Rationing Under Sticky Prices

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Paper and slides available at https://www.tholden.org/.

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Motivation

- Stockouts are common in normal times and were particularly high during and post Covid.
  - 11% of all consumer goods in the US were out of stock in 2019, pre-Covid (Cavallo & Kryvtsov 2023).
  - 23% of consumer goods were out of stock in 2022 (January-July). My story: high inflation ⇒ high stockouts.

- Stockouts are forms of **rationing**. They are ultimately a choice of the supplier.
  - If a supplier is prepared to pay a high enough amount for production inputs, it can always increase production.

- If prices were flexible, they would have increased proportionally to the increase in MC ⇒ no stockouts!
  - But with sticky prices, firms ration demand to avoid selling below MC.

Sticky prices inevitably lead to rationing

- If a firm cannot adjust its nominal price, then its **real price will decline** over time at the rate of inflation.
- A lower real price implies **higher demand** for its good. Higher demand means **higher marginal costs**.
- Eventually, its marginal costs (rising) will be greater than its price (falling) if it continues to meet all demand.
- No firm wants to sell at a price below marginal cost. Instead, it should stop producing, **rationing demand**.
- Yet essentially all the prior sticky price literature (Calvo or menu cost) assumes that firms always meet all demand.

This paper

- What are the macroeconomic implications of allowing firms to ration?

- I allow for random rationing in a continuous time NK model with endogenous price rigidity, and find:

1. Rationing generates a convex Phillips curve.
2. Rationing massively reduces the welfare costs of positive trend inflation.
3. True output falls following “expansionary” monetary shocks (but measured output increases).

- Basic mechanism:

  High inflation $\Rightarrow$ low markups $\Rightarrow$ high rationing $\Rightarrow$ fewer varieties consumed $\Rightarrow$ downward pressure on output.

Does rationing matter in practice?

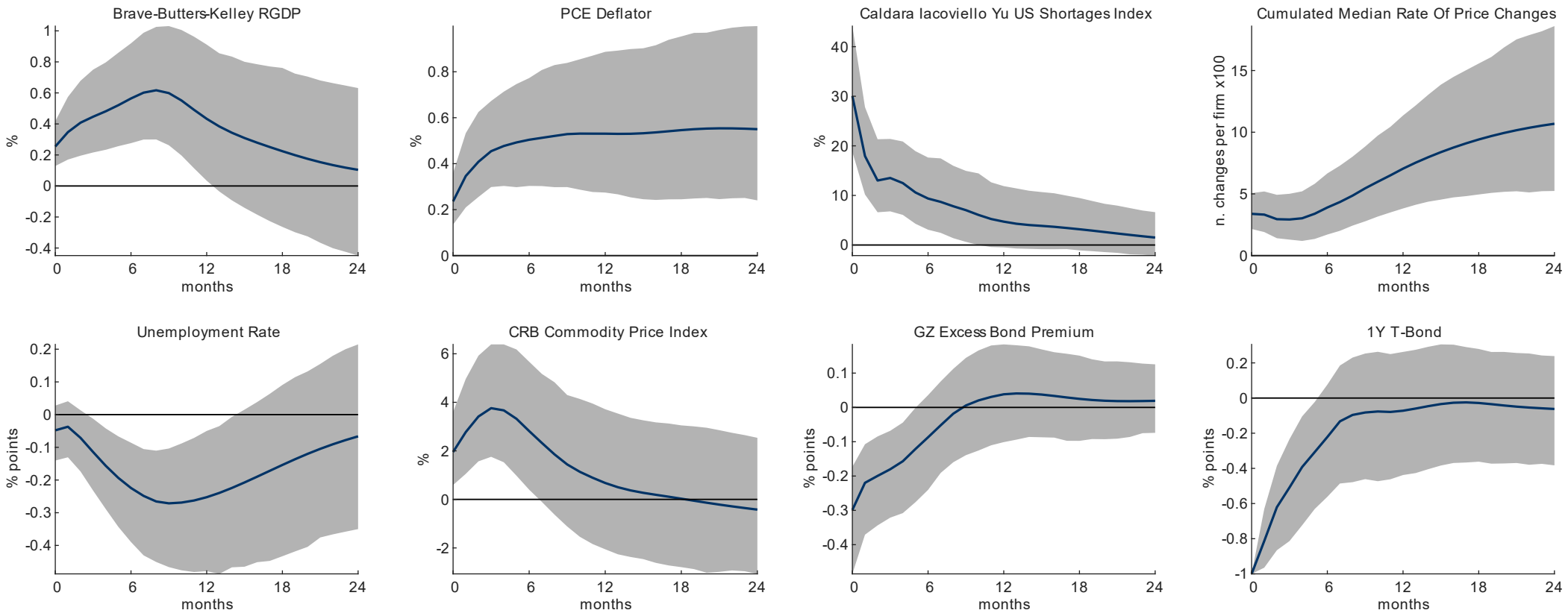
"Mark-ups are 10%, inflation is 2%, prices are updated at least once per year, real prices will not hit marginal cost."

- But: firm demand: $y \propto \left(\frac{p}{P}\right)^{-\epsilon}$ ($\epsilon \approx 10$) and marginal costs: $mc \propto y^{\frac{\alpha}{1-\alpha}}$ ($\alpha \approx \frac{3}{5}$), so $mc \propto \left(\frac{p}{P}\right)^{-\epsilon\frac{\alpha}{1-\alpha}} \approx \left(\frac{p}{P}\right)^{-15}$.
  - In the short run, some labour and intermediate inputs are fixed ($\approx \frac{2}{5}$ at annual freq. (Abraham et al. 2024)) $\Rightarrow \alpha \approx \frac{3}{5}$.
- So: A 2% fall in real prices increases marginal costs by 30%. Good-bye mark-ups! Hello rationing!
- This calculation understates firms' reasons to ration:
  - Firms face high frequency demand fluctuations. Mark-ups are much lower at times of high demand.
  - Inflation can be much higher than 2%. It was around 7% post-Covid!

Empirical evidence for rationing

- **1** (Not today)**:** Increases in price adjustment rates are followed by reductions in stockouts.
- **2** (Not today)**:** Quantities sold are concave in price age, in line with goods with old prices being rationed.
- **3:** Rationing helps match the convexity of the Phillips curve (Forbes, Gagnon & Collins 2022).
- **4:** Expansionary monetary shocks increase a measure of rationing and other shortages.
- **5:** Rationing helps match the fast response of prices to high-frequency identified monetary policy shocks.

Impulse response to an expansionary monetary shock



- Estimated following Figure 3 of Miranda-Agrippino & Ricco (2021) (95% bands) but:
  - with Brave-Butters-Kelley RGDP (Brave, Cole & Kelley 2019; Brave, Butters & Kelley 2019) not IP, with PCEPI not CPI,
  - with the Caldara-Iacoviello-Yu US Shortages Index (Caldara, Iacoviello & Yu 2025),
  - and with the cumulation of the median rate of price changes, excluding sales (Montag & Villar 2025).

The model

Setup

- The model is in continuous time, with no aggregate uncertainty, just MIT shocks.
- Assume firm price change opportunities arrive at rate $\lambda_t > 0$.
- The time t density of firms that last updated at τ is $\lambda_\tau e^{-\int_\tau^t \lambda_v \, \mathrm{d}v}$.
- Index firms/products by the time they last updated their price, τ, and by their demand shock ζ. Firm output: $y_{\zeta,\tau,t}$.
- $g(\zeta)$ is the PDF of the demand shock, which is independent across time and firms.
- For tractability, I assume $g(\zeta) = \theta\zeta^{\theta-1}$ where $\theta > 0$ (so $\zeta \sim$ Beta$(\theta, 1)$). Mean $\approx 1 - \frac{1}{\theta}$. Std. dev. $\approx \frac{1}{\theta}$.

Consumer rationing and aggregation

- $\psi \in [0,1]$ denotes a purchaser-good-time-specific shock controlling if a given purchaser can buy a given good.

- $y_{\psi,\zeta,\tau,t}$: sales to buyer with shock ψ, at t, of good produced by firm that updated price at τ, with demand shock ζ.

