A Robust Monetary Rule

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Paper: https://is.gd/ARobustMonetaryRule

Slides: https://is.gd/SlidesARobustMonetaryRule

Motivation: Fragility of the Taylor principle

- The Taylor principle requires the response of nominal rates to inflation to be greater than one.
  - Sufficient for determinacy in simple models. (I.e. guarantees no sunspots.)
- Insufficient if there is e.g.:
  - A fraction of hand-to-mouth households (Gali, Lopez-Salido & Valles 2004).
  - Firm-specific capital (Sveen & Weinke 2005).
  - High government spending (Natvik 2009).
  - A positive inflation target (Ascari & Ropele 2009), particularly in the presence of trend growth and sticky wages (Khan, Phaneuf & Victor 2019).
- Inverts if there are e.g.:
  - Enough hand-to-mouth households (Bilbiie 2008).
  - Certain financial frictions (Manea 2019).
  - Non-rational expectations (Branch & McGough 2010; 2018).

This paper

- Presents a family of monetary rules which guarantee determinate inflation under the weakest possible assumptions on the rest of the economy.
  - Robust to heterogeneity, non-rational households/firms, nature of nominal rigidities, existence/slope of the Phillips curve, etc.

- These rules enable the determinate implementation of an arbitrary path for inflation (optionally a function of the history of structural shocks), robustly across models.
  - Thus, rules within the family can automatically match observed inflation dynamics, or any model's optimal policy.
  - The rules can attain high welfare even when restricted to be "simple", and such "simple" rules also help explain observed US Fed behaviour.

A first example

- Nominal bond: $\$1$ bond purchased at t returns $\$(1 + i_t)$ at $t + 1$.
- Real bond (e.g. TIPS): $\$1$ bond purchased at t returns $\$(1 + r_t + \pi_{t+1})$ at $t + 1$.
  - π_{t+1} is realized inflation between t and $t + 1$.
- Arbitrage between these two implies the Fisher equation:

$$i_t = r_t + \mathbb{E}_t \pi_{t+1}$$

  - Abstracting from inflation risk / term / liquidity premia for now.

- Central bank uses the “real rate rule”:

$$i_t = r_t + \phi \pi_t$$

- With $\phi > 1$. Then:

$$\mathbb{E}_t \pi_{t+1} = \phi \pi_t$$

- Unique non-explosive solution, $\pi_t = 0$. Determinate inflation!

Why is this robust? No need for Euler!

- Does not require an aggregate Euler equation to hold.
  - Robust to heterogeneous households and hand-to-mouth agents.
  - Robust to non-rational household expectations.

- For the Fisher equation to hold we just need either:
  - Two deep pocketed, fully informed, rational agents in the economy, OR
  - A large market of rational agents with dispersed information. (Hellwig 1980; Lou et al. 2019)

- Much more likely financial market participants have RE than households.
  - We can even partially relax the RE requirement for financial market participants.

Why is this robust? No need for Phillips!

- Does not require an aggregate Phillips curve to hold.
  - Robust to slope of the Phillips curve (if it exists).
  - Robust to forward/backward looking degree of Phillips curve equation.
  - Robust to non-rational firm expectations.

- If CB is unconcerned with output and unemployment then they do not need to care at all about the Phillips curve or its slope.
  - Under this monetary rule, the Phillips curve is irrelevant for inflation dynamics.
  - The Phillips curve (if it exists) determines the output gap, given inflation.

- Only require that at least some prices are adjusted each period using current information.

Real rate rules elsewhere in the literature

- Papers with rules responding to real rates, for analytic convenience e.g.:
  - Adão, Correia & Teles (2011), Holden (2019), Lubik, Matthes & Mertens (2019).

- Closest prior work: Cochrane (2017; 2018) on spread targeting.
  - Cochrane briefly considers a rule of the form $i_t = r_t + \phi \pi_t$ before setting $\phi = 0$.
  - Determinacy in Cochrane's world comes from the Fiscal Theory of the Price Level.

- Other related work:
  - Hetzel (1990) proposes using nominal bond, real bond spread to guide policy.
  - Dowd (1994) proposes targeting the price of price level futures contracts.
  - Hall & Reis (2016) propose making IOR a function of price level deviations from target, e.g. nominal return from $\$1$ of $\$(1 + r_t)\frac{p_{t+1}}{p_t^*}$ or $\$(1 + i_t)\frac{p_t}{p_t^*}$.

Monetary policy shocks

- Adding a monetary policy shock permits inflation to move.

- Suppose the CB uses the rule:

$$i_t = r_t + \phi\pi_t + \zeta_t$$

- with $\phi > 1$, and ζ_t drawn from an $\mathrm{AR}(1)$ process with persistence ρ.
- Then from the Fisher equation:

$$\mathbb{E}_t\pi_{t+1} = \phi\pi_t + \zeta_t$$

- This has the unique solution: $\pi_t = -\frac{1}{\phi-\rho}\zeta_t$.
  - Contractionary (positive) monetary policy shocks reduce inflation.
  - If the CB is more aggressive (ϕ is larger) inflation is less volatile.
  - Can understand inflation dynamics without knowing the rest of the economy.

Explaining observed inflation dynamics

- Extensive body of empirical evidence finds no role for the Phillips curve in forecasting inflation.
  - Data: Atkeson & Ohanian (2001), Ang, Bekaert & Wei (2007), Stock & Watson (2009), Dotsey, Fujita & Stark (2018).
  - Theoretical explanations: McLeay & Tenreyro (2019) and papers cited therein.

- E.g. In post-1984 period, Dotsey, Fujita & Stark (2018) find that an $\mathrm{IMA}(1{,}1)$ model beats Phillips curve based forecasts (both conditionally and unconditionally).

- Supportive of models in which the causation in the Phillips curve only runs in one direction: *from inflation to the output gap*.

Output dynamics in a simple model

- As before, suppose CB sets $i_t = r_t + \phi\pi_t + \zeta_t$, so $\pi_t = -\frac{1}{\phi-\rho}\zeta_t$.
- Rest of model 1: Phillips curve (PC), with mark-up shock ω_t:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa x_t + \kappa\omega_t$$

- Rest of model 2: Discounted/compounded Euler equation (EE) (Bilbiie 2019), with n_t exogenous natural rate ($\delta = 1$, $\varsigma = \text{EIS}$ recovers standard Euler equation):

$$x_t = \delta\mathbb{E}_t x_{t+1} - \varsigma(r_t - n_t)$$

- PC implies: $x_t = -\frac{1}{\kappa}\frac{1-\beta\rho}{\phi-\rho}\zeta_t - \omega_t$. x_t does not help forecast inflation as $\mathbb{E}_t\pi_{t+1} = \rho\pi_t$.
  - Once you know π_t, there is no extra useful information in x_t.

Real rate dynamics in a simple model

- In the model of the last slide, if ω_t is IID, EE implies:

$$r_t = n_t + \frac{1}{\varsigma}\left[\frac{1}{\kappa}\frac{(1-\beta\rho)(1-\delta\rho)}{\phi-\rho}\zeta_t + \omega_t\right]$$

- Derived without solving EE forward!
  - Implies degree of discounting/compounding (δ) has no impact on determinacy.
  - Also implies robustness to missing transversality conditions.
  - Contrasts with Bilbiie (2019) who found the Taylor principle was only sufficient for determinacy in the discounting case with $\delta \leq 1$ (under a standard Taylor rule).
  - Also contrasts with Bilbiie (2008) who found the Taylor principle was only sufficient for determinacy in the $\varsigma > 0$ case (again, under a standard Taylor rule).

- Under real rate rule, the Taylor principle is always necessary and sufficient!
  - (Given $\phi \geq 0$.) Also robust to non-unit response to real rates!

Implementing arbitrary inflation dynamics

- A slight generalization of our rule permits the determinate implementation of arbitrary inflation dynamics.
- Suppose CB uses the rule:

$$i_t = r_t + \mathbb{E}_t \pi^*_{t+1} + \phi(\pi_t - \pi^*_t)$$

- π^*_t: an exogenous stochastic process, possibly a function of economy's other shocks.

- So from the Fisher equation: $\mathbb{E}_t(\pi_{t+1} - \pi^*_{t+1}) = \phi(\pi_t - \pi^*_t)$.
- With $\phi > 1$, unique, determinate solution: $\pi_t = \pi^*_t$.
- The CB can hit an arbitrary path for inflation! E.g. optimal policy.

