathbb{I}_t| - \left(1 - \delta_{\mathbb{I},t}\right) |\mathbb{I}_{t-1}| \right] + \int_{i \in \mathbb{I}_t} \left(L_t^R(i) + L_t^A(i) + L^F \right) J_t(i) di$$

$$+ {W_t^W}^{-\frac{1+\lambda}{\lambda}} \frac{X_t}{|\mathbb{I}_{t-1}|} \int_{i \in \mathbb{I}_{t-1}} \frac{1}{A_t(i)} \left(\frac{A_t(i)}{1 + \mu_{t-1}(i)} \right)^{\frac{1+\lambda}{\lambda}} di$$

- Output definition (including R&D as investment, in line with the 2013 NIPA revision):

$$Y_t^W = C_t + \left[\frac{\mathcal{L}_t^I}{E_t^I} \left[|\mathbb{I}_t| - \left(1 - \delta_{\mathbb{I},t}\right) |\mathbb{I}_{t-1}| \right] + \int_{i \in \mathbb{I}_t} \left(L_t^R(i) + L_t^A(i) \right) J_t(i) di \right] W_t^W$$

Proposition 1

- There are a strictly positive measure of structural parameters (including g_S and g_N) such that:
  - With probability 1, for all t and all $i \in \mathbb{I}_t$, the second order and uniqueness conditions are satisfied.
  - With probability 1, ${A_t^*}^{-\zeta^R} |\mathbb{I}_t|^{\phi^R} \to 0$ and ${A_t^*}^{-\zeta^A} |\mathbb{I}_t|^{\phi^A} \to 0$ as $t \to \infty$, so research and invention are indeed getting harder over time.
  - With probability 1, $J_t(i) \geq 2$ for all t and all $i \in \mathbb{I}_t$, so the number of firms in protected industries is indeed bounded below asymptotically
  - $g_{|\mathbb{I}|} > \log(1 - \delta_{\mathbb{I}})$, so invention does not stop asymptotically, and consequently neither does research.
  - Asymptotically catch-up to the frontier is instantaneous in protected industries
  - Research and appropriation are not performed in non-protected industries asymptotically.
  - The asymptotic growth rate of consumption and output per capita is equal to g_{A^*}, where:

$$0 < \frac{\wp\eta\lambda}{1 - \gamma\wp\eta\lambda} < g_{A^*} < \frac{3}{2}\wp\lambda.$$