- ψ is uniform. $y_{\psi,\zeta,\tau,t} = \begin{cases} y^*_{\zeta,\tau,t}, & \psi \leq \bar{\psi}_{\zeta,\tau,t} \\ 0, & \psi > \bar{\psi}_{\zeta,\tau,t} \end{cases}$. $y^*_{\zeta,\tau,t}$ sales when not rationed.

- $\bar{\psi}_{\zeta,\tau,t}$ is the probability a consumer is not rationed at the firm. Adjusts to ensure: $y_{\zeta,\tau,t} = \int_0^1 y_{\psi,\zeta,\tau,t}\,\mathrm{d}\psi = y^*_{\zeta,\tau,t}\bar{\psi}_{\zeta,\tau,t}$.

- The aggregate good is produced from intermediates by a perfectly competitive industry with technology:

$$Y_t = D^{-\frac{\epsilon}{\epsilon-1}}\left[\int_{-\infty}^{t} \lambda_\tau e^{-\int_\tau^t \lambda_v\,\mathrm{d}v} \int_0^1 \zeta g(\zeta) \int_0^1 y_{\psi,\zeta,\tau,t}^{\frac{\epsilon-1}{\epsilon}}\,\mathrm{d}\psi\,\mathrm{d}\zeta\,\mathrm{d}\tau\right]^{\frac{\epsilon}{\epsilon-1}}, \qquad D = \frac{\theta}{\theta+1}$$

Firm production and rationing

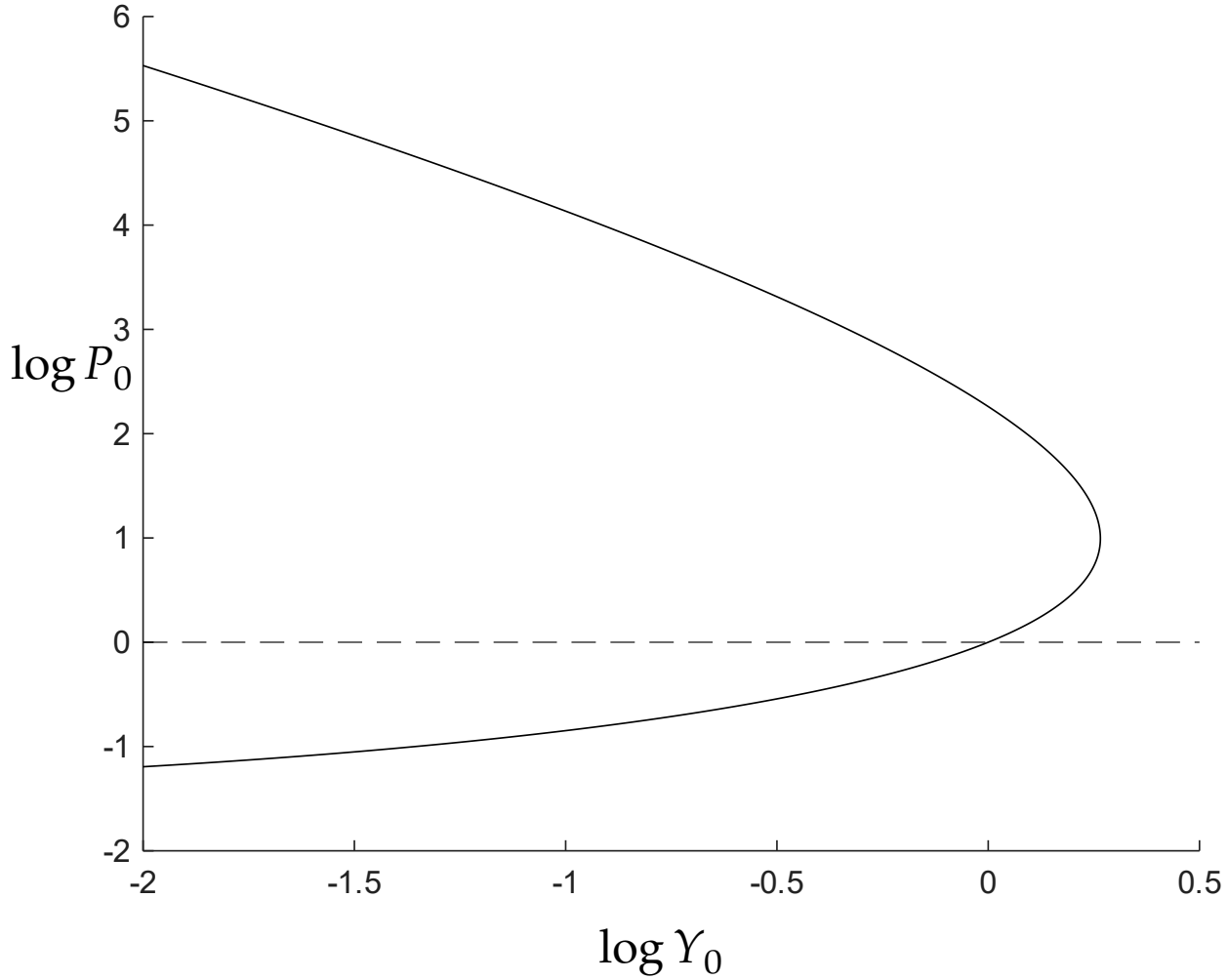
- The FOC of the aggregators imply demand must satisfy: $y_{\zeta,\tau,t} \leq y^*_{\zeta,\tau,t} := \left(\frac{D}{\zeta}\frac{p_\tau}{P_t}\right)^{-\epsilon} Y_t$.

- Firms produce using the production technology: $y_{\zeta,\tau,t} = \left(A_t l_{\zeta,\tau,t}\right)^{1-\alpha}$.

- Firm flow real production profits are guaranteed to be positive for small enough $l_{\zeta,\tau,t}$.

- Optimal production: There is a quantity $\bar{\zeta}_{\tau,t} > 0$ such that:
  - If $\zeta < \bar{\zeta}_{\tau,t}$, there is no rationing, and the firm sells on the demand curve.
  - If $\zeta > \bar{\zeta}_{\tau,t}$, there is rationing, and the firm sells up to the point at which P=MC.
  - $\bar{\psi}_{\zeta,\tau,t} = \min\left\{1, \left(\frac{\bar{\zeta}_{\tau,t}}{\zeta}\right)^{\epsilon}\right\}$. High $\bar{\zeta}_{\tau,t}$, less likely to be rationed.

Plotting the short-run Phillips curve (true price/output)



- Assume: $P_t = \exp(\pi t)$ for $t < 0$.
- And: $P_t = P_0 \exp(\pi t)$ for $t \geq 0$.
- So, prices jump at time 0.
- Graphs plot possible (Y_0, P_0) (%, relative to s.s.).

- Solid line is short-run PC allowing rationing.
- Dashed line is short-run PC without rationing.

- Independent of price setting! From aggregation.

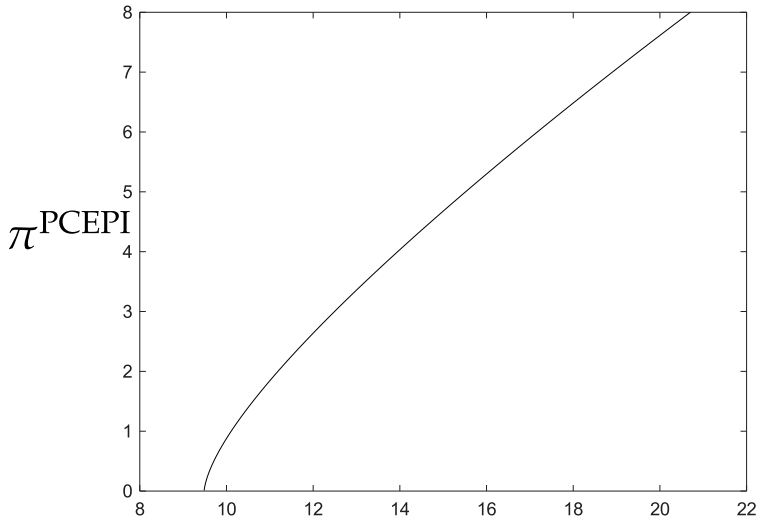
- Calibration will be given shortly.