- Related literature on implementation of optimal policy: Svensson & Woodford (2003), Dotsey & Hornstein (2006), Evans & Honkapohja (2006), Evans & McGough (2010).

Practical implementation 1/3

- In most countries, inflation protected securities are only issued occasionally, so the CB would struggle to control overnight returns on such securities.
  - Suppose as an example they instead target five-year returns.
  - Long maturities may have substantial risk/term/liquidity premia.
  - Extra complication: Inflation may be observed with a lag. One month for US CPI.
- Notation:
  - i_t: nominal yield per period on a five-year sovereign (nominal) bond at t.
  - r_t: real yield per period on a five-year sovereign inflation protected bond at t.
  - T: number of periods in five years. E.g. if t is measured in months, $T = 60$.
  - L: information lag. Market participants use the $t - L$ information set in period t.
  - ν_{t-L} risk (etc.) premia on five-year nominal bonds relative to five-year real bonds at t. (Lagged subscript as participants use $t - L$ dates variables at t.)
  - $\bar{\nu}_{t-L}$ central bank's period t belief about level of ν_{t-L} (possibly correlated with ν_{t-L}).

Practical implementation 2/3

- Fisher equation:

$$i_t - r_t = \nu_{t-L} + \mathbb{E}_{t-L} \frac{1}{T} \sum_{k=1}^{T} \pi_{t+k}$$

- CB uses the rule:

$$i_t - r_t = \bar{\nu}_{t-L} + \mathbb{E}_{t-L} \frac{1}{T} \sum_{k=1}^{T} \pi^*_{t+k} + \phi(\pi_{t-L} - \pi^*_{t-L})$$

- Combining implies:

$$\mathbb{E}_t \frac{1}{T} \sum_{k=1}^{T} (\pi_{t+k+L} - \pi^*_{t+k+L}) = (\bar{\nu}_t - \nu_t) + \phi(\pi_t - \pi^*_t)$$

- With $\phi > 1$ this has a unique solution of the form:

$$\pi_t = \pi^*_t + \mathbb{E}_t \sum_{j=0}^{\infty} A_j (\bar{\nu}_{t+j} - \nu_{t+j}), \qquad A_0 = -\frac{1}{\phi}, \qquad A_j = \mathrm{O}\left(\phi^{-\frac{j}{T+L}}\right) \text{ as } j \to \infty$$

Practical implementation 3/3

- CB's inflation error $\pi_t - \pi_t^*$ is stationary as long as $\bar{\nu}_{t+j} - \nu_{t+j}$ is stationary.
- If ϕ is large enough, $\pi_t \approx \pi_t^*$.
  - I.e. if the central bank is aggressive enough, neither limited knowledge of risk premia, nor information lags make any difference to CB's ability to hit $\pi_t = \pi_t^*$.

- Note: CB's trading desk should hold $i_t - r_t$ constant between meetings.
  - This requires i_t to move between meetings, in response to observed changes in r_t.
  - No reason this should be significantly harder than holding i_t fixed.

- CB could also offer to exchange $\$1$ face value of real debt for $\$(1 + i_t - r_t)$ face value of nominal, as proposed by Cochrane (2017; 2018).
- Or to buy/sell portfolios with $\$1$ nominal debt, $-\$1$ real debt for $\$(i_t - r_t)$.

Welfare

- We've seen: Real rate rules can determinately implement an arbitrary path for inflation, including optimal policy. Automatic that they can attain high welfare!

- Makes sense to limit to "simple" real rate rules though.
  - "Simple" here means simple dynamics of targeted inflation.
  - Claim: Looking for optimal simple inflation dynamics is a useful approach to policy.

- Two exercises in paper/full slides:
  - $\mathrm{MA}(0)$, $\mathrm{MA}(1)$ and $\mathrm{ARMA}(1{,}1)$ inflation policy in a simple NK model. Latter is sufficient to attain unconditional optimal.
  - Examination of optimal policy in the Justiniano, Primiceri & Tambalotti (2013) model. Multiple shock $\mathrm{ARMA}(1{,}2)$ inflation policy is very close to fully optimal.

Empirical support

- As before: Real rate rules can determinately implement an arbitrary path for inflation, including observed inflation dynamics. Automatic that they can explain the data!

- Question is whether simple real rate rules could explain the data.
  - Central bankers do not describe themselves as following a real rate rule.
  - They may act *as if* they did though!

- Two exercises in paper/full slides:
  - Performance of $\mathrm{ARMA}(1{,}1)$ rules in the Smets & Wouters (2007) model. These rules beat original model in inflation and nominal rate RMSE and likelihood.
  - Direct estimation of a real rate rule on monthly data. Reconciles TIPS breakeven inflation and SPF expectations. (MLE) Estimated ϕ is 1.56!

Conclusion

- The Taylor principle does not guarantee determinacy with standard monetary rules.
- Real rate rules guarantee determinacy no matter the rest of the economy.

- Real rate rules enable the determinate implementation of arbitrary inflation dynamics.
- As such, they can attain high welfare / explain observed dynamics.

- Even simple real rate rules can attain high welfare.
- Optimal policy can be well approximated by an ARMA process with few MA terms.

- Even simple real rate rules can explain the US data.
- They reconcile TIPS & SPF expectations measures, and better explain nominal rates.

Extra slides

- But don’t price setters determine inflation?
- Original outline (“Rest of this talk”)
- Welfare details
- Empirics details
- References
- Appendix slides

But don't price setters determine inflation?

- Suppose all firms doubled their price today. What would happen?
- The CB observes high inflation, so offers a deposit facility paying $i_t = r_t + \phi\pi_t > r_t$ (continuously adjusting i_t as r_t moves).
  - Alternatively: CB pays interest on reserves or issues central bank securities.
- Financial market participants still expect zero future inflation, so they are happy to deposit and receive $i_t > r_t$.
- The entirety of the money stock ends up being transferred to this deposit facility (and r_t almost certainly rises).
- Consumers have no cash ⇒ at least some goods are not sold ⇒ goods markets do not clear.
- At least some firms reduce their price until markets clear.
  - This will only occur when $\pi_t = 0$.

Rest of this talk

- Generalizations and generality.
  - Monetary policy shocks and observed inflation dynamics.
  - Example in a simple model.
  - Determinate implementation of arbitrary inflation dynamics.
- Practical implementation.
  - Dealing with information lags and risk (etc.) premia.
- Welfare.
  - Simple models, simple rules.
  - Richer models.
- Empirical support.
  - Our rules vs that of Smets & Wouters (2007).
  - Direct estimation on monthly data.

A simple NK model for policy analysis

- We'll look at welfare in a simple model with the Phillips curve (ω_t IID):

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + \kappa \omega_t$$

- And the policy objective to minimise:

$$(1-\beta)\mathbb{E}\sum_{k=0}^{\infty}\beta^k\left(\pi_{t+k}^2 + \lambda x_{t+k}^2\right) = \mathbb{E}\left(\pi_t^2 + \lambda x_t^2\right)$$

  - Equality under the constraint that policy must be time-invariant.

- Optimal policy must have an $\mathrm{MA}(\infty)$ representation ($\theta_1, \theta_2, \ldots$ TBD):

$$\pi_t = \kappa \sum_{k=0}^{\infty} \theta_k \omega_{t-k}$$

  - Implies objective is:

$$\mathbb{E}\left(\pi_t^2 + \lambda x_t^2\right) = \mathbb{E}\left[\omega_t^2\right]\sum_{k=0}^{\infty}\left[\kappa^2\theta_k^2 + \lambda(\theta_k - \beta\theta_{k+1} - \mathbb{1}[k=0])^2\right]$$

