# Aggregation bias in investment and capital

Tom Holden

Deutsche Bundesbank

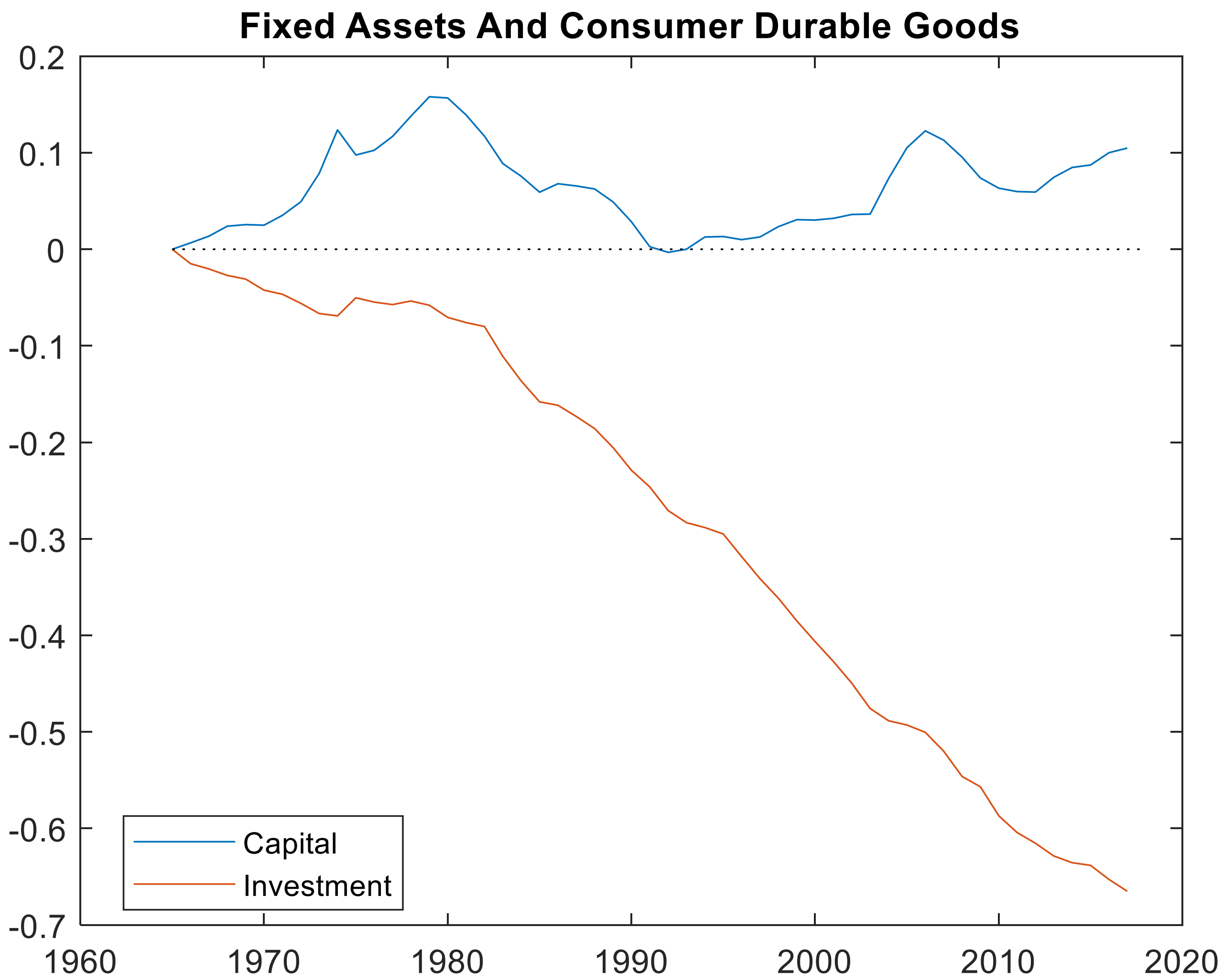
# Overview

- Observed (US BEA) prices of **capital** are **increasing** in units of consumption.
- Observed (US BEA) prices of **investment** are **falling** in units of consumption.

- The BEA calculates these prices using the Fisher Index to aggregate across capital types and prices.
  - For non-durable goods, this is (roughly) the right thing to do. See e.g. Diewert (1993).
  - For durable goods this may lead to biases. See e.g. Jorgenson & Griliches (1972), Diewert & Lawrence (2000) for discussion of biases relating to depreciation schedules.

- We explain the gap between observed relative capital price growth and observed relative investment price growth through a new measurement bias.
  - The investment bundle is more skewed towards new varieties than the capital bundle.
  - New varieties experience faster productivity growth rates, so have faster falling prices.
- Correctly measured …
  - Capital and investment prices are identical. (Definition of investment units!)
  - US GDP (investment) growth is $\approx$ 0.10 (0.47) percentage points per year lower.
  - Investment specific technological change only explains 6.9% of aggregate growth.