Other model features

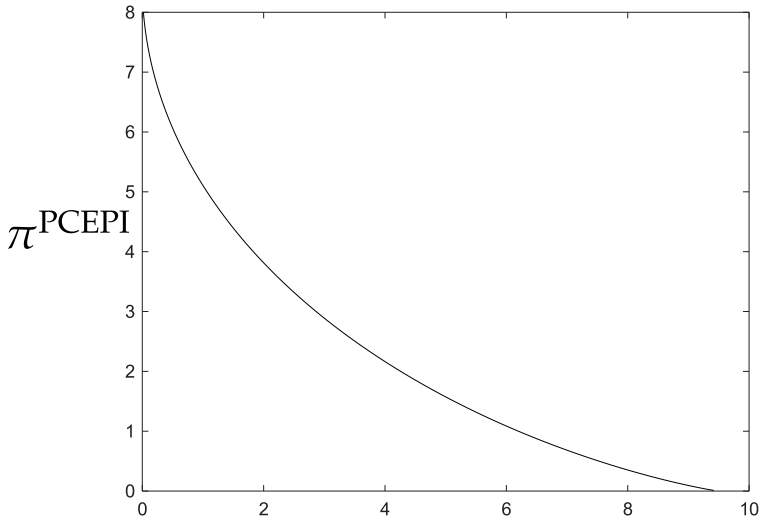
- Price change arrival rate λ_t is endogenized broadly following Blanco et al. (2024).
  - Gives aggregate state dependence, ensuring comparative statics are reasonable. Three params to match three facts.
  - Calibration $\Rightarrow$ With rationing: 0.1% of labour is used for price adjustment. Without: 2.0% of labour is used for price adjustment.
- Households + parameterization pretty standard: $\rho = 2\%$. $\pi^{\text{PCEPI}} := 2\%$. $\epsilon := 10$, $\nu := 2$. $\alpha := \frac{3}{5}$ (Abraham et al. 2024).
- $\theta := 27$. Matching 11% stockouts in 2019 from Cavallo & Kryvtsov (2023) to $1 - \bar{\psi}$. ζ mean 0.96. ζ s.d. 0.03.
- Monetary policy ensures that π_t always hits an exogenous target π_t^*. Look at monetary shocks = shocks to π_t^*.

Results

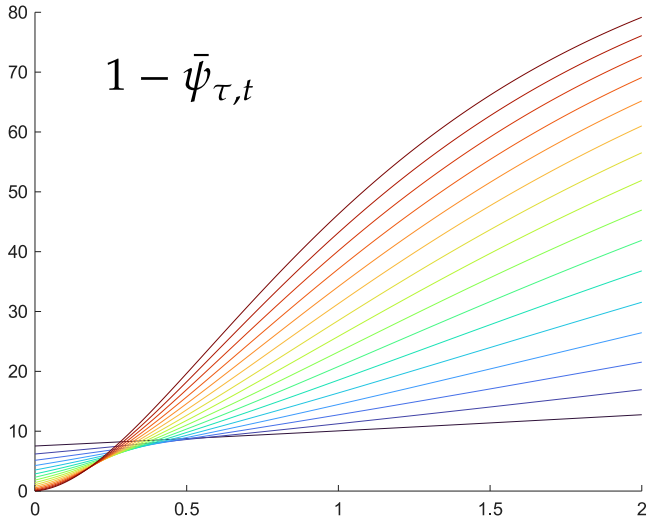
Stockouts and rationing as a function of inflation



Average stockout level, $1 - \bar{\psi}$ (percent).



Stockout rate at firms with new prices, $1 - \bar{\psi}_{t,t}$ (percent).



Stockout levels (percent) as a function of price age (years), with varying steady state inflation levels.

Dark blue corresponds to 0.5% inflation.

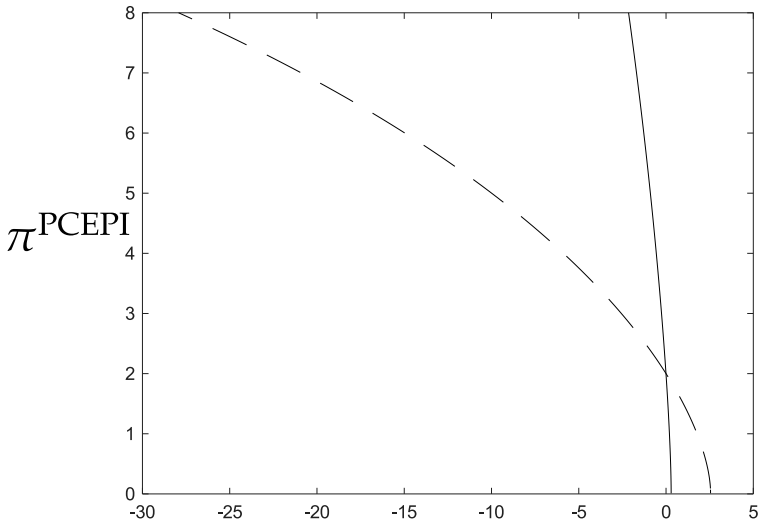
Dark red corresponds to 8% inflation.

Figure 6: Stockouts and rationing as a function of PCEPI inflation (percent).

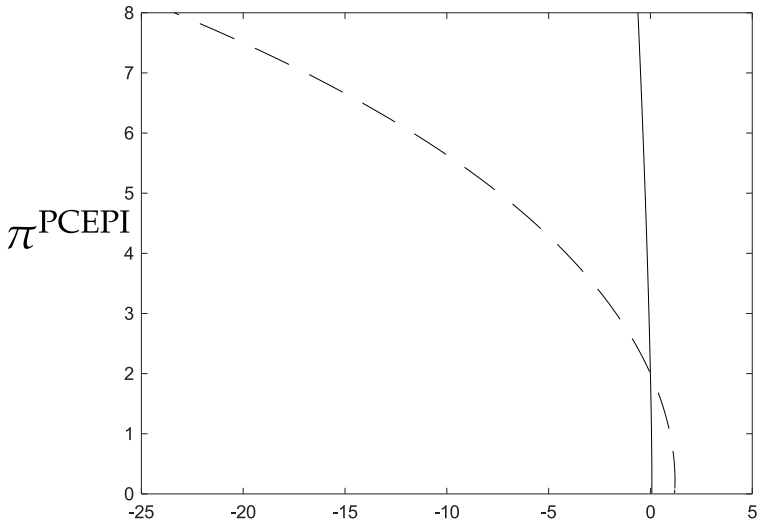
When inflation is high rationing is high. High inflation quickly erodes mark-ups.

Firms with new prices ration less with high inflation as they set high initial mark-ups.

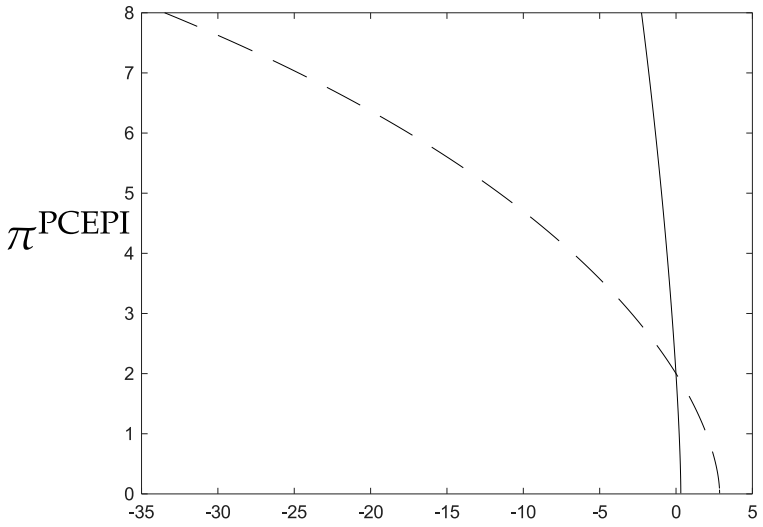
Output and welfare as a function of inflation



Relative output: $\log Y$ (percent).



Relative production labour supply: $\log L$ (percent).