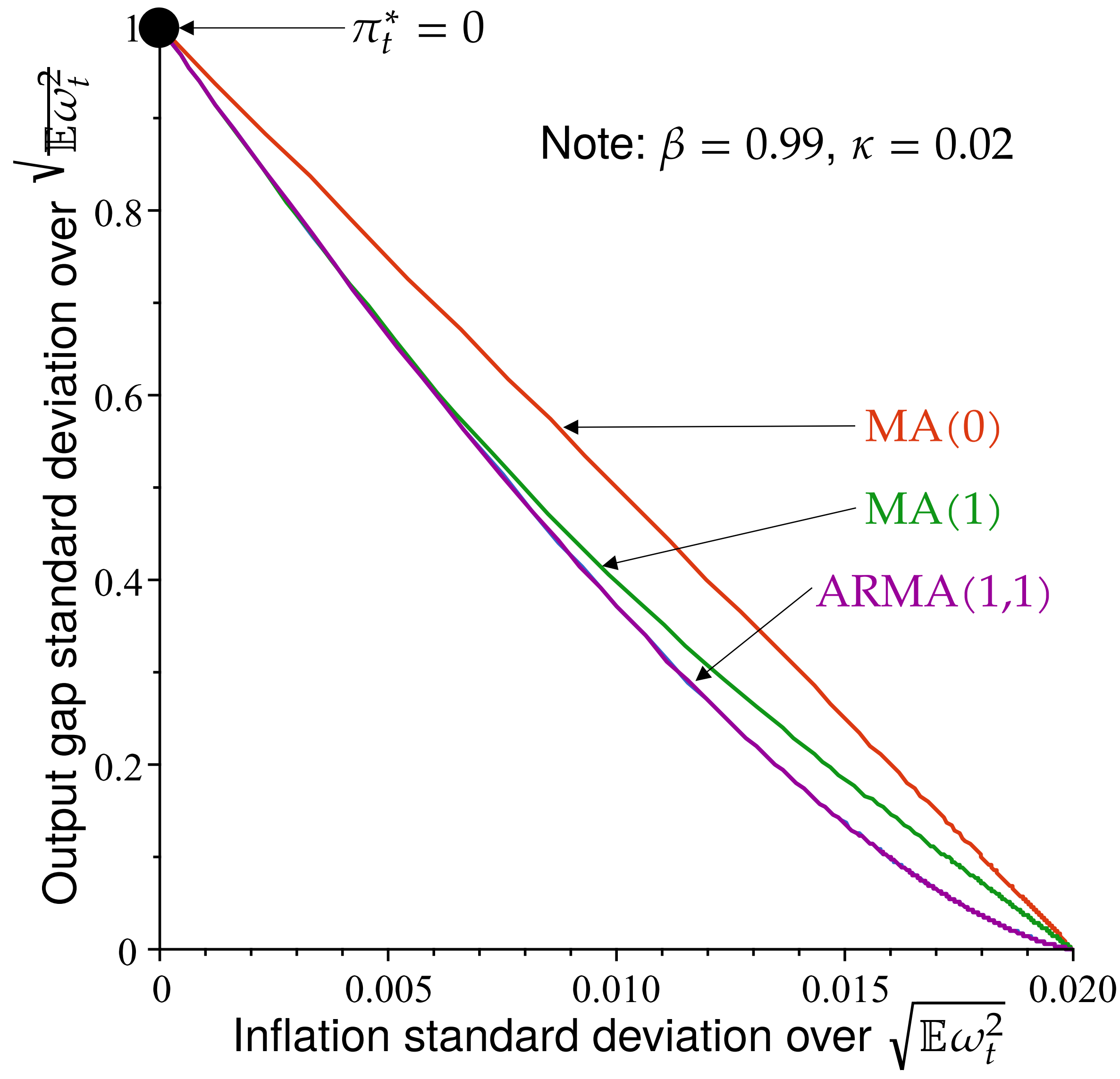
Welfare of real rate rules in a simple model

- Optimising subject to $\pi_t = \pi_t^*$ being an $\mathrm{MA}(0)$ gives the discretionary optimum with $\pi_t = \kappa \frac{\lambda}{\lambda+\kappa^2} \omega_t$ and $\pi_t + \frac{\lambda}{\kappa} x_t = 0$.

- Optimising subject to $\pi_t = \pi_t^*$ being an $\mathrm{MA}(1)$ gives a solution with $\pi_t = \kappa\theta_0\omega_t + \kappa\theta_1\omega_{t-1}$ where $\theta_0 \geq 0$ and $\theta_1 \leq 0$.
  - Thus ω_t increases π_t while reducing $\mathbb{E}_t \pi_{t+1}$, lessening output gap movements.

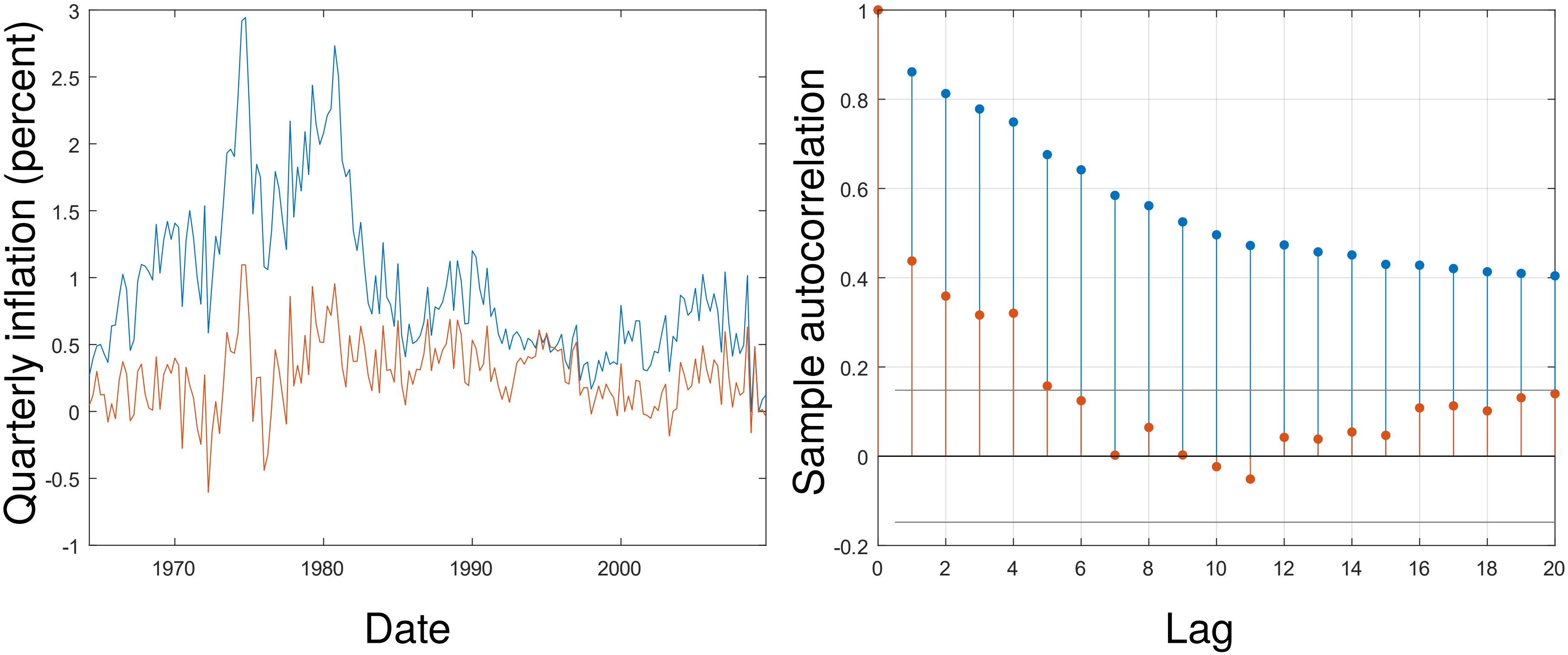
- Optimising subject to $\pi_t = \pi_t^*$ being an $\mathrm{ARMA}(1{,}1)$ give the unconditionally optimal solution from the prior literature (e.g. Damjanovic, Damjanovic & Nolan (2008)) with $\pi_t + \frac{\lambda}{\kappa}(x_t - \beta x_{t-1}) = 0$.
  - Optimal MA coefficient equals $-\beta \approx -0.99$. Close to the $\mathrm{IMA}(1{,}1)$ processes estimated by Dotsey, Fujita & Stark (2018) for the post-1984 period.

Policy frontiers (varying λ)



Relative variances

Optimal inflation dynamics in a richer model



Using the Justiniano, Primiceri & Tambalotti (2013) model and replication files.

Blue: actual US inflation dynamics.

Red: inflation dynamics under optimal policy and US historical shocks. Less persistent!