# Capital & inv. prices, units of cons. goods

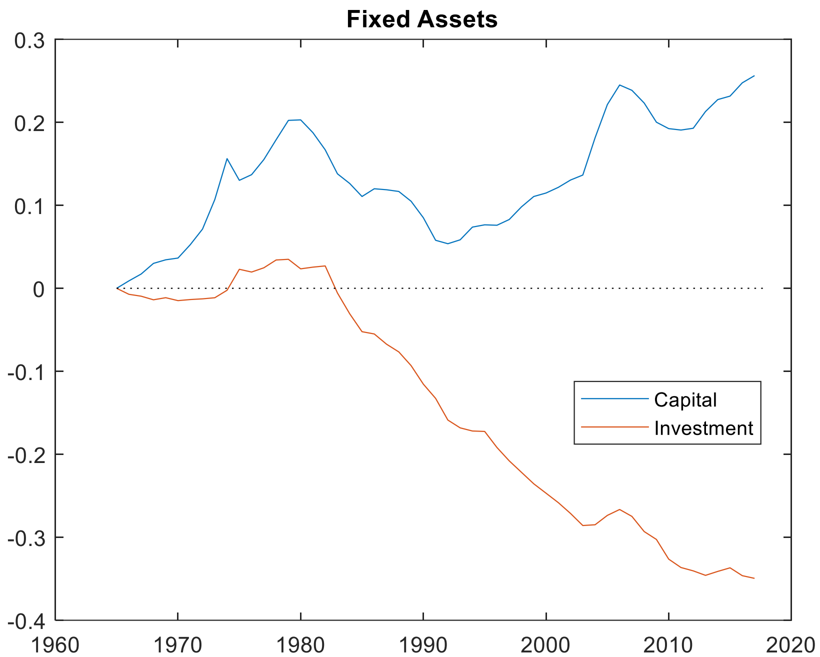



*Average age (2017): 21.7 years*

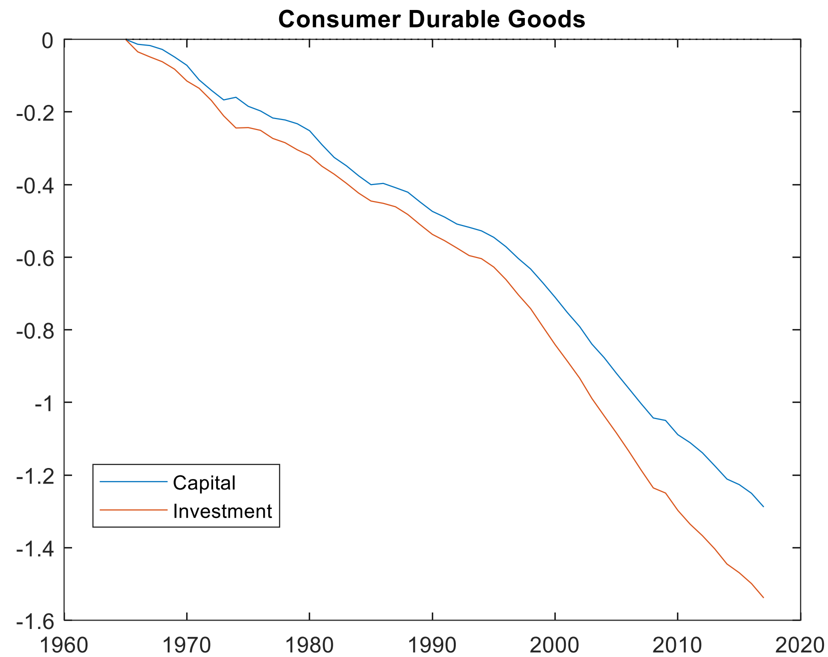
# Digression: Individual good price measurement

- This is data from the US BEA Fixed Asset tables.
  - Note that this data enters into standard (NIPA) national accounts via CFC.
- The literature from the 90s was concerned with mismeasurement of individual investment goods.
  - Gordon (1990) produced a new measure which was partly extended by Greenwood, Hercowitz & Krusell (1997) (GHK), Cummins & Violante (2002) and Basu, Fernald, Fisher & Kimball (2013).
- Since then the national accounts have been thoroughly revised, and the problems raised by Gordon are mostly solved.
  - In part using Gordon’s methodology.
- For the duration, we take the NIPA measures of the prices of individual goods as correct.
  - Our issue is entirely one of aggregation.