Relative welfare:

$$100\left(\log Y - \Psi\frac{1}{1+\nu}\left(L + \frac{\kappa_1}{1+\kappa_2}(\lambda - \underline{\lambda})^{1+\kappa_2}\right)^{1+\nu}\right)$$

Figure 7: Output and welfare as a function of inflation (percent).

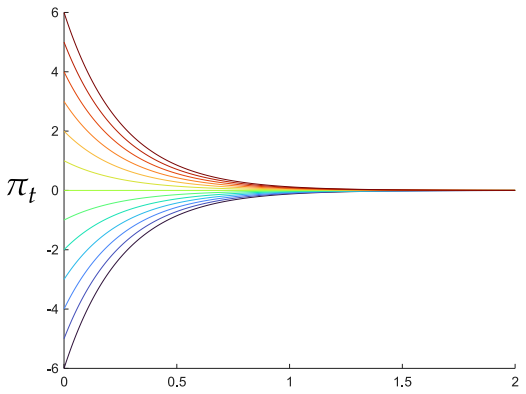
Black solid lines are the model with rationing. Black dashed lines are the model without rationing.

All plots are normalized to hit 0% on the horizontal axis when $\pi^{\text{PCEPI}} = 2\%$.

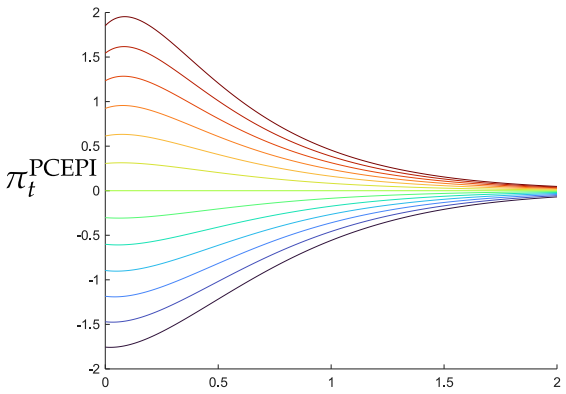
Welfare costs of inflation are much lower under rationing.

Firms with old prices are no longer making losses on every unit sold. Less labour is used in price adjustment.

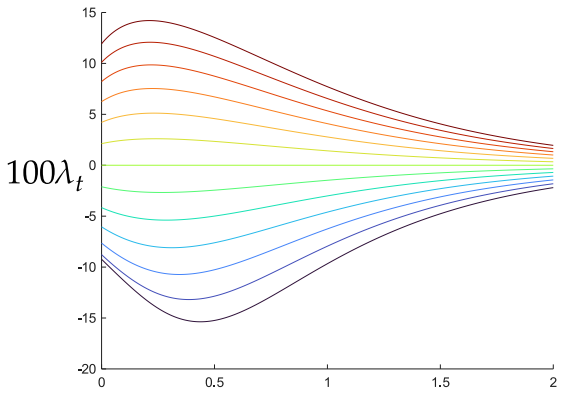
IRFs to π^* shocks (persistence matched to VAR)



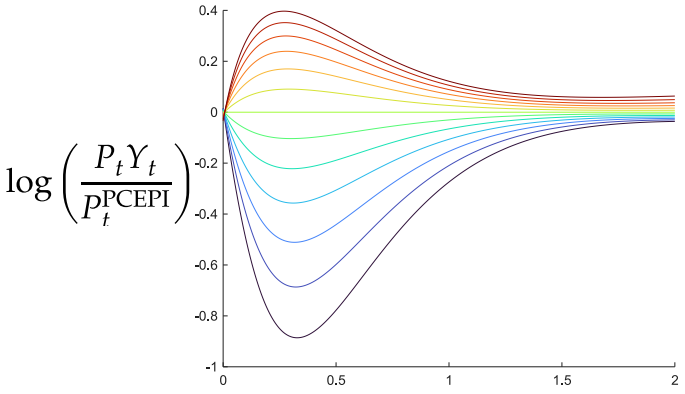
Driving inflation shocks (percent) as a function of time (years).



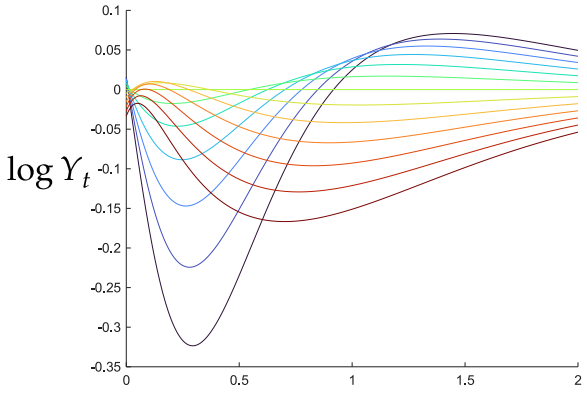
PCEPI inflation (percent) as a function of time (years).



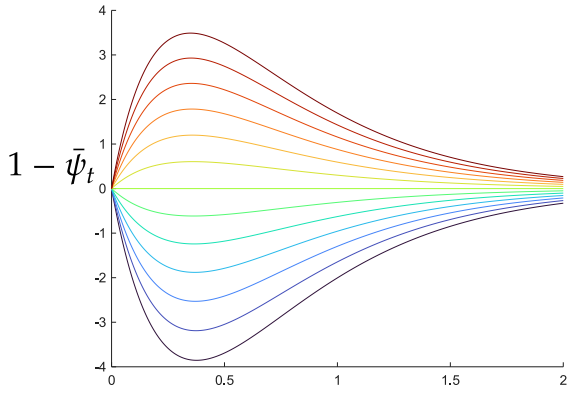
Price adjustment rate (× 100) as a function of time (years).



Measured real GDP response (percent) as a function of time (years).



True output response (percent) as a function of time (years).

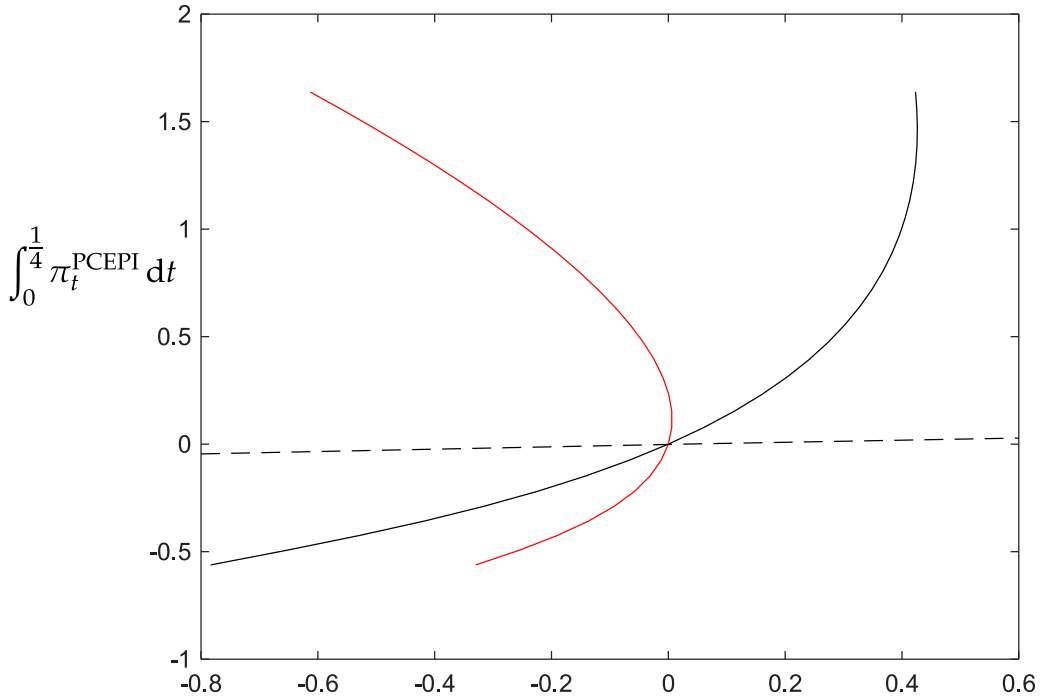


Stockout share response (percentage points) as a function of time (years).

Figure 10: Impulse responses to monetary shocks, with rationing.

Colours are consistent across subplots. All responses are relative to the no-shock counterfactual.