Simple approximation to optimal policy 1/2

- For any $\rho \in (-1,1)$, the solution for optimal inflation has a multiple shock, $\mathrm{ARMA}(1,\infty)$ representation:

$$\pi_t - \pi = \rho(\pi_{t-1} - \pi) + \sum_{k=0}^{\infty} \sum_{n=1}^{N} \theta_{n,k}^{(\rho)} \varepsilon_{n,t-k}$$

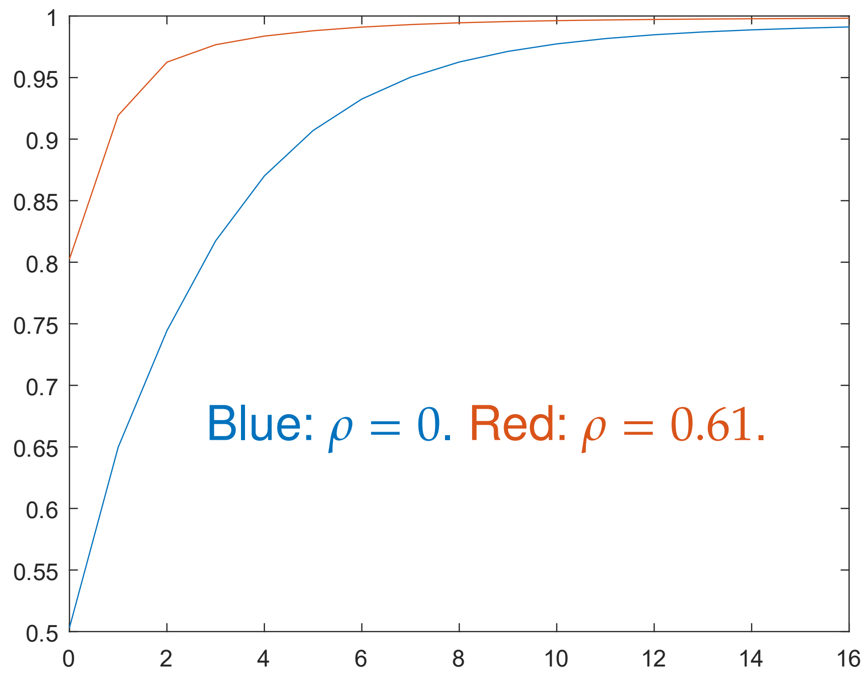
  - $\varepsilon_{1,t}, \ldots, \varepsilon_{N,t}$ are the model's structural shocks.

- Approximate by truncating MA terms at some point: E.g. multiple shock $\mathrm{ARMA}(1,K)$:

$$\pi_t^{(K)} - \pi = \rho\left(\pi_{t-1}^{(K)} - \pi\right) + \sum_{k=0}^{K} \sum_{n=1}^{N} \theta_{n,k}^{(\rho)} \varepsilon_{n,t-k}$$

- Henceforth: "multiple shock ARMA" = "MSARMA".

Simple approximation to optimal policy 2/2



Number of MA lags

Proportion of the variance of optimal inflation in the Justiniano, Primiceri & Tambalotti (2013) model explained by truncating the number of MA lags.

MSARMA(1,1) explains $> 90\%$ of optimal inflation variance, MSARMA(1,2) $> 95\%$!

ARMA(1,1) inflation in the S&W model 1/3

- Reduced-form empirical evidence supports ARMA(1,1) or IMA(1,1) inflation dynamics, so natural to look at the performance of monetary rules generating this.
- Based on the Smets & Wouters (2007) model and data with a few very minor fixes / changes. (We re-estimate the original model with these changes.)

- Effectively, we replace the monetary rule with:

$$\pi_t = \rho^{\mathrm{M}} \pi_{t-1} + \varepsilon_t^{\mathrm{M}} + \theta^{\mathrm{M}} \varepsilon_{t-1}^{\mathrm{M}}$$

- Allow $\varepsilon_t^{\mathrm{M}}$ to be correlated with model's structural shocks (6 new parameters).

- To keep the same number of parameters as original model:
  - Fix $\rho^{\mathrm{M}} = 0.99$.
  - Remove the MA components from price and wage mark-up shocks.

ARMA(1,1) inflation in the S&W model 2/3

- Estimates for the model with a standard monetary rule are very close to those of Smets & Wouters (2007).

- Estimates for the model with ARMA(1,1) inflation are drastically different:
    - Much more flexible prices and wages. Increased price-flexibility in line with some micro-data for posted prices (e.g. Bils & Klenow 2004; Klenow & Kryvtsov 2008).
    - Much more prominent role for technology shocks (TFP & IST).
    - Largest shock impact is from the TFP shock which raises output by 0.7 p.p.
    - Estimated MA coefficient on inflation is negative in line with reduced-form evidence and optimal policy in the simple NK model.

ARMA(1,1) inflation in the S&W model 3/3

		Re-estimated S&W	Modified S&W
Root mean squared one quarter forecast error (in-sample)	Inflation	0.2899	**0.2876**
	Nominal interest rates	0.2442	**0.2414**
Likelihood of inflation and nominal interest rates		-51.57	**-48.57**

- Given prior empirical findings it is unsurprising that the model with ARMA(1,1) inflation beats the S&W model in inflation RMSE.
- More surprising:
  - The model with ARMA(1,1) inflation beats the S&W model in nominal rate RMSE!
  - It also attains higher likelihood for inflation and nominal interest rates!
- Fed may be targeting a desired inflation path, rather than using a Taylor rule?!?

Is estimated Fed behaviour reasonable?

- From estimated correlations between $\varepsilon_t^{\mathrm{M}}$ and other shocks, we can express:

$$\varepsilon_t^{\mathrm{M}} = \theta_a \varepsilon_t^a + \theta_b \varepsilon_t^b + \theta_g \varepsilon_t^g + \theta_I \varepsilon_t^I + \theta_p \varepsilon_t^p + \theta_w \varepsilon_t^w + \hat{\varepsilon}_t^{\mathrm{M}}$$

- Estimated parameters:

$$\theta_a = -0.15, \theta_b = -0.93, \theta_g = 0.0090, \theta_I = 0.00088, \theta_p = 0.054, \theta_w = -0.013$$

$$\rho^{\mathrm{M}} = 0.99 \text{ (fixed)}, \qquad \theta^{\mathrm{M}} = -0.37$$

- To see if this is reasonable, turn off $\hat{\varepsilon}_t^{\mathrm{M}}$ shock, and numerically optimise over $\rho^{\mathrm{M}}, \theta^{\mathrm{M}}, \varepsilon_t^a, \varepsilon_t^b, \varepsilon_t^g, \varepsilon_t^I, \varepsilon_t^p, \varepsilon_t^w$ to minimise: $\sqrt{\mathbb{E}\left[\pi_t^2 + \frac{1}{16} x_t^2\right]}$

- Optimal reduces objective from 1.18 to 0.92. All but one sign identical. Values:

$$\theta_a = -0.0047, \theta_b = -0.00019, \theta_g = 0.0047, \theta_I = 9.26 \times 10^{-6}, \theta_p = 0.016, \theta_w = 0.0088$$

$$\rho^{\mathrm{M}} = 0.26, \qquad \theta^{\mathrm{M}} = -0.38$$

A real rate rule model for US inflation 1/4

- Sample:
  - Start pre-sample with first quarter covered in SPF CPI: 4/1981.
  - Start actual sample at the end of the Volker disinflation recession: 11/1982.
  - Final period: 1/2020.

- Observed (US) variables (1/2):
  - π_t: Realtime monthly CPI (All Urban Consumers, All Items). π_t is month t CPI, observed at $t+1$. Monthly from: 5/1981.
  - $\mathcal{E}_{60,t}, \mathcal{E}_{84,t}, \mathcal{E}_{120,t}, \mathcal{E}_{240,t}, \mathcal{E}_{360,t}$: Observed break-even inflation from 5,7,10,20,30-year TIPS. Monthly from: 1/2003 (5,7,10), 7/2004 (20), 2/2010 (30)

A real rate rule model for US inflation 2/4

- Observed (US) variables (2/2):
  - Median quarter-on-quarter CPI inflation expectations from the SPF, horizons -1, 0, 1, 2, 3, 4 quarters. Quarterly from: 8/1981.
  - Median Q4-to-Q4 CPI inflation expectations from the SPF, horizons 0, 1, 2, 5, 10 years. Quarterly from: 8/1981 (0, 1), 8/2005 (2), 11/1991 (5, 10). Different forecast quarters treated as different variables as horizon differs. (Effectively $5 \times 4 = 20$ annual variables.)

- All SPF forecasts are put into quarterly terms.
- Assumed observed with common $\mathrm{AR}(2)$ bias/m.e., plus IID m.e. (common s.d.).
- Allow for a non-unit (common) response to true expectations.
- Allow for SPF forecasters to have a linear combination of $t, t-1, t-2$ information.