# Capital & inv. prices, units of cons. goods

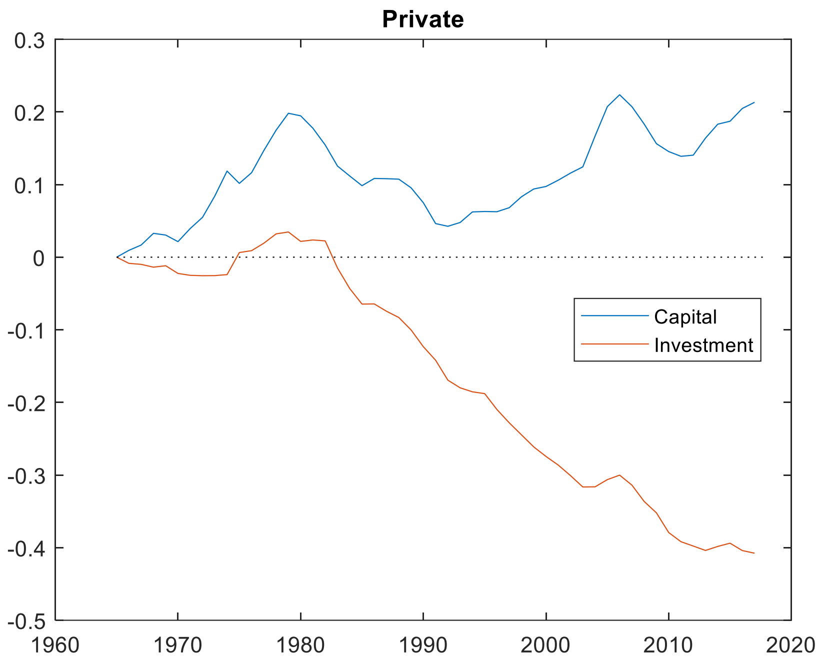



*Average age (2017): 23.2 years*

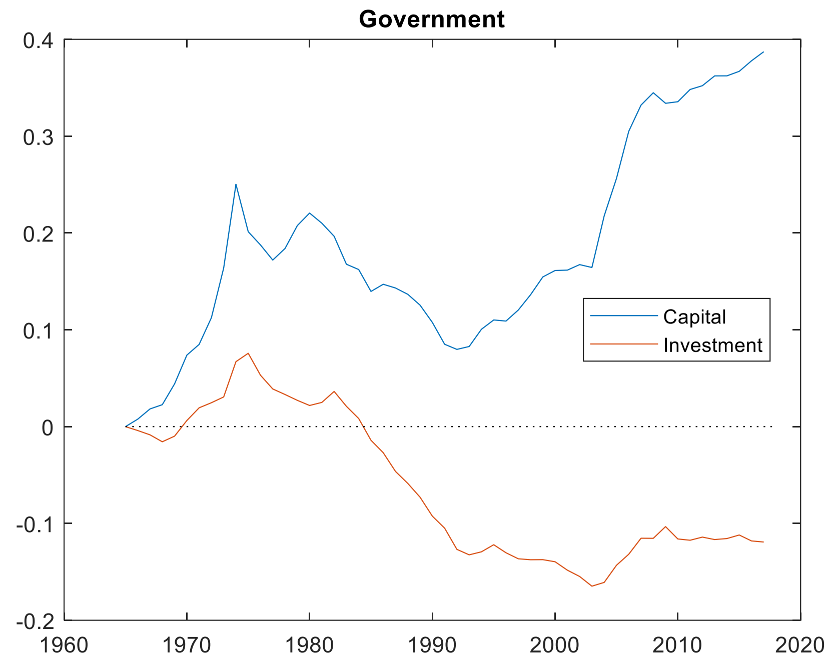



*Average age (2017): 4.5 years*

# Capital & inv. prices, units of cons. goods

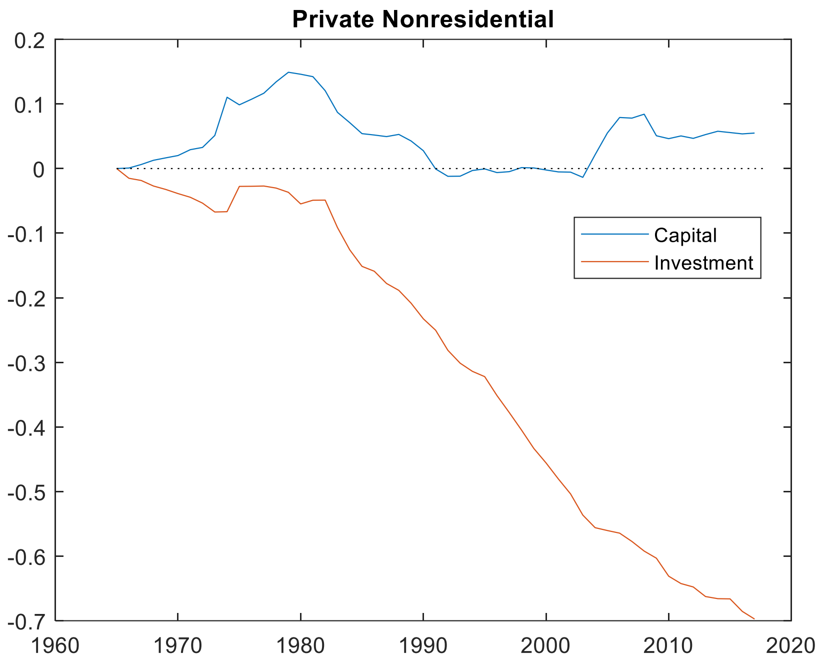



*Average age (2017): 22.8 years*

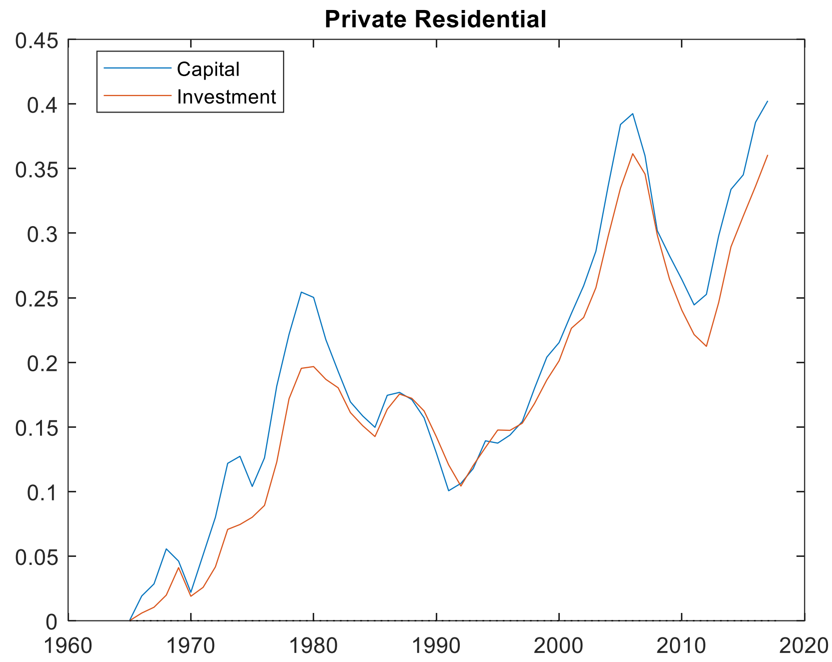



*Average age (2017): 24.3 years*

# Capital & inv. prices, units of cons. goods

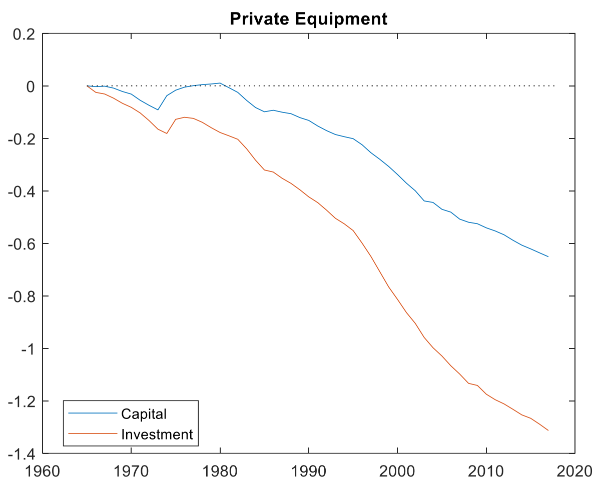



*Average age (2017): 16.0 years*

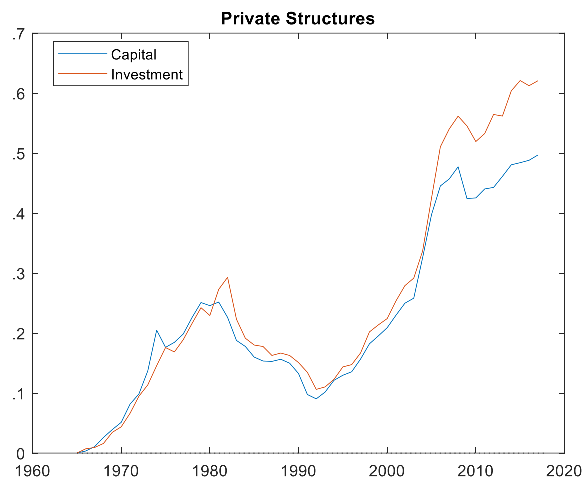



*Average age (2017): 30.6 years*

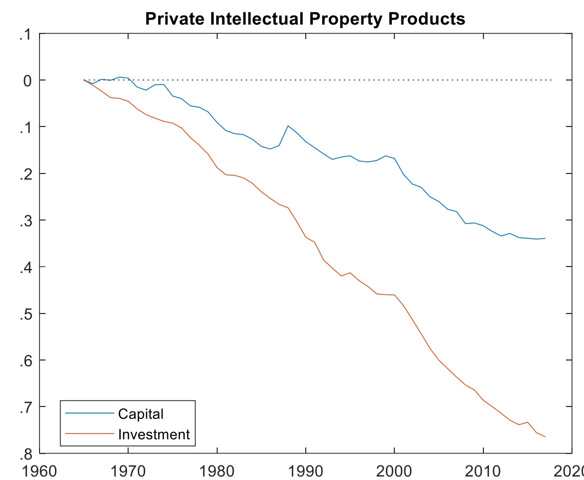
# Capital & inv. prices, units of cons. goods