The three-month Phillips curve



Black solid line: measured cumulated real GDP, $\log \int_0^{\frac{1}{4}} \frac{P_t Y_t}{P_t^{\text{PCEPI}}} \mathrm{d}t$, with rationing.

Red solid line: cumulated true output, $\log \int_0^{\frac{1}{4}} Y_t \, \mathrm{d}t$, with rationing.

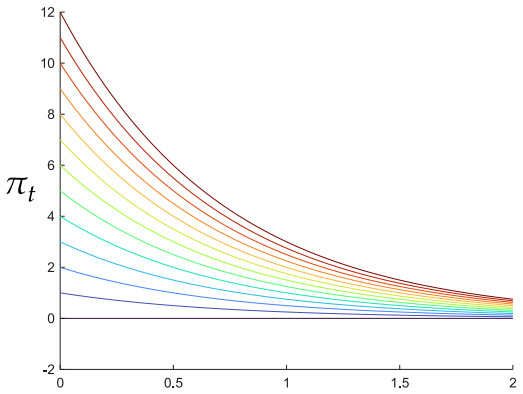
Black dashed line: measured cumulated real GDP, $\log \int_0^{\frac{1}{4}} \frac{P_t Y_t}{P_t^{\text{PCEPI}}} \mathrm{d}t$, without rationing.

- Black line (measured output with rationing) matches the PC slope of 1.2 derived from applying the same calculations to the VAR IRFs I showed previously.
- Note that expansionary monetary policy reduces true output but increases measured output.
- The PCEPI deflator does not capture gains from variety, so it misses the losses coming from rationing reducing the measure of consumed varieties.

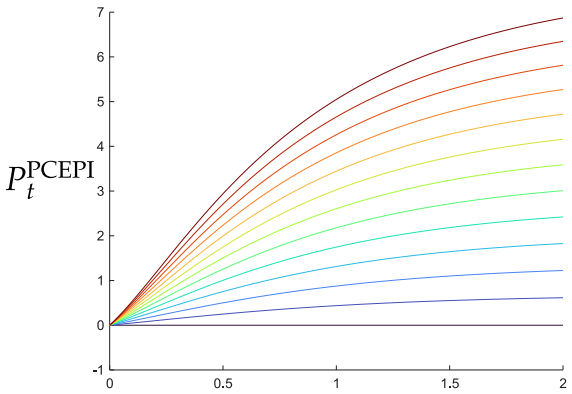
Figure 9: The three-month Phillips curve with (solid lines) and without (dashed lines) rationing.

All variables in percent. All variables are relative to the no-shock counterfactual.

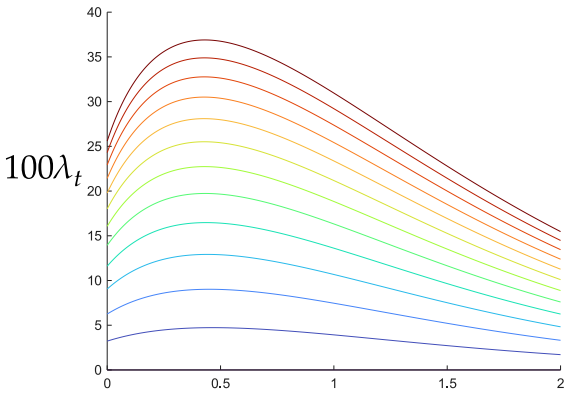
IRFs to π^* shocks (matching 7% post-Covid inflation)



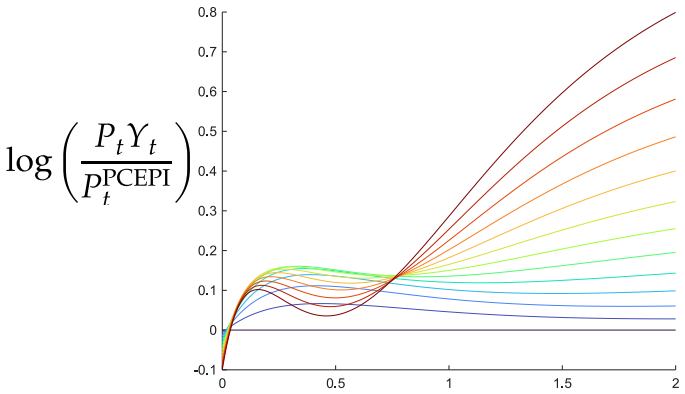
Driving inflation shocks (percent) as a function of time (years).



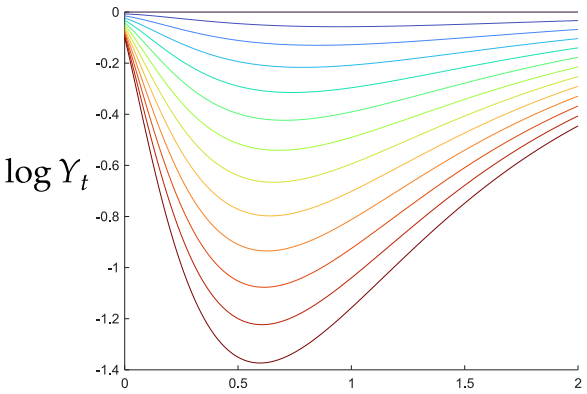
PCEPI **level** (percent) as a function of time (years).



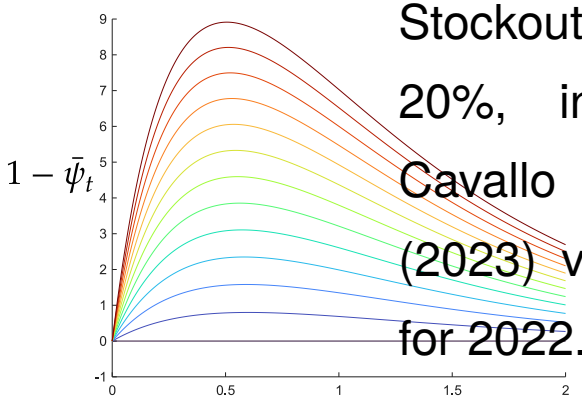
Price adjustment rate (× 100) as a function of time (years).



Measured real GDP response (percent) as a function of time (years).



True output response (percent) as a function of time (years).



Stockout share response (percentage points) as a function of time (years).

Stockout rates hit 20%, in line with Cavallo & Kryvtsov (2023) value of 23% for 2022.

Figure 11: Impulse responses to more persistent monetary shocks, with rationing.

Colours are consistent across subplots. All responses are relative to the no-shock counterfactual.

Conclusion

- The standard assumption that firms always satisfy all demand is **not innocuous**.
- Allowing rationing produces a model that **fits the data** better and performs more reasonably in extreme conditions.
- Allowing rationing drastically **reduces the welfare costs** of positive trend inflation.
- But when rationing is allowed, **welfare relevant output declines** following “expansionary” monetary shocks.
- Monetary policy may be **less useful for stabilisation** than we previously thought.

Extra slides

Price adjustment

- Why don't firms just change prices, rather than rationing?
  - Firms can benefit from tolerating P<MC if they anticipate lower MC (mean reversion?) or lower menu costs in future.
  - Modern menu cost models rely heavily on random menu costs and/or free price change opportunities.
  - By revealed preference, firms that can ration make higher profits than firms that cannot. Under rationing, profits always >0.
  - Since profits are higher when rationing is allowed, lower menu (etc.) costs are needed to justify the observed price stickiness.

- I take a tractable approach to state-dependent pricing broadly following Blanco et al. (2024).
  - Firms are owned by conglomerates. Conglomerates choose the rate of price adjustment, not which firms adjust.
  - Provides aggregate state dependence.
  - Matches flat adjustment hazard rate found by Klenow & Kryvtsov (2008), Nakamura & Steinsson (2008), Klenow & Malin (2010).