A real rate rule model for US inflation 3/4

- Targeted monthly CPI inflation π_t^* is $\mathrm{IID} + \mathrm{AR}(1) + \mathrm{I}(1) + \mathrm{News\ term}$.
  - $\mathrm{IID} + \mathrm{AR}(1)$ gives an $\mathrm{ARMA}(1,1)$ so this is in line with prior empirical evidence.

- The "$\mathrm{News\ term}$" is necessary in order to explain the evidence on the performance of survey expectations from Ang, Bekaert & Wei (2007).
  - If SPF participants are able to beat standard time-series models, they must be receiving signals informative about future inflation.
  - One approach: add standard news shocks. Computationally intractable at these horizons. (Number of state variables is $\mathrm{O}(\mathrm{horizon}^2)$.)
  - Instead, the "$\mathrm{News\ term}$" is a sum of repeated-root $\mathrm{AR}(2)$ processes, with IRF peaks at 0, 3, 6, 9, 12, 24, 36, 48, 60, 108, 156, 204, 252 months.
  - Standard deviations are a spline in the horizon with knots at 0, 12, 60, 252 months.

A real rate rule model for US inflation 4/4

- Break-even inflation determination:

$$\mathcal{E}_{T,t} = \tau \mathcal{E}_{0,T,t} + (1-\tau)\mathcal{E}_{1,T,t-1}$$

$$\mathcal{E}_{0,T,t} = \gamma_K \nu_{C,t} + \nu_{T,t} + \mathbb{E}_t \frac{1}{T}\sum_{k=1}^{T} \pi_{t+k}\,, \qquad \mathcal{E}_{1,T,t} = \gamma_K \nu_{C,t} + \nu_{T,t} + \mathbb{E}_t \frac{1}{T}\sum_{k=1}^{T} \pi_{t+k+1}$$

  - $\nu_{C,t}$ is a common $\mathrm{AR}(2)$ risk/term/liquidity premium. $\nu_{K,t}$ is an IID idiosyncratic one.
  - $\tau = 1 \Rightarrow$ market participants have no information lag. $\tau = 0 \Rightarrow$ one-month lag.

- Monetary rule:

$$\begin{aligned} &\tau \mathcal{E}_{0,60,t} + (1-\tau)\mathcal{E}_{1,60,t} \\ &\quad = \gamma_K \bar{\nu}_{C,t} + \bar{\nu}_{60,t} + \tau\left(\mathbb{E}_t \frac{1}{T}\sum_{k=1}^{T} \pi^*_{t+k}\right) + (1-\tau)\left(\mathbb{E}_t \frac{1}{T}\sum_{k=1}^{T} \pi^*_{t+k+1}\right) \\ &\quad + \phi(\pi_t - \pi^*_t) \end{aligned}$$

  - $\bar{\nu}_{C,t}$ and $\bar{\nu}_{60,t}$ follow processes with the same form and parameters as corresponding un-barred variables. Allow correlated shocks between un-/barred.

Estimation results 1/2

- Total of 31 estimated parameters. (For 2904 data points from 466x32 dataset).
  - Two (π_t^* $\mathrm{I}(1)$ component s.d. & horizon 0 news s.d.) were driven to zero.
  - Standard errors below (in brackets) are conditional on these parameters.
- Mean risk (etc.) premia at short horizons (10 years and below) are insignificant.
  - Liquidity premium balancing out risk premium?
  - At 30 years, mean annualized risk (etc.) premia are 0.51 p.p. (0.24 p.p.).
- The $\mathrm{AR}(2)$ common risk premia process is quite persistent (roots: 0.96 (0.01) 0.47 (0.14)) and reasonably volatile (annualized s.d. 0.88 p.p. (0.17 p.p.)).
  - IID component of risk premia has annualized s.d. between 0.02 and 0.09 p.p.
- The SPF bias process is less persistent (roots: 0.42 (0.21), repeated) and less volatile (annualized s.d. 0.17 p.p. (0.02 p.p.)).
  - IID component of SPF error has annualized s.d. 0.17 p.p. (0.01 p.p.).

Estimation results 2/2

- Estimated correlation between shocks to $\nu_{C,t}$ & $\nu_{60,t}$ and corresponding shocks to $\bar{\nu}_{C,t}$ & $\bar{\nu}_{60,t}$ is 0.11, but insignificant. $\bar{\nu}_{C,t}$ & $\bar{\nu}_{60,t}$ may be mopping up policy shocks.

- ϕ is 1.56 (0.41). Quite standard! Large s.e. reflect weak identification.

- The response of observed SPF expectations to rational expectations is 0.95 (0.01).
  - SPF forecasters are significantly under responsive.
- SPF forecasts have a mean upwards bias of 0.11 p.p. $(0.03$ p.p.$)$.
- SPF forecasters information set is 32% (4%) current, -8% (5%) one month lagged, and 76% two period lagged.
  - Negative share of month lagged info suggests over-sensitivity to current signals.
- Financial market participants information is 72% (2%) current, 28% one month lagged.

Variance decomposition (%)

Shock to→	$\nu_{C,t}$	$\bar{\nu}_{C,t}$	$\nu_{60,t}$	$\bar{\nu}_{60,t}$	$\nu_{>60,t}$	π_t^* IID	π_t^* AR(1)	π_t^* News term
π_t	2.41	2.98	0.01	0.02	0	47.08	8.91	38.6
π_t^*	0	0	0	0	0	49.78	9.42	40.81
$\mathbb{E}_t \frac{1}{60} \sum_{k=1}^{60} \pi_{t+k}^*$	0	0	0	0	0	0	0.01	99.99
$\mathbb{E}_t \frac{1}{60} \sum_{k=1}^{60} \pi_{t+k}$	1.12	1.38	0	0	0	0	0.01	97.49
$\mathbb{E}_t \frac{1}{120} \sum_{k=1}^{120} \pi_{t+k}$	0.38	0.46	0	0	0	0	0	99.16
$\mathbb{E}_t \frac{1}{240} \sum_{k=1}^{120} \pi_{t+k}$	0.11	0.14	0	0	0	0	0	99.74
$\mathcal{E}_{60,t}$	15.49	1.14	0.05	0	0	0	0.01	83.31
$\mathcal{E}_{120,t}$	10.04	0.41	0	0	0.05	0	0	89.51
$\mathcal{E}_{240,t}$	6.81	0.13	0	0	0.02	0	0	93.04

Filtered (2,96) variance decomposition (%)

Shock to→	$\nu_{C,t}$	$\bar{\nu}_{C,t}$	$\nu_{60,t}$	$\bar{\nu}_{60,t}$	$\nu_{>60,t}$	π_t^* IID	π_t^* AR(1)	π_t^* News term
π_t	1.26	1.56	0.02	0.03	0.00	82.09	14.58	0.46
π_t^*	0.00	0.00	0.00	0.00	0.00	84.52	15.01	0.47
$\mathbb{E}_t \frac{1}{60} \sum_{k=1}^{60} \pi_{t+k}^*$	0.00	0.00	0.00	0.00	0.00	0.00	2.21	97.79
$\mathbb{E}_t \frac{1}{60} \sum_{k=1}^{60} \pi_{t+k}$	26.11	32.27	0.00	0.00	0.00	0.00	0.92	40.70
$\mathbb{E}_t \frac{1}{120} \sum_{k=1}^{120} \pi_{t+k}$	12.35	15.27	0.00	0.00	0.00	0.00	0.35	72.04
$\mathbb{E}_t \frac{1}{240} \sum_{k=1}^{120} \pi_{t+k}$	3.26	4.03	0.00	0.00	0.00	0.00	0.09	92.63
$\mathcal{E}_{60,t}$	83.80	6.57	0.95	0.00	0.00	0.00	0.13	8.54
$\mathcal{E}_{120,t}$	78.81	3.39	0.00	0.00	1.28	0.00	0.06	16.47
$\mathcal{E}_{240,t}$	66.59	1.33	0.00	0.00	0.73	0.00	0.02	31.32