*Average age (2017): 7.0 years*

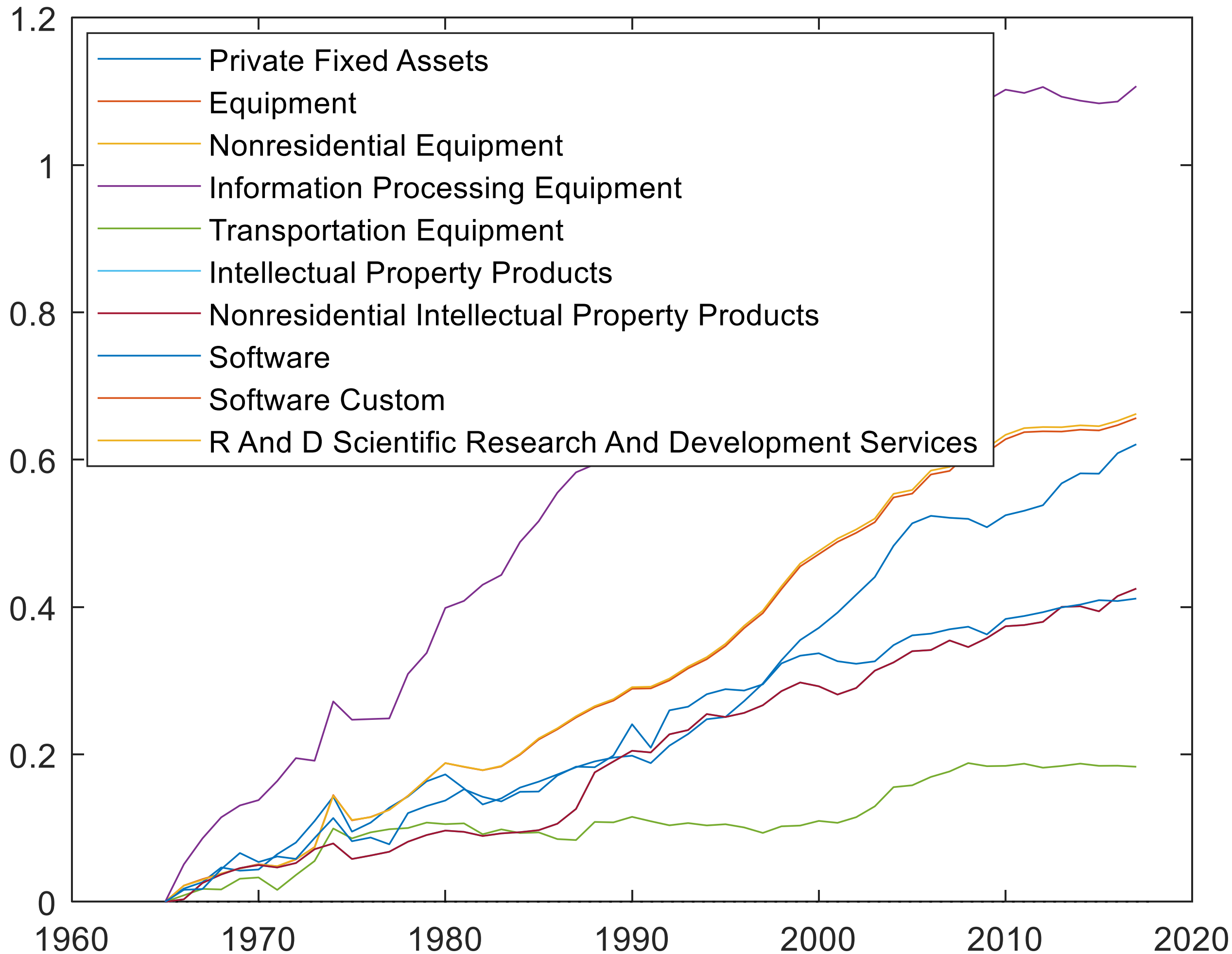



*Average age (2017): 22.9 years*




*Average age (2017): 4.5 years*

# Price of capital in units of inv. by type




Types are plotted for which the graph has a slope statistically significantly different from 0 at 10% (HAC adjusted)

# Lessons from these graphs

- Across all but one of these broad categories, the price of capital is growing faster than the price of investment.
- The price of fixed assets in units of consumption goods is increasing.

- The categories with the highest average ages have experienced the fastest increases in capital prices.
  - Only the categories with average ages below 10 years have experienced capital price declines.

- Particularly for capital types with relatively low shelf lives, the existing capital stock appears more valuable than you would expect from investment prices.
- Intuition:
  - For a capital type with high product turn-over, producing old varieties soon becomes relatively costly.
  - If new varieties are not perfect substitutes for old varieties, then the stock of old varieties can become valuable.

# Additional related Literature

- **Cambridge capital controversy:** Robinson, Sraffa, Samuelson, Solow, etc.
- **Vintage capital:** See e.g. Boucekkine, de la Croix & Licandro (2008; 2011).
    - Note that unlike the standard vintage capital literature, in the model here it will be possible to produce old varieties as well as new ones.
- **Relative price of investment & investment specific technological change:** Greenwood, Hercowitz & Krusell (1997), Krussell (1998), Licandro, Ruiz-Castillo & Duran (2002), Justiniano, Primiceri & Tambalotti (2011).
- **Biases in the national accounts:** Broda & Weinstein (2006; 2010), Redding & Weinstein (2016).
- **Micro capital prices:** Lanteri (2018).
- **Variety specific growth:** Adam & Weber (2019).

# Capital in the model

- The aggregate capital good is produced by a perfectly competitive industry with the technology:

$$K(t) = \left[\int_{-\infty}^{t} K_s(t)^{\frac{1}{1+\lambda}} \, ds\right]^{1+\lambda}.$$

- Note: the set of varieties/vintages evolves exogenously.

- Capital producers choose demand for specific vintages to maximise their profits (zero in equilibrium):

$$R(t)K(t) - \int_{-\infty}^{t} R_s(t)K_s(t) \, ds \, .$$

- $R(t)$ and $R_s(t)$ are aggregate and vintage $s$ rental rates, respectively.

- In equilibrium:

$$K_s(t) = K(t)\left(\frac{R_s(t)}{R(t)}\right)^{-\frac{1+\lambda}{\lambda}}, \qquad R(t) = \left[\int_{-\infty}^{t} R_s(t)^{-\frac{1}{\lambda}} \, ds\right]^{-\lambda}.$$

# Growth in rental rates

- For now we assume $R_t(t)K_t(t) = 0$.
  - This will be true in the model as new varieties will start off with zero productivity.
  - Removes biases coming from love of (new) varieties.
  - See e.g. Broda & Weinstein (2010).

- Then, we have:

$$\frac{\dot{R}(t)}{R(t)} = \frac{\int_{-\infty}^{t} R_s(t)K_s(t)\frac{\dot{R}_s(t)}{R_s(t)}\,ds}{\int_{-\infty}^{t} R_s(t)K_s(t)\,ds}.$$

# Digression: Price indices in continuous time

- The BEA use Fisher aggregators, for which (for an arbitrary good):

$$\frac{P(t)}{P(t-\zeta)} = \sqrt{\frac{\int_{-\infty}^{t} P_s(t) Y_s(t)\, ds}{\int_{-\infty}^{t-\zeta} P_s(t-\zeta) Y_s(t)\, ds} \frac{\int_{-\infty}^{t-\zeta} P_s(t) Y_s(t-\zeta)\, ds}{\int_{-\infty}^{t-\zeta} P_s(t-\zeta) Y_s(t-\zeta)\, ds}}.$$

- Thus if $P_t(t) Y_t(t) = 0$:

$$\lim_{\zeta \to 0} \left[ \frac{1}{\zeta} \log \frac{P(t)}{P(t-\zeta)} \right] = \frac{\int_{-\infty}^{t} P_s(t) Y_s(t) \frac{\dot{P}_s(t)}{P_s(t)}\, ds}{\int_{-\infty}^{t} P_s(t) Y_s(t)\, ds}.$$

- I.e. if applied to capital quantities and rental rates, this gives the theoretically correct aggregate price.
  - This is not what the BEA does as rental rates are hard to infer.
- In general, the biases we find in this paper do not occur for non-durable goods.
  - Capital services are a non-durable good with price equal to the rental rate of capital.

# Further model primitives (1/2)

- The law of motion for capital good vintage $s$ is:

$$\dot{K}_s(t) = I_s(t) - \delta K_s(t).$$

- Capital and labour are combined to produce an intermediate good $X_t$, the numeraire. (Details later.)

- One unit of the intermediate good may be converted into $e^{\gamma t}$ units of the consumption good in period $t$.

  - Produced under perfect competition.