Prior literature

- Early:
  - Barro & Grossman (1971), Drèze (1975), Svensson (1984), Corsetti & Pesenti (2005) (restrict shocks to ensure no rationing).
- NK rationing models (all with sales-capped rationing):
  - Under sticky wages: Huo & Ríos-Rull (2020), Gerke et al. (2023): Infinite dimensional state, numerical.
  - Under sticky prices: Hahn (2022): Only steady state results. No dynamics. No idiosyncratic shocks.
- Alternatives capturing some of the effects of rationing:
  - Michaillat & Saez (2015), Liu (2025): search and matching, smoothing out the Barro & Grossman (1971) model.
  - Kharroubi and Smets (2024): shutdown if profits negative at demanded quantity.
- Stockouts in inventory models:
  - Alessandria, Kaboski & Midrigan (2010), Kryvtsov & Midrigan (2013), Bils (2016).
  - In these papers, firms always meet demand if they have stock available, even if marginal value of that stock > price.

Price-adjustment rates and stockouts

- Data from replication package of Cavallo & Kryvtsov (2023).
- Specification:

$$y_{i,j,t} = \beta x_{i,j,t-1} + \alpha_{i,j} + \gamma_{i,t} + \delta_{j,t} + \varepsilon_{i,j,t}$$

  - i indexes 7 countries (Canada, China, France, Germany, Japan, Spain, USA).
  - j indexes 37 product categories (3-digit). t indexes 11 months in 2019.

  - $y_{i,j,t}$ is the **average stockout rate** (for a country, category, month), in percent.
  - $x_{i,j,t}$ is $100 \times$ the **average annualized rate of price adjustment** (for a country, category, month).
  - Standard errors 3-way clustered. Clusters indexed by (i,j), (i,t), (j,t).

- Result: $\hat{\beta} = -0.155$, std. err. 0.044, p-value < 0.001. Increased price flexibility reduces stockouts.

Average output over the life of a price: Setup

- Data: Dominick's Finer Foods (1989-1994).
- 21,474,126 observations after dropping for each product and store:
  - First/last price, any price $\neq$ cumulative max price, one week after each price change/missing, any price older than four years, one observation due to differencing.
- Specification, estimated via FGLS:

$$\frac{y_{i,j,k,l,t} - y_{i,j,k,l,t-1}}{\bar{y}_{i,j,k,l}} = \beta_{A(i,j,k,l,t)} + \alpha_{i,j,k} + \gamma_{i,j,t} + \delta_{i,k,t} + \sigma^{(1)}_{i,j,A(i,j,k,l,t)} \sigma^{(2)}_{i,j,k} \sigma^{(3)}_{i,j,t} \sigma^{(4)}_{i,k,t} \varepsilon_{i,j,t}$$

i indexes narrow categories (86) k indexes products (10,166) t indexes weeks (398)

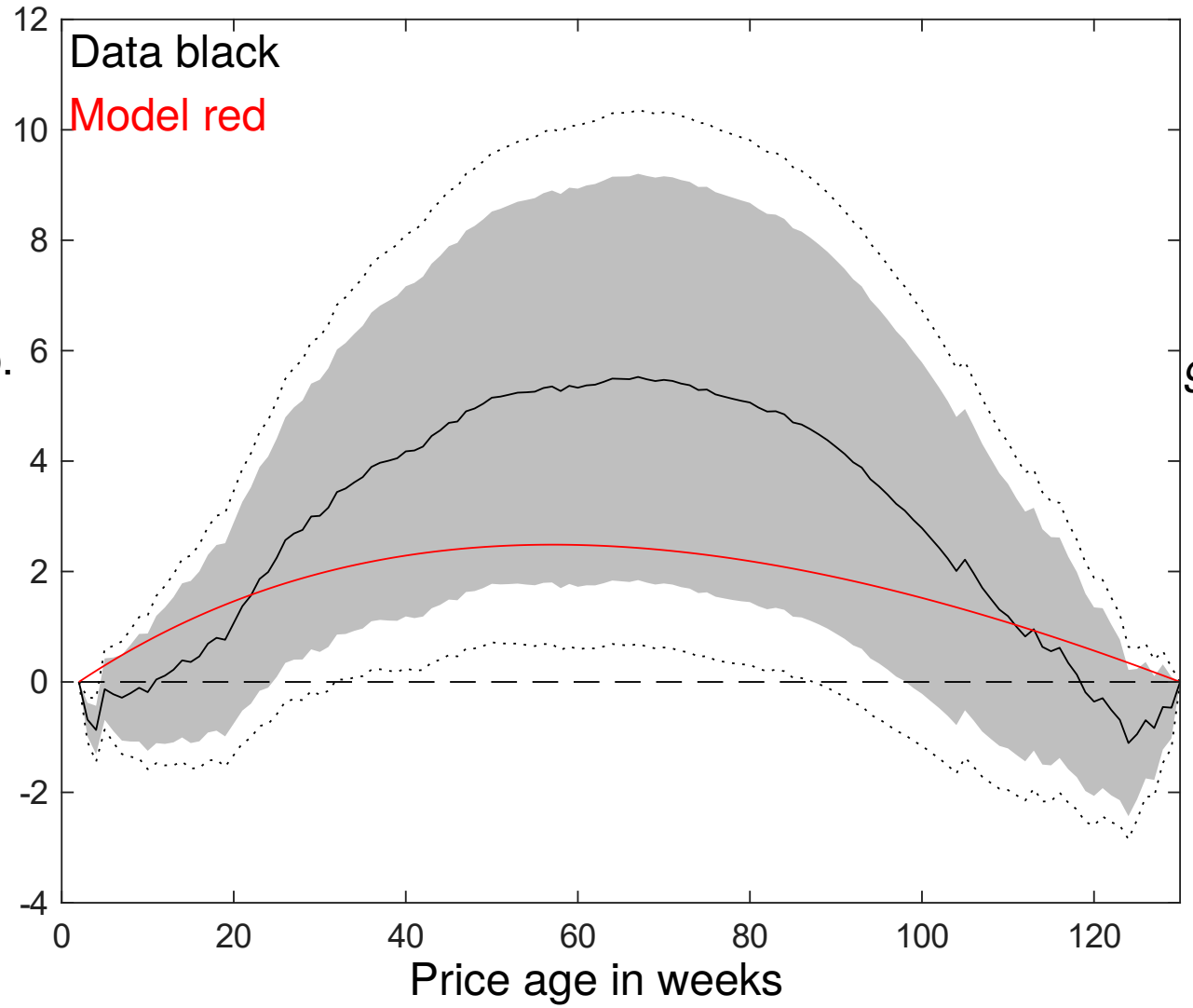
j indexes stores (93) l indexes prices (947,660)

  - $A(i,j,k,l,t)$ is the "age" of the i,j,k,l price at t. $y_{i,j,k,l,t}$ gives units sold. $\bar{y}_{i,j,k,l}$ is average of $y_{i,j,k,l,t}$ over the life of the price.
  - Standard errors 4-way clustered. Clusters indexed by: $\big(i,j,A(i,j,k,l,t)\big)$, (i,j,k), (i,j,t), (i,k,t).

Average output over the life of a price: Results

Relative output by age p.p.

$$100 \sum_{a=3}^{\text{AGE}} \beta_a$$



Normalized to 0 at 2 & 130.
95% confidence band shaded.
99% confidence band dotted.

Other aggregates

- Average probability that a buyer from a particular firm receives order: $\bar{\psi}_{\tau,t} := \int_0^1 \bar{\psi}_{\zeta,\tau,t} g(\zeta)\, d\zeta = \frac{\theta \bar{\zeta}_{\tau,t}^{\epsilon} - \epsilon \bar{\zeta}_{\tau,t}^{\theta}}{\theta - \epsilon}$.

- Average probability of receiving order across all firms: $\bar{\psi}_t := \int_{-\infty}^{t} \lambda_\tau e^{-\int_\tau^t \lambda_v \,\mathrm{d}v} \bar{\psi}_{\tau,t} \,\mathrm{d}\tau$.
  - Equal weighted for comparability with Cavallo & Kryvtsov (2023) evidence.