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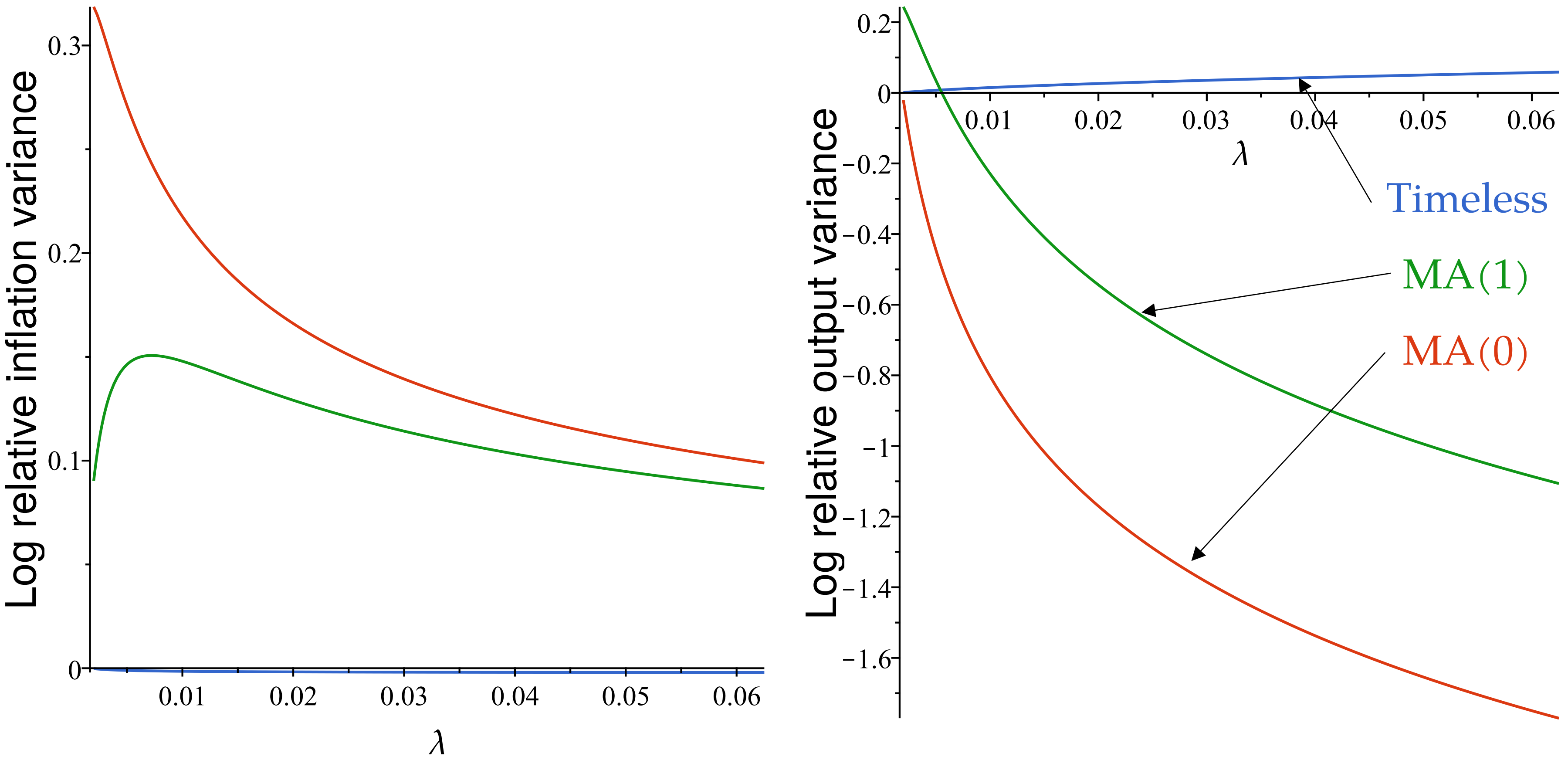
Log relative variances to ARMA(1,1) policy



Note: $\beta = 0.99$, $\kappa = 0.02$.

$\mathrm{MA}(0)$ and $\mathrm{MA}(1)$ policies generate too much inflation variance.

Back

Changes to Smets & Wouters (2007) model

- Use uniform-priors on shock standard deviations, following Rabanal & Rubio-Ramírez (2005; 2008).
    - Makes easier to compare predictive likelihoods across models.
    - Avoids biasing estimates away from finding a dominant shock.

- Initialize the state-covariance with the stationary distribution and 74 quarters of pre-sample information, rather than a diffuse prior and 4 quarters.
    - Ensures predictive likelihood is reasonable early in the sample.

- Correct a type pointed out by Del Negro & Schorfheide (2012) which changes the coefficient multiplying IST in the evolution of the capital stock.
    - Particularly important in modified model as we estimate a big role for IST.

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Estimates from S & W model and modified

Variable	Prior Shape	Prior Mean	Prior SD	S&W Mode	S&W Re-estimated Mode	Modified Model Mode
φ	Normal	4.00	1.50	5.48	5.17	0.02
σ_c	Normal	1.50	0.38	1.39	1.42	1.23
λ	Beta	0.70	0.10	0.71	0.73	0.24
ξ_w	Beta	0.50	0.10	0.73	0.75	0.07
σ_l	Normal	2.00	0.75	1.92	2.01	3.32
ξ_p	Beta	0.50	0.10	0.65	0.64	0.13
ι_w	Beta	0.50	0.15	0.59	0.58	0.47
ι_p	Beta	0.50	0.15	0.22	0.25	0.18
ψ	Beta	0.50	0.15	0.54	0.40	0.90
Φ_p	Normal	1.25	0.13	1.61	1.55	1.36
r_π	Normal	1.50	0.25	2.03	2.04	
ρ	Beta	0.75	0.10	0.81	0.82	
r_y	Normal	0.12	0.05	0.08	0.12	
$r_{\Delta y}$	Normal	0.12	0.05	0.22	0.22	
$\bar{\pi}$	Gamma	0.62	0.10	0.81	0.65	0.61
$100(\beta^{-1} - 1)$	Gamma	0.25	0.10	0.16	0.13	0.18
$\bar{l}$	Normal	0.00	2.00	-0.1	1.22	-1.06
$\bar{\gamma}$	Normal	0.40	0.10	0.43	0.51	0.32
α	Normal	0.30	0.05	0.19	0.19	0.29
σ_a	Uniform	50.0	28.87	0.45	0.47	0.51
σ_b	Uniform	50.0	28.87	0.24	0.23	0.14
σ_g	Uniform	50.0	28.87	0.52	0.52	0.55

Variable	Prior Shape	Prior Mean	Prior SD	S&W Mode	S&W Re-estimated Mode	Modified Model Mode
σ_I	Uniform	50.0	28.87	0.45	0.45	31.91
σ_r	Uniform	50.0	28.87	0.24	0.24	
σ_p	Uniform	50.0	28.87	0.14	0.14	0.67
σ_w	Uniform	50.0	28.87	0.24	0.25	2.90
ρ_a	Beta	0.50	0.20	0.95	0.98	0.99
ρ_b	Beta	0.50	0.20	0.18	0.27	0.86
ρ_g	Beta	0.50	0.20	0.97	0.97	0.96
ρ_I	Beta	0.50	0.20	0.71	0.73	0.98
ρ_r	Beta	0.50	0.20	0.12	0.13	
ρ_p	Beta	0.50	0.20	0.90	0.99	0.91
ρ_w	Beta	0.50	0.20	0.97	0.97	0.97
μ_p	Beta	0.50	0.20	0.74	0.87	
μ_w	Beta	0.50	0.20	0.88	0.90	
ρ_{ga}	Normal	0.50	0.25	0.52	0.55	0.52
$\theta^{\mathbf{M}}$	Uniform	0.00	0.58			-0.37
$\sigma^{\mathbf{M}}$	Uniform	50.0	28.87			0.29
$\mathrm{corr}(\varepsilon_t^{\mathbf{M}}, \varepsilon_t^a)$	Uniform	0.00	0.58			-0.27
$\mathrm{corr}(\varepsilon_t^{\mathbf{M}}, \varepsilon_t^b)$	Uniform	0.00	0.58			-0.46
$\mathrm{corr}(\varepsilon_t^{\mathbf{M}}, \varepsilon_t^g)$	Uniform	0.00	0.58			0.02
$\mathrm{corr}(\varepsilon_t^{\mathbf{M}}, \varepsilon_t^I)$	Uniform	0.00	0.58			0.10
$\mathrm{corr}(\varepsilon_t^{\mathbf{M}}, \varepsilon_t^p)$	Uniform	0.00	0.58			0.13
$\mathrm{corr}(\varepsilon_t^{\mathbf{M}}, \varepsilon_t^w)$	Uniform	0.00	0.58			-0.13

Back

IRFs from the modified S & W model

- “y” refers to output, “c” refers to consumption, “w” refers to real wages, “lab” refers to total hours worked, “pinf” refers to inflation, “r” refers to nominal interest rates.