  - I.e. the price of a unit of consumption is $P_C(t) \coloneqq e^{-\gamma t}$.

  - Effectively: $\gamma$ is the TFP growth of the consumption good producing sector.

# Further model primitives (2/2)

- One unit of the intermediate good becomes:

$$\psi(t-s)^{\phi}e^{\kappa s}$$

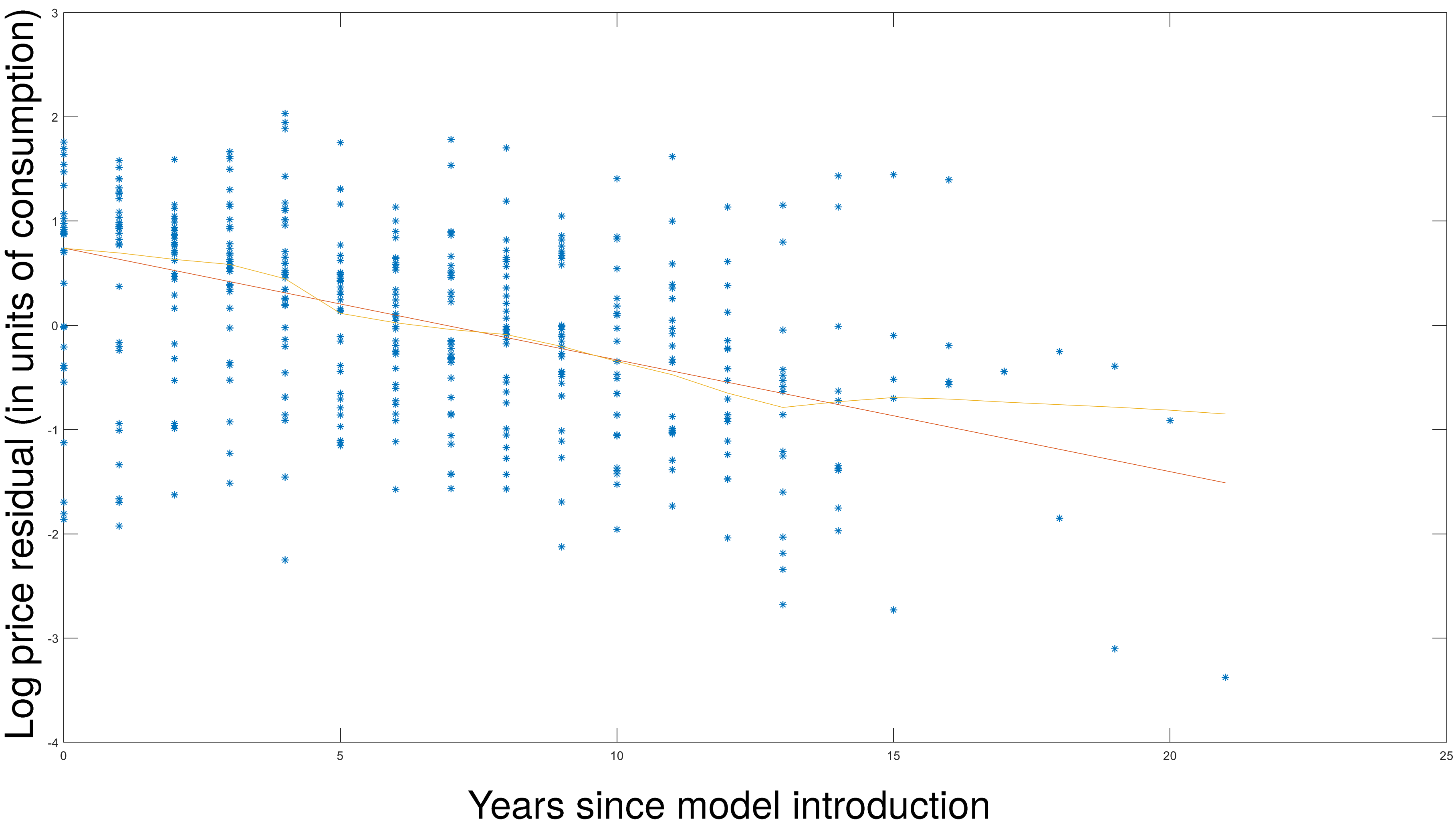
  units of capital variety $s$ in period $t$. Assume $\kappa > 0$.

  - Again produced under perfect competition.
  - I.e. the price of investment goods of vintage $s$ is:

$$Q_s(t) := \frac{1}{\psi(t-s)^{\phi}e^{\kappa s}} \Rightarrow \frac{\dot{Q}_s(t)}{Q_s(t)} = -\frac{\phi}{t-s}.$$

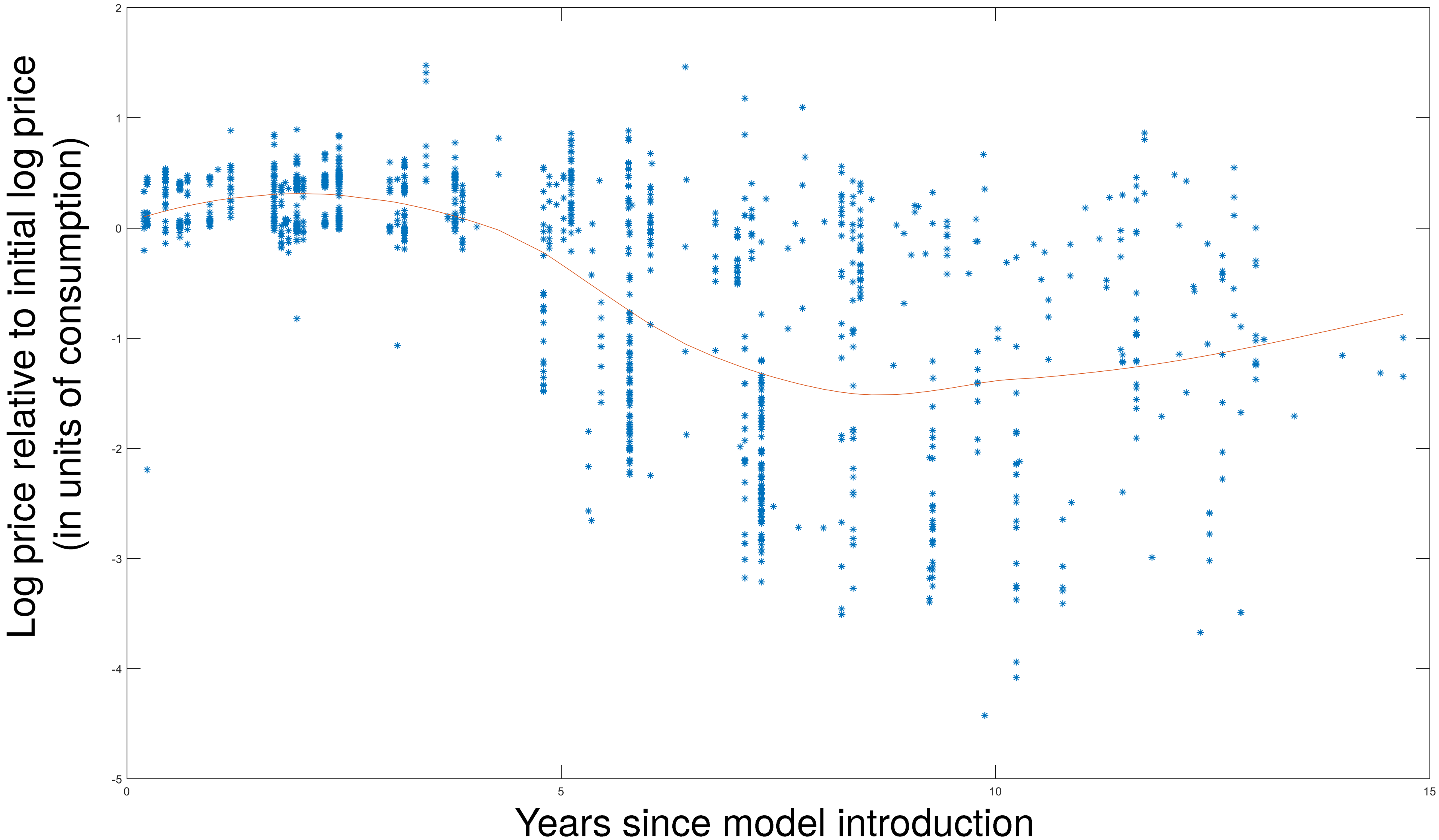
  - $\psi > 0$ just scales productivity.
  - $\phi > 0$ determines the growth rate of young products. (All products start with zero productivity, but then experience rapid growth.)
  - $\kappa > 0$ determines how productivity grows over time, holding fixed variety age $(t-s)$.