- Model PCEPI, following BLS imputation procedure (unobserved price changes assumed equal average change):
  - $\frac{1}{\Delta}\left(\log P_t^{\text{PCEPI}} - \log P_{t-\Delta}^{\text{PCEPI}}\right) = \frac{1}{\Delta}\int_{-\infty}^{t-\Delta} \lambda_\tau e^{-\int_\tau^{t-\Delta} \lambda_v \,\mathrm{d}v} \left[\bar{\psi}_{\tau,t-\Delta}\lambda_t \Delta \left[\bar{\psi}_{t,t} \log \frac{p_t}{p_\tau} + (1-\bar{\psi}_{t,t}) \log \frac{P_t^{\text{PCEPI}}}{P_{t-\Delta}^{\text{PCEPI}}}\right] + \bar{\psi}_{\tau,t-\Delta}(1-\lambda_t\Delta)\left[\bar{\psi}_{\tau,t} 0 + (1 - \bar{\psi}_{\tau,t}) \log \frac{P_t^{\text{PCEPI}}}{P_{t-\Delta}^{\text{PCEPI}}}\right] + (1-\bar{\psi}_{\tau,t-\Delta}) \log \frac{P_t^{\text{PCEPI}}}{P_{t-\Delta}^{\text{PCEPI}}}\right] \mathrm{d}\tau$
  - So: $\pi_t^{\text{PCEPI}} := \frac{\mathrm{d}\log P_t^{\text{PCEPI}}}{\mathrm{d}t} = \lambda_t \bar{\psi}_{t,t} \frac{\int_{-\infty}^{t} \lambda_\tau e^{-\int_\tau^t \lambda_v \,\mathrm{d}v} \bar{\psi}_{\tau,t}^2 \frac{1}{\bar{\psi}_{\tau,t}} \log \frac{p_t}{p_\tau} \,\mathrm{d}\tau}{\int_{-\infty}^{t} \lambda_\tau e^{-\int_\tau^t \lambda_v \,\mathrm{d}v} \bar{\psi}_{\tau,t}^2 \,\mathrm{d}\tau}$.

Random rationing vs sales-capped rationing

- Two potential models of rationing:
  - Sales-capped rationing: The firm caps the quantity it sells to any given consumer.
  - Random rationing: Some consumers get lucky and buy their entire order. Others go home with nothing.

- While we saw some sales caps during Covid, random rationing seems more natural.
  - It also fits the aggregate data better. (Sales-capped rationing generates an excessively steep Phillips curve.)
  - While with tight consumer storage constraints, if goods are semi-durable and consumers shop frequently, the result of random rationing can look a lot like quantity-capped rationing, without such constraints the result is random rationing again.

- With random rationing, changes in rationing change measure of varieties consumed. Consumers love variety!

State variables and the short-run Phillips curve

- All of the model's state variables are of the form: $X_{j,t} := \int_{-\infty}^{t} \lambda_\tau e^{-\int_\tau^t \lambda_v dv} p_\tau^{\chi_j} d\tau$, so: $\dot{X}_{j,t} = \lambda_t \left[p_t^{\chi_j} - X_{j,t} \right]$ (for $j \in \mathbb{Z}$).

- The definition of total labour demand is: $L_t := \int_{-\infty}^{t} \lambda_\tau e^{-\int_\tau^t \lambda_v dv} \int_0^1 l_{\zeta,\tau,t}\, g(\zeta)\, \mathrm{d}\zeta\, \mathrm{d}\tau$.
- This implies an equilibrium condition relating L_t, A_t, $\widehat{W}_t$, Y_t, P_t, $X_{1,t}$ & $X_{2,t}$, with $\chi_1 := \theta + \frac{1}{\alpha} + \frac{\theta}{\epsilon}\frac{1-\alpha}{\alpha}$, $\chi_2 := \frac{1}{\alpha}$.
- The definition of aggregate output implies an equilibrium condition relating $\widehat{W}_t$, Y_t, P_t, $X_{1,t}$ & $X_{2,t}$.
- Combined with the household labour FOC, these two equations give a short-run Phillips curve, holding states fixed.

- Without rationing, the equivalent first equation relates L_t, A_t, $\widehat{W}_t$, Y_t, P_t & $X_{-1,t}$ with $\chi_{-1} := -\frac{\epsilon}{1-\alpha}$.
- And the second just relates P_t & $X_{-2,t}$ with $\chi_{-2} := -(\epsilon - 1)$.
  - Thus, if $X_{-2,t}$ is fixed, P_t is fixed. The short-run Phillips curve is horizontal in the NK model without rationing!

Instability without rationing

- I stationarize $X_{j,t}$ by defining $\widehat{X}_{j,t} := \frac{X_{j,t}}{P_t^{\chi_j}}$. And I define: $\hat{p}_t := \frac{p_t}{P_t}$. Then: $\dot{\widehat{X}}_{j,t} = \lambda_t \hat{p}_t^{\chi_j} - (\lambda_t + \chi_j \pi_t)\widehat{X}_{j,t}$.

- So: $\lambda_t + \chi_j \pi_t$ determines the stability of $\widehat{X}_{j,t}$. It is stable if $\lambda_t + \chi_j \pi_t > 0$.

- For the model with rationing, $\chi_1 = \theta + \frac{1}{\alpha} + \frac{\theta}{\epsilon}\frac{1-\alpha}{\alpha} > 0$ and $\chi_2 = \frac{1}{\alpha} > 0$. Stability guaranteed!

- For the model without rationing, $\chi_{-1} = -\frac{\epsilon}{1-\alpha} < 0$ and $\chi_{-2} = -(\epsilon - 1) < 0$.

- If ϵ, α or π_t are large enough, then $\lambda_t + \chi_{-1}\pi_t < 0$ or $\lambda_t + \chi_{-2}\pi_t < 0$. Potential instability!

New prices

- For $j \in \mathbb{N}$, define: $z_{j,\tau} := \int_\tau^\infty e^{-\int_\tau^t (\lambda_v + r_v)\,\mathrm{d}v} D^{\omega_{j,1}} \widehat{W}_t^{\omega_{j,2}} Y_t^{\omega_{j,3}} P_t^{\omega_{j,4}}\,\mathrm{d}t$, so $\dot{z}_{j,\tau} = -D^{\omega_{j,1}} \widehat{W}_t^{\omega_{j,2}} Y_\tau^{\omega_{j,3}} P_\tau^{\omega_{j,4}} + (\lambda_\tau + r_\tau) z_{j,\tau}$.

- Allowing rationing, updating firms optimally set: $p_\tau^{\theta + \frac{\theta}{\epsilon}\frac{1-\alpha}{\alpha}} \propto \frac{z_{2,\tau}}{z_{1,\tau}}$.
  - Where: $\omega_{1,1} := \theta$, $\omega_{1,2} := -\frac{\theta+\epsilon}{\epsilon}\frac{1-\alpha}{\alpha}$, $\omega_{1,3} := -\frac{\theta}{\epsilon}$, $\omega_{1,4} := -\chi_1$, $\omega_{2,1} := 0$, $\omega_{2,2} := -\frac{1-\alpha}{\alpha}$, $\omega_{2,3} := 0$, $\omega_{2,4} := -\chi_2$.

- Without rationing, updating firms optimally set: $p_\tau^{1+\epsilon\frac{\alpha}{1-\alpha}} \propto \frac{z_{-2,\tau}}{z_{-1,\tau}}$.
  - Where: $\omega_{-1,1} := 0$, $\omega_{-1,2} := -\epsilon$, $\omega_{-1,3} := 1$, $\omega_{-1,4} := -\chi_{-2}$, $\omega_{-2,1} := -\frac{\epsilon}{1-\alpha}$, $\omega_{-2,2} := 1$, $\omega_{-2,3} := \frac{1}{1-\alpha}$, $\omega_{-2,4} := -\chi_{-1}$.

- I stationarize by defining: $\hat{z}_{j,t} := \frac{z_{j,t}}{P_t^{\omega_{j,4}}}$. Again, with rationing the $\hat{z}_{j,t}$ are (backwards) stable, but not without.

Price change opportunity arrival rate choice

- Aggregate state dependence is necessary for reasonable comparative static results.
  - I broadly follow Blanco et al. (2024) in modelling an endogenous rate of price change opportunities.
- All firms are owned by conglomerates. Each conglomerate owns countably many firms.
- Each conglomerate chooses the price adjustment rate λ_t for the firms it owns (the same rate for all firms).
  - The conglomerate cannot control which particular firms update at any point in time, only the total quantity.
  - The conglomerate maximizes its firms' total profit, minus a cost of $\frac{\kappa_1}{1+\kappa_2}(\max\{0, \lambda_t - \underline{\lambda}\})^{1+\kappa_2}$ labour units.