Dynamic behaviour

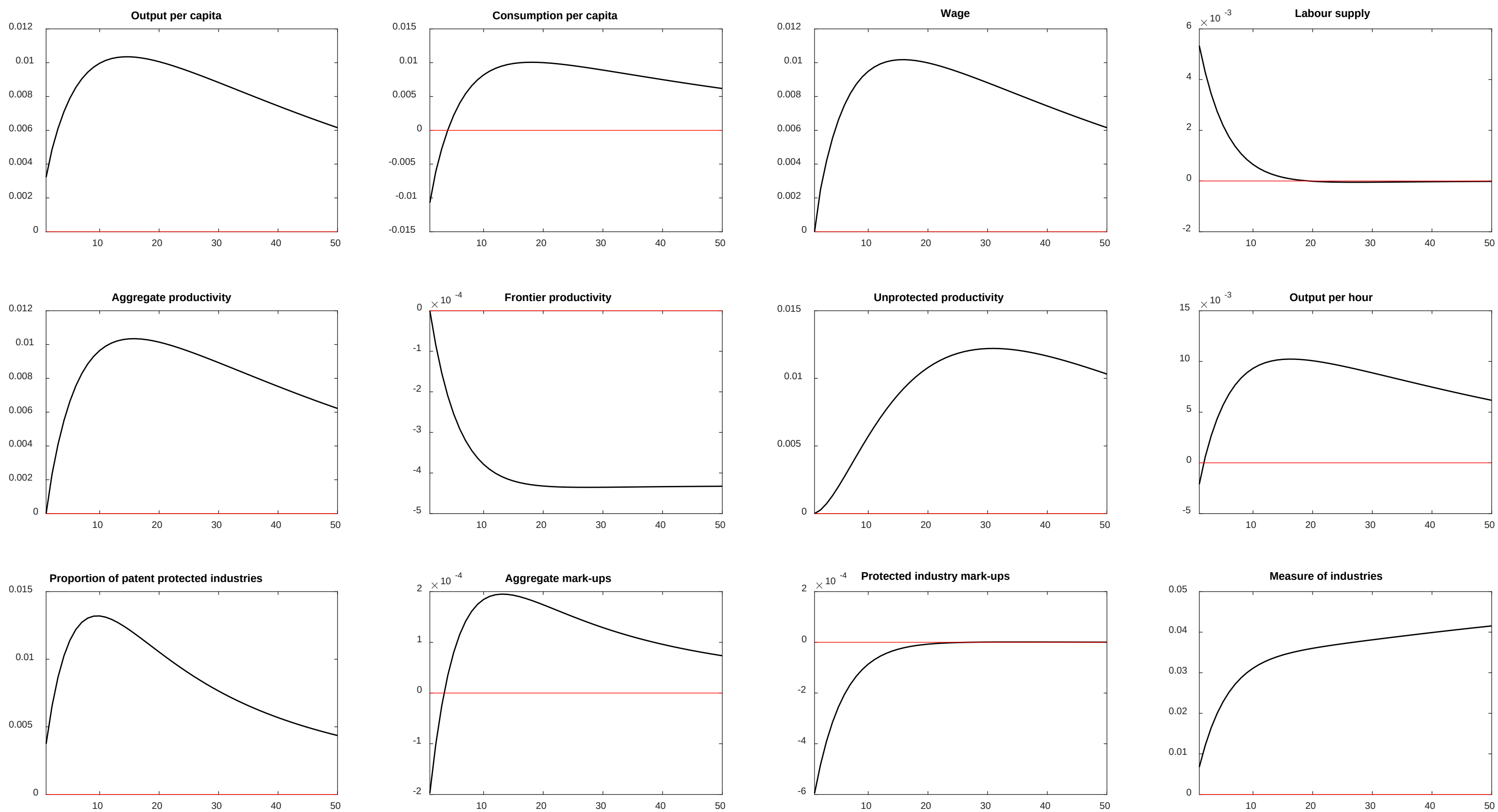
Simulating the model

- The model converges towards one exhibiting balanced growth.
- In the absence of a knife-edge assumption, mark-ups will converge to $\eta\lambda$.

- Here we assume that the relative productivity of prototypes (S_t) is such that $\frac{E_t^R(\sup \mathbb{I}_t)}{E_t^I}$ is stationary, which gives stationary mark-ups in $(\eta\lambda, \lambda)$.
  - More informative about the model's non-asymptotic behaviour.

- We simulate the balanced growth model to which our model converges, asymptotically.
- For this talk, the parametrisation is merely illustrative.
  - $\beta = 0.96$ for an annual model.
  - Some values taken from a richer estimated version.

Illustrative IRF to a 5% population shock



Related work

- The next version of this paper will include a richer model estimated on a long span of annual US data.

- For a EC H2020 project, I've embedded this model within a rich, many-country open economy model with capital and assorted other frictions.
  - Estimated.
  - Designed for policy exercises examining e.g. the value of R&D funding.
  - Contribution to the practical estimation of many country models and to the global imbalances literature.

- A separate paper with Kemal Ozhan (JMC!) looks at the behaviour of an open economy in which firms only perform appropriation (towards the global frontier), not research.
  - Explains "The Cycle is the Trend" result, endogenously.

Conclusion and recap of key mechanisms

- Variance at frequency zero of differenced GDP is minimal, as fluctuations in the number of firms per industry absorb short-run demand fluctuations, ensuring firms stay roughly constant size over the cycle.

- Medium frequency cycles are generated as increased demand means higher returns to invention, meaning more industries are patent protected, and so more have high productivity and mark-ups.

- We generate endogenous growth without knife edge assumptions.
- The growth rate is not a function of the growth rate of population, unlike in semi-endogenous growth models.

- Currently building and estimating a richer model for more extensive empirical validation.