# New aircraft prices




Residuals after standardizing at the good level. Data kindly provided by A. Lanteri.

# Intel CPU Prices




Current price data web scraped from NewEgg.com. Initial price data web scraped from Wikipedia.

# Optimal capital variety holdings (1/2)

- Suppose a firm holds capital vintage $s$.

- If $r(t)$ is the deterministic interest rate at $t$, the firm's value is:

$$\int_0^\infty e^{-\int_0^\xi r(t+\tau)\,d\tau}[R_s(t+\xi)K_s(t+\xi) - Q_s(t+\xi)I_s(t+\xi)]\,d\xi\,.$$

- Optimisation implies:

$$\frac{R_s(t)}{Q_s(t)} = \delta + r(t) + \frac{\phi}{t-s}.$$

- (Unsurprisingly) Price of capital good $s$ is:

$$\int_0^\infty e^{-\int_0^\xi r(t+\tau)\,d\tau - \delta\xi} R_s(t+\xi)\,d\xi = Q_s(t).$$

# Optimal capital variety holdings (2/2)

- If we define:

$$R^*(\tau) := \frac{\delta + r^* + \frac{\phi}{\tau}}{\psi \tau^{\phi} e^{-\kappa\tau}}, \qquad R^* := \left[\int_0^{\infty} R^*(\tau)^{-\frac{1}{\lambda}} \, d\tau\right]^{-\lambda},$$

- Then if $r(t) \equiv r^*$ (i.e. we are on the BGP):

$$R(t) = e^{-\kappa t} R^*, \qquad R_s(t) = e^{-\kappa t} R^*(t - s).$$

- Both individual and aggregate rental rates are declining at rate $\kappa$.

- Note:

$$\frac{R^{*\prime}(\tau)}{R^*(\tau)} = \kappa - \frac{\phi}{(\delta + r^*)\tau^2 + \phi\tau} - \frac{\phi}{\tau}.$$

  - So $R^*(\tau)$ is U-shaped.

# Some notation

- Suppose throughout that the random variable $\mathrm{T}$ has p.d.f.:

$$\tilde{\tau} \mapsto \frac{Q_{t-\tilde{\tau}}(t) K_{t-\tilde{\tau}}(t)}{\int_0^\infty Q_{t-\tau}(t) K_{t-\tau}(t)\, d\tau}.$$

- $\mathbb{E}\mathrm{T}$ is the average years since variety introduction of the capital stock with weights given by value shares of that age.
    - Not the same as average age from the national accounts.
    - Chiefly convenient for stating results.

- Lemma 1 (exercise in integration by parts):

$$\kappa = \mathbb{E}\left[(1+\lambda)\frac{\phi}{(\delta + r^*)\mathrm{T}^2 + \phi \mathrm{T}} + \frac{\phi}{\mathrm{T}}\right].$$

# The true price of capital

- The true price of the aggregate capital stock satisfies:

$$P_K(t) = \frac{\int_{-\infty}^{t} Q_s(t) K_s(t)\, ds}{K(t)} = \frac{e^{-\kappa t} R^*}{\delta + r^* + \phi \mathbb{E}\mathrm{T}^{-1}},$$

- **Shrinking at rate $\boldsymbol{\kappa}$.**

- By Lemma 1:

$$P_K(t) > \frac{e^{-\kappa t} R^*}{\delta + r^* + \kappa} = \int_0^{\infty} e^{-\int_0^{\xi} r(t+\tau)\, d\tau - \delta \xi} R(t+\xi)\, d\xi\,,$$

- With equality in limit $\kappa \to 0$.

- Buying a unit of the aggregate capital stock at $t$ gets you period $t+\xi$ returns of:

$$\int_{-\infty}^{t} e^{-\delta \xi} R_s(t+\xi) \frac{K_s(t)}{K(t)}\, ds \neq R(t+\xi)!$$

# Investment and effective depreciation

- Define $\Delta(t)$ ($= \Delta^*$ on BGP) to solve:

$$\frac{e^{-\kappa t}R^*}{\delta + r^* + \phi\mathbb{E}\mathrm{T}^{-1}} = P_K(t) = \int_0^\infty e^{-\int_0^\xi (r(t+\tau)+\Delta(t+\tau))\,d\tau} R(t+\xi)\,d\xi$$

$$= e^{-\kappa t}R^* \int_0^\infty e^{-(\Delta^*+r^*+\kappa)\xi}\,d\xi = \frac{e^{-\kappa t}R^*}{\Delta^* + r^* + \kappa}$$

- I.e.:

$$\Delta^* = \delta - (\kappa - \phi\mathbb{E}\mathrm{T}^{-1}) < \delta.$$

- We then **define** aggregate investment by:

$$I(t) \coloneqq \dot{K}(t) + \Delta(t)K(t).$$

- These definitions will ensure:
  1. The aggregate capital good is equivalent to a homogeneous capital good with depreciation $\Delta(t)$ and rental rate $R(t)$.
  2. The aggregate price of investment equals that of capital.

# The true price of investment

- Let $z(t) \coloneqq \frac{I(t)}{K(t)}$ $(= z^*$ on the BGP), so $\frac{\dot{K}(t)}{K(t)} = z^* - \Delta$ on the BGP.

- Lemma 2: $\mathbb{E}\left[z^* + \delta - \Delta^* - \frac{1+\lambda}{\lambda} \frac{R^{*\prime}(\mathrm{T})}{R^*(\mathrm{T})}\right] = z^*$.