Households and monetary policy

- In period τ, households maximize: $\int_\tau^\infty e^{-\int_\tau^t \rho_v \, \mathrm{d}v} \left[\log Y_t - \Psi_t \frac{1}{1+\nu} \left(L_t + \frac{\kappa_1}{1+\kappa_2} (\lambda_t - \underline{\lambda})^{1+\kappa_2} \right)^{1+\nu} \right] \mathrm{d}t$.

- They face the budget constraint: $Y_t + \frac{\dot{B}_t^{(i)}}{P_t} + \dot{B}_t^{(r)} = W_t \left(L_t + \frac{\kappa_1}{1+\kappa_2} (\lambda_t - \underline{\lambda})^{1+\kappa_2} \right) + i_t \frac{B_t^{(i)}}{P_t} + r_t B_t^{(r)} + \mathrm{T}_t$.

  - $B_t^{(i)}$ nominal bonds. $B_t^{(r)}$ real bonds. Y_t output = consumption, at price P_t. W_t wage. L_t labour. T_t profits from owning firms.

- FOCs imply $\Psi_t \left(L_t + \frac{\kappa_1}{1+\kappa_2} (\lambda_t - \underline{\lambda})^{1+\kappa_2} \right)^\nu = \frac{W_t}{Y_t}$, $r_t = \rho_t + \frac{\dot{Y}_t}{Y_t}$, $i_t = r_t + \pi_t$, where $\pi_t = \frac{\dot{P}_t}{P_t}$.

- Monetary policy sets $i_t = r_t + \pi_t^* + \phi(\pi_t - \pi_t^*)$ with $\phi > 1$ and π_t^* an exogenous target (Holden 2024).

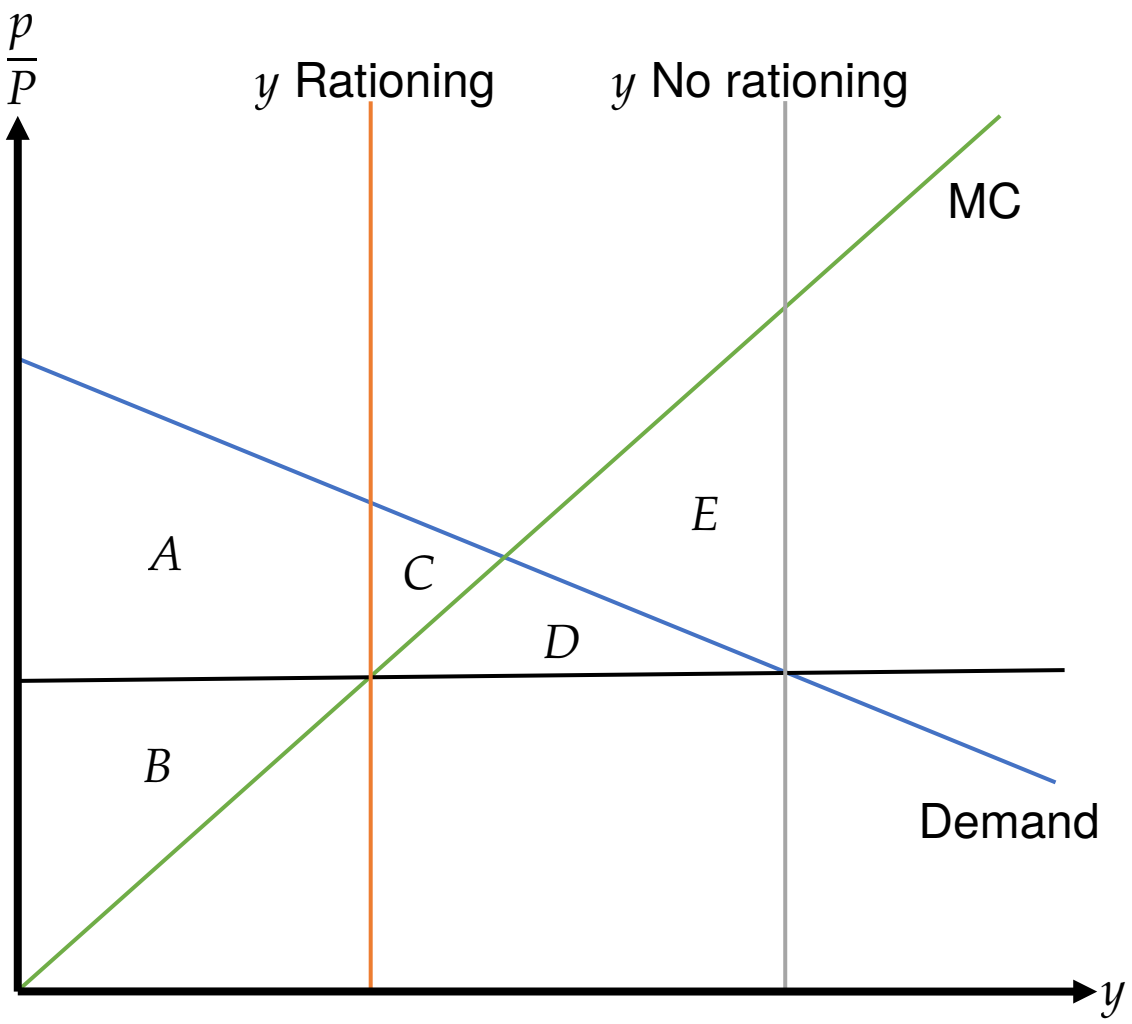
- From Fisher equation, $r_t + \pi_t = i_t = r_t + \pi_t^* + \phi(\pi_t - \pi_t^*)$, so $\pi_t = \pi_t^*$ for all t. Inflation is effectively exogenous.

Parameterization / calibration

- $\rho = 2\%$. $\pi^{\text{PCEPI}} := 2\%$ unless otherwise stated. With rationing, requires $\pi^* := 2.04\%$, without $\pi^* := 2.00\%$.
- $\epsilon := 10$, $\nu := 2$, Smets & Wouters (2007). $\alpha := \frac{3}{5}$, Abraham et al. (2024).
- $\theta := 27$. Matching 11% stockouts in 2019 from Cavallo & Kryvtsov (2023) to $1 - \bar{\psi}$. ζ mean 0.96. ζ s.d. 0.03.
- $\underline{\lambda} := 0.73$, minimum annual median price adjustment rate in the Montag & Villar (2025) data.

- With rationing, $\kappa_1 = 0.016$ and $\kappa_2 = 3.75$, without rationing $\kappa_1 = 0.105$ and $\kappa_2 = 2.06$. Imply $\lambda = 1.48$.
  - Matching time series mean of the median rate of price adjustment from the Montag & Villar (2025) data, $\lambda = 1.48$.
  - And matching $\int_0^{\frac{1}{4}} (\lambda_t - \lambda)\, \mathrm{d}t \Big/ \int_0^{\frac{1}{4}} \left(\pi_t^{\text{PCEPI}} - \pi^{\text{PCEPI}}\right) \mathrm{d}t$ following a monetary policy shock to 8.2 as estimated from Figure 1.
  - With rationing: 0.1% of labour is used for price adjustment. Without: 2.0% of labour is used for price adjustment.
  - Rationing reduces the price adjustment frictions needed to match the data!

The microeconomics of rationing vs excess production



- Without rationing: CS is $A + C + D$. PS is $B - D - E$.
- Without rationing: Welfare is $A + B + C - E$.
- With rationing: CS is A. PS is B. Welfare is $A + B$.
- Welfare is higher with rationing when $E > C$.
- Plausible as demand ($\propto y^{-\frac{1}{\epsilon}}$) is flatter than MC ($\propto y^{\frac{\alpha}{1-\alpha}}$).
- The economy with rationing should be less distorted!

The macroeconomics of rationing vs excess production

- If too much is produced by some firms (with old prices), other firms face higher marginal costs, so produce less.
- Demand is shifted from undistorted firms (with new prices) to distorted ones (with old prices).
- Bad!

- If demand is rationed for some goods (with old prices), other firms face lower marginal costs, so produce more.
- Demand is shifted from distorted firms (with old prices) to undistorted ones (with new prices).
- Good!