- All graphs are in percentage points.

- Monetary policy shock is ordered last before taking Cholesky decomposition.
    - Monetary policy reacts contemporaneously to all of the other shocks in the model, while the other shocks do not respond to monetary policy.

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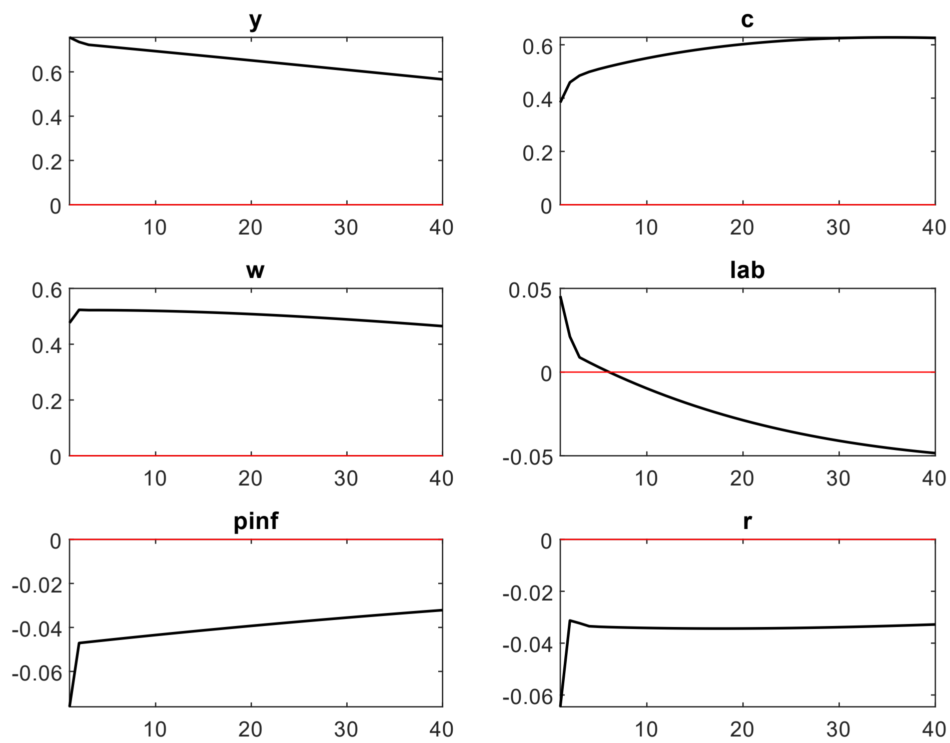
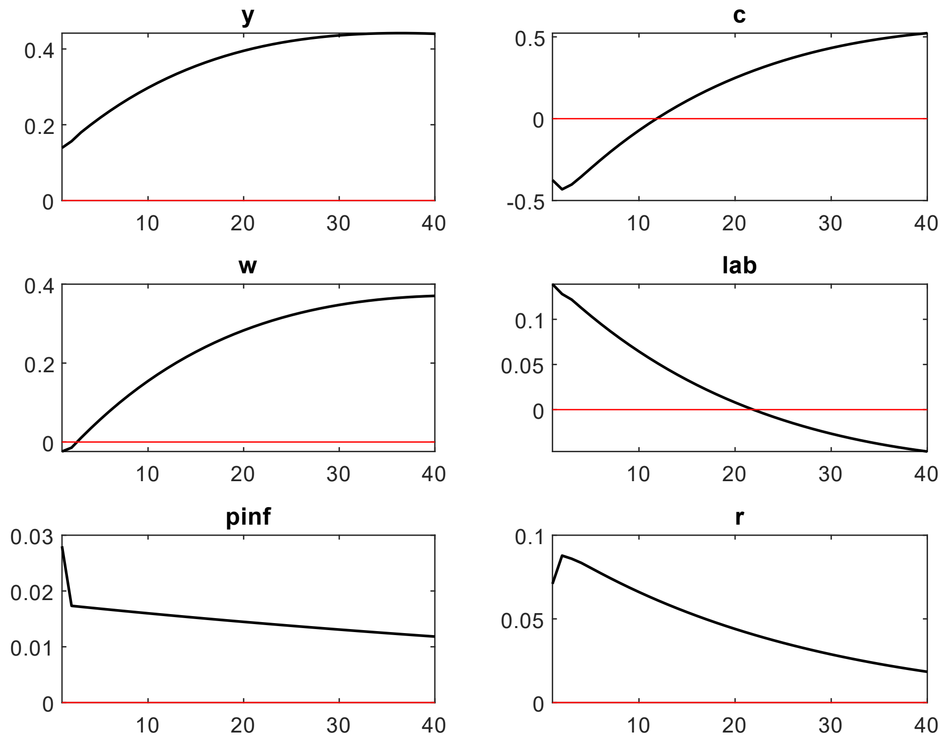
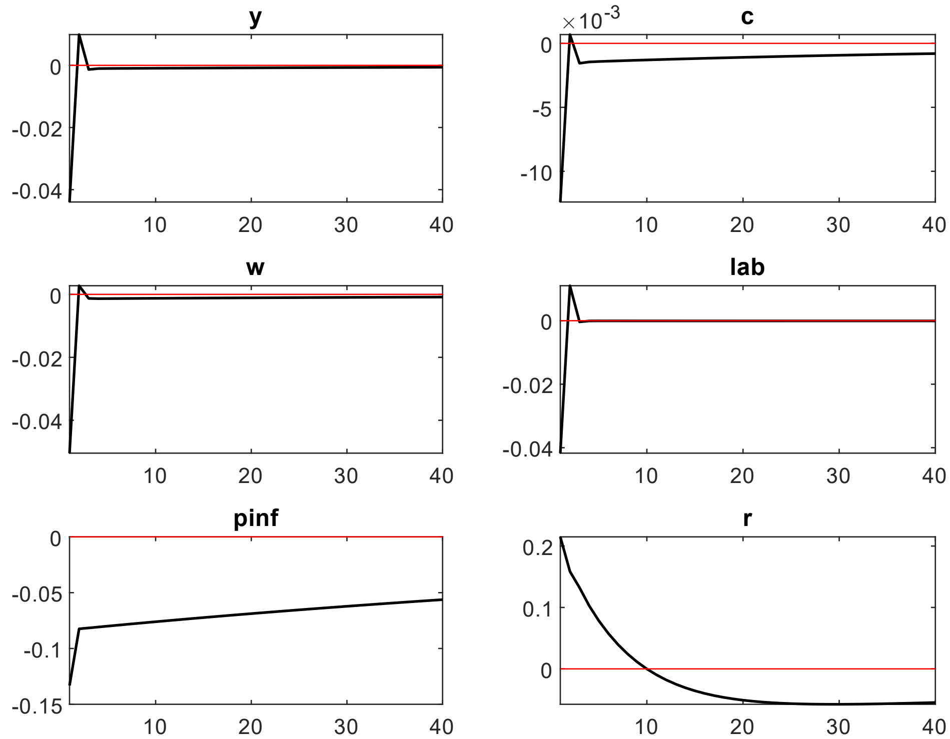