- True price of aggregate investment is given by:

$$P_I(t) \coloneqq \frac{\int_{-\infty}^{t} Q_s(t) I_s(t)\, ds}{I(t)} = \frac{P_K(t)}{z^*} \mathbb{E}\left[z^* + \delta - \Delta^* - \frac{1+\lambda}{\lambda} \frac{R^{*\prime}(\mathrm{T})}{R^*(\mathrm{T})}\right] = P_K(t).$$

- The true investment price is **equal** to the capital price.

# The observed price of capital

- The observed price of capital grows at rate:

$$
\begin{aligned}
\frac{\dot{P}_K^{\mathrm{OBS}}(t)}{P_K^{\mathrm{OBS}}(t)} := \frac{\int_{-\infty}^{t} Q_s(t) K_s(t) \frac{\dot{Q}_s(t)}{Q_s(t)}\, ds}{\int_{-\infty}^{t} Q_s(t) K_s(t)\, ds} &= -\mathbb{E}\,\frac{\phi}{\mathrm{T}} \\
&= -\kappa + \mathbb{E}\left[(1+\lambda)\,\frac{\phi}{(\delta + r^*)\mathrm{T}^2 + \phi \mathrm{T}}\right] \\
&\geq -\kappa.
\end{aligned}
$$

- Using Lemma 1.

- The observed price of capital is growing more quickly than the true price!

# The observed price of investment

- The observed price of investment grows at rate:

$$\frac{\dot{P}_I^{\mathrm{OBS}}(t)}{P_I^{\mathrm{OBS}}(t)} := \frac{\int_{-\infty}^{t} Q_s(t) I_s(t) \frac{\dot{Q}_s(t)}{Q_s(t)}\, ds}{\int_{-\infty}^{t} Q_s(t) I_s(t)\, ds} = -\frac{\mathbb{E}\frac{\phi}{\mathrm{T}}\left[z^* + \delta - \Delta^* - \frac{1+\lambda}{\lambda}\frac{R^{*\prime}(\mathrm{T})}{R^*(\mathrm{T})}\right]}{\mathbb{E}\left[z^* + \delta - \Delta^* - \frac{1+\lambda}{\lambda}\frac{R^{*\prime}(\mathrm{T})}{R^*(\mathrm{T})}\right]}$$

$$\leq -\mathbb{E}\frac{\phi}{\mathrm{T}} = \frac{\dot{P}_K^{\mathrm{OBS}}(t)}{P_K^{\mathrm{OBS}}(t)}.$$

- Thus observed investment growth is below observed capital growth!

- Providing $\Delta^* + r^* + \kappa - z^* > 0$ (which will hold automatically in our GE setup), we can prove that for sufficiently large $\lambda$:

$$\frac{\dot{P}_I^{\mathrm{OBS}}(t)}{P_I^{\mathrm{OBS}}(t)} \leq -\kappa \leq \frac{\dot{P}_K^{\mathrm{OBS}}(t)}{P_K^{\mathrm{OBS}}(t)}$$

- The observed price of investment is growing more slowly than the true price!

# “$\phi$ bias”

- “$\phi$ bias” disappears when $\phi = 0$. If $\phi = 0$ then:

$$\frac{\dot{P}_I^{\mathrm{OBS}}(t)}{P_I^{\mathrm{OBS}}(t)} = \frac{\dot{P}_I(t)}{P_I(t)} = -\kappa = \frac{\dot{P}_K(t)}{P_K(t)} = \frac{\dot{P}_K^{\mathrm{OBS}}(t)}{P_K^{\mathrm{OBS}}(t)}.$$

- With $\phi$ positive, a wedge is driven between observed and true price growth, pushing down observed investment price growth, and pushing up observed capital price growth.

- $\phi$ bias comes from the facts that:

    1. The investment bundle is skewed towards newer goods than the capital bundle.

    2. New goods experience rapid productivity growth.

# Closing the model

- Let $C(t)$ be aggregate consumption, $N(t)$ population, $w(t)$ labour income per capita in units of consumption goods, $A(t)$ household total asset holdings.

- Assume on the BGP $w(t) = w^* e^{gt}$, where we will solve for $g$.

- At $t$, households maximise:

$$\int_0^\infty e^{-\rho\tau} N(t+\tau) \frac{\left(\frac{C(t+\tau)}{N(t+\tau)}\right)^{1-\sigma} - 1}{1-\sigma} d\tau\,,$$

- Subject to:

$$\dot{A}(t) = r(t)A(t) + P_C(t)w(t)N(t) - P_C(t)C(t).$$

- Assume the intermediate good is produced under perfect competition with the technology:

$$\begin{aligned} K(t)^\alpha N(t)^{1-\alpha} = X(t) &= P_C(t)C(t) + P_I(t)I(t) + E(t) \\ &= R(t)K(t) + P_C(t)w(t)N(t). \end{aligned}$$

# In equilibrium

- Consumption per capita grows at rate:

$$g \coloneqq \gamma + \frac{\alpha}{1-\alpha}\kappa.$$

- Capital and investment per capita grow at rate:

$$h \coloneqq \frac{\kappa}{1-\alpha}.$$

- We have:

$$r^* = \rho + \sigma g - \gamma = \rho + \frac{\alpha}{1-\alpha}\sigma\kappa - (1-\sigma)\gamma,$$
$$z^* = h + n + \delta.$$

- Investment bias condition holds:

$$\Delta^* + r^* + \kappa - z^* = \rho - n - g(1-\sigma) > 0,$$

  - As the RHS of the equality is the effective discount rate.

# Calibration (1/3)

- Fix:
  - $\lambda \leftarrow \frac{1}{2.2-1} \approx 0.83$. Broda & Weinstein (2006), 2.2 is median e.o.s. at SITC-3 level, 1990-2001, also e.o.s. for "automatic data process machines", quite similar to e.o.s. for vehicles, 3.0.
  - $\sigma \leftarrow \frac{1}{0.594} \approx 1.68$. Havranek et al. (2015), unimportant for results.

- Calibrate (LHS are observed geometric means 1965 to now):
  - $n \leftarrow \frac{d}{dt} \log N(t) \approx 0.0098$.
  - $\alpha \leftarrow \frac{Y^{\mathrm{GDP}}(t) - Y^{\mathrm{PROP}}(t) - Y^{\mathrm{LAB}}(t)}{Y^{\mathrm{GDP}}(t) - Y^{\mathrm{PROP}}(t)} \approx 0.36$.
    - $Y^{\mathrm{PROP}}(t)$ is proprietor's income.
    - $Y^{\mathrm{LAB}}(t)$ is compensation of employees, plus government social benefits to persons, less contributions for government social insurance.
  - $g \leftarrow \frac{d}{dt} \log \left( \frac{Y^{\mathrm{GDP}}(t)}{N(t) P_C(t)} \right) \approx 0.017$.
  - $z^* \leftarrow \frac{P_I(t) I(t)}{P_K(t) K(t)} \approx 0.073$.
  - $\mathcal{R} \leftarrow \frac{R(t) K(t)}{P_K(t) K(t)} \approx 0.12$.

# Calibration (2/3)

- Calibrate (LHS are observed geometric means 1965 to now):
  - $g_{PK} \leftarrow \frac{d}{dt}\log\left(\frac{P_K^{\mathrm{OBS}}(t)}{P_C(t)}\right) \approx 0.0049.$
  - $g_{PI} \leftarrow \frac{d}{dt}\log\left(\frac{P_I^{\mathrm{OBS}}(t)}{P_C(t)}\right) \approx -0.0067.$
  - $\delta^{DATA} \leftarrow \frac{CFC(t)}{P_K(t)K(t)} \approx 0.050.$
    - Only used to check the over-identifying restriction: $\delta^{DATA} = z^* - n - g + g_{PK}$.
    - We use $\delta \approx 0.052$ (the RHS of the previous restriction).
    - Error: $0.15$ percentage points, $2.9$ percent.

# Calibration (3/3)

- Guess $\phi$, $\Delta^*$, then evaluate:
  - $h \leftarrow z^* - n - \Delta^* \approx 0.019.$
  - $\kappa \leftarrow h(1-\alpha) \approx 0.012.$
  - $\gamma \leftarrow g - \alpha h \approx 0.010.$
  - $\phi \mathbb{E}\mathrm{T}^{-1} \leftarrow -\mathscr{g}_{PK} + \gamma \approx 0.0050.$
  - $\dfrac{\mathbb{E}\frac{\phi}{\mathrm{T}}\left[z^*+\delta-\Delta^*-\frac{1+\lambda}{\lambda}\frac{R^{*\prime}(\mathrm{T})}{R^*(\mathrm{T})}\right]}{\mathbb{E}\left[z^*+\delta-\Delta^*-\frac{1+\lambda}{\lambda}\frac{R^{*\prime}(\mathrm{T})}{R^*(\mathrm{T})}\right]} \leftarrow -\mathscr{g}_{PI} + \gamma \approx 0.017.$
  - $\mathscr{r}^* \leftarrow \mathcal{R} - \Delta^* - \kappa \approx 0.066.$
  - $\rho \leftarrow \mathscr{r}^* + \gamma - \sigma g \approx 0.048.$

- Calculate residuals from directly evaluating the expectations above.
  - We evaluate the required integrals numerically, by treating them as expectations of appropriate gamma distributed random variables, mapping these into uniforms, and then using a Fejer Type 1 rule.
  - If they are not zero, adjust our guesses of $\phi$, $\Delta^*$.

# Calibration consequences

- Annual growth of capital price in units of consumption goods:
  - Observed: $0.49\%$.
  - True: $-0.21\%$.
  - Bias: $-0.70$ percentage points
- Annual growth of investment price in units of consumption goods:
  - Observed: $-0.67\%$
  - True: $-0.21\%$
  - Bias: $0.47$ percentage points
- Annual growth of output price in units of consumption goods:
  - Observed: $-0.15\%$
  - True: $-0.05\%$
  - Bias: $0.10$ percentage points

- Annual real output growth is overstated by 0.10 percentage points in the NIPA!
- Annual real investment growth is overstated by 0.47 percentage points!

# Further consequences

- Lower role for investment specific technological change as an engine of growth.
  - IST ($\kappa > \gamma$) explains 6.9% of growth in our calibration.
  - Compared to 60% in Greenwood, Hercowitz & Krusell (1997).
  - IST may still drive growth in particular sectors (e.g. equipment, IPP, durable cons.).

- Unlikely can explain the decline in the labour share with falling capital prices and capital/labour substitution (Neiman and Karabarbounis 2014).
  - In fact, capital and labour appear to be gross compliments:
    - Best practice estimates: León-Ledesma, McAdam & Willman (2010).
    - Surveys: Chirinko (2008), Klump, McAdam & Willman (2012).
    - Meta-study: Knoblach, Rößler & Zwerschke (2016)
  - Were capital prices falling rapidly, a rise in the labour share would be expected.
  - We at least reduce this puzzle.

- Piketty (2014) / Piketty & Zucman (2014) via Rognlie (2014).
  - With lower real investment growth, we should expect $\frac{P_K K}{P_Y Y} \to \frac{s}{g_I + \delta}$ to be larger.

# Conclusions

- We built a simple model to explain the apparent divergence of capital and investment prices.
  - All of this divergence can be explained by incorrect aggregation of the prices of fixed assets.
  - Traditional price aggregates are inappropriate for durable goods.
  - Aggregation should be on implied rental rates, not prices.
- Model implies a greatly reduced role for IST.
- Model predicts output growth in units of consumption goods is overstated by about 0.10 percentage points per year.
- Currently working on reconstructing US national accounts from lowest level up.

# Appendix: IST Literature

- Nominal capital LOM:

$$P_{K,t}K_t = (1-\delta)P_{K,t}K_{t-1} + P_{I,t}I_t$$
$$\Rightarrow K_t = (1-\delta)K_{t-1} + \frac{P_{I,t}}{P_{K,t}}I_t$$

- Greenwood, Hercowitz & Krusell (1997) call $\frac{P_{I,t}}{P_{K,t}}$ "investment specific technological change" (IST).
  - GHK identify it with $\frac{P_{C,t}}{P_{I,t}}$, the price of consumption in units of investment.
  - This ratio is rising in the US, so could be an engine of growth.
  - $\frac{P_{I,t}}{P_{K,t}}$ is roughly inversely proportional to Tobin's Q $= \frac{\text{market}}{\text{replacement}}$.

- BUT $\frac{P_{I,t}}{P_{K,t}}$ is the price of investment in units of capital goods, not $\frac{P_{C,t}}{P_{I,t}}$!
  - Even if $\frac{P_{I,t}}{P_{K,t}}$ and $\frac{P_{I,t}}{P_{C,t}}$ are falling, $\frac{P_{K,t}}{P_{C,t}}$, the price of capital in units of consumption goods, may still be rising, in which case IST would not be an engine of growth.

# Appendix: What should the BEA do?

- To (roughly) correctly measure the price of capital and investment, the BEA could proceed as follows:

1. Guess an interest rate $r(t)$.
2. Measure individual good prices and depreciation rates as at present.
3. From the good prices $Q_s(t)$, depreciation rates $\delta_s(t)$ and guessed interest rate $r(t)$, construct implied rental rates $R_s(t)$ using:
$$\frac{R_s(t)}{Q_s(t)} = \delta_s(t) + r(t) - \frac{\dot{Q}_s(t)}{Q_s(t)}.$$
3. Aggregate $R_s(t)$ and $K_s(t)$ to obtain $R(t)$ and $K(t)$ using the Fisher index formula.
   - Note that this is implicitly using second derivatives of $Q_s(t)$!
4. Evaluate $P_K(t) = \frac{\int_{-\infty}^{t} Q_s(t)K_s(t)\,ds}{K(t)}$ and $I(t) = \frac{\int_{-\infty}^{t} Q_s(t)I_s(t)\,ds}{P_K(t)}$ (so $P_I(t) = P_K(t)$).
5. Evaluate $\Delta(t) = \frac{I(t)-\dot{K}(t)}{K(t)}$, and check $\frac{R(t)}{P_K(t)} - \Delta(t) = r(t) - \frac{\dot{P}_K(t)}{P_K(t)}$, if not, adjust $r(t)$.