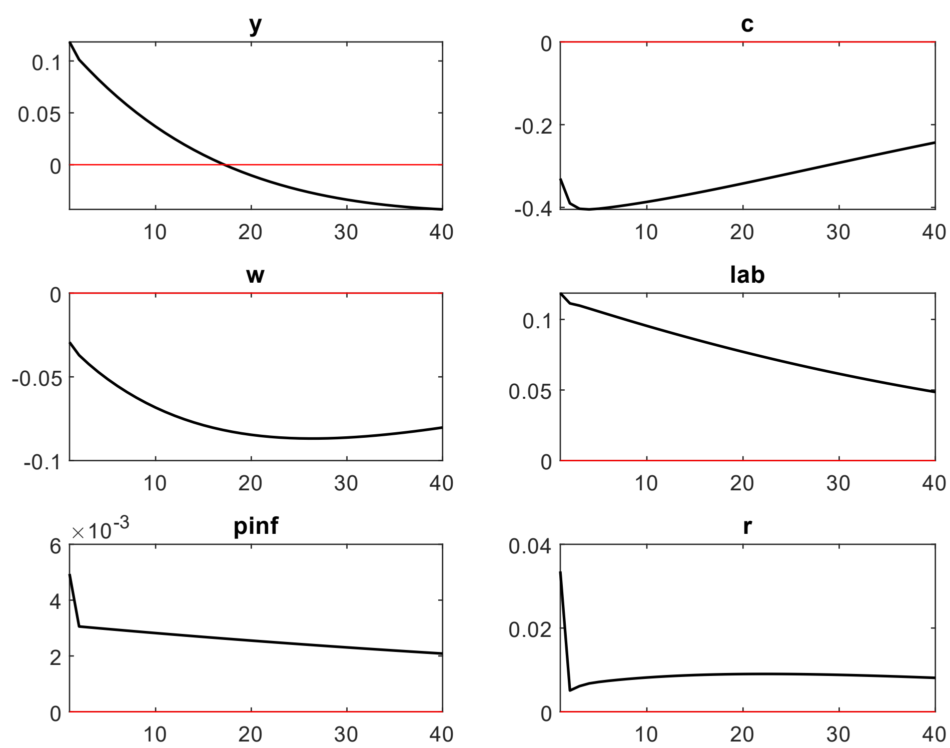
Response to a total factor productivity shock



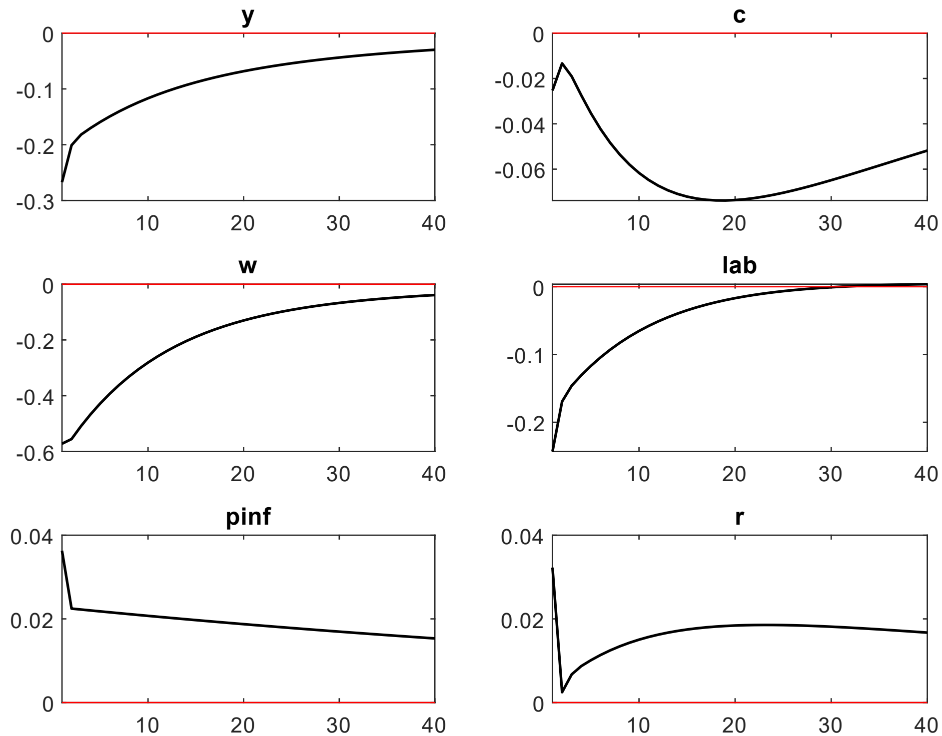
Response to an investment specific technology shock



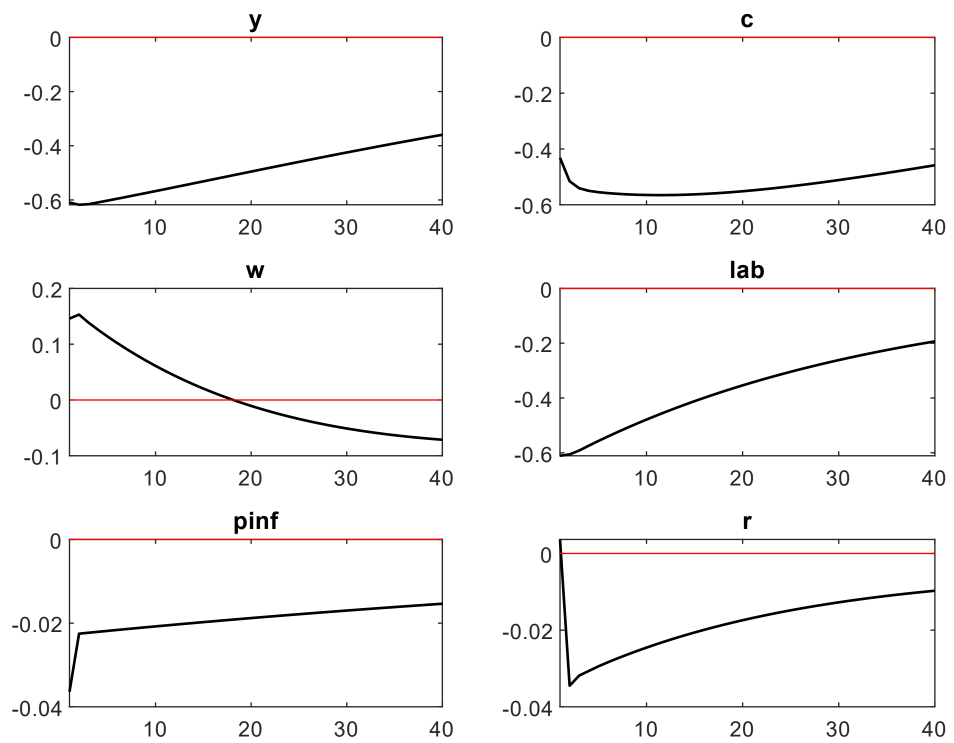
Response to a risk premium shock



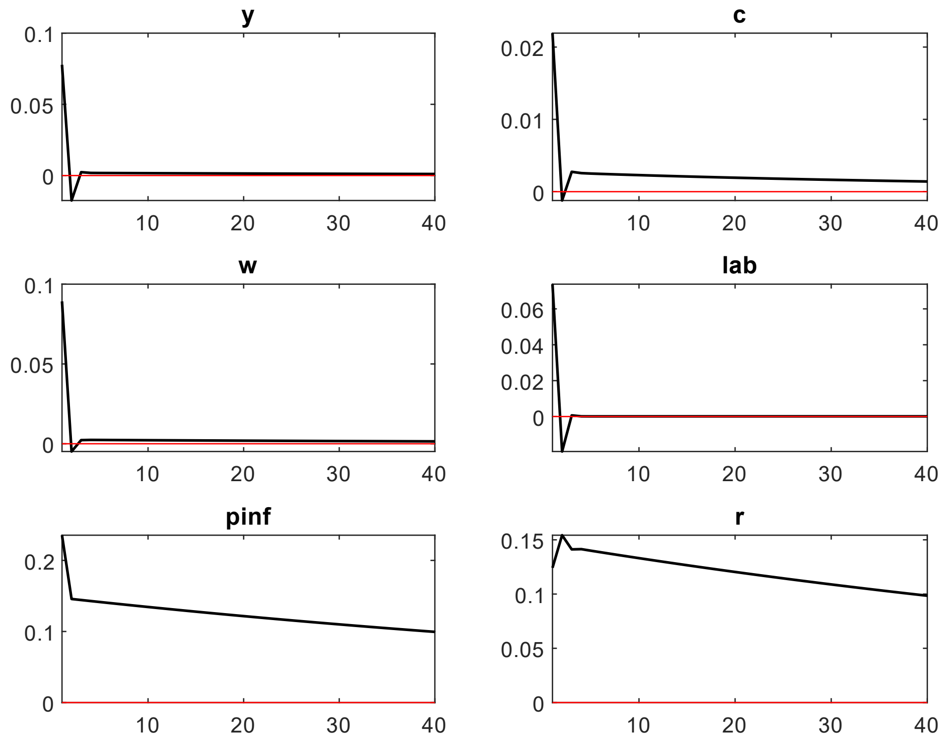
Response to an exogenous spending shock



Response to a price mark-up shock



Response to a wage mark-up shock



Response to a monetary policy shock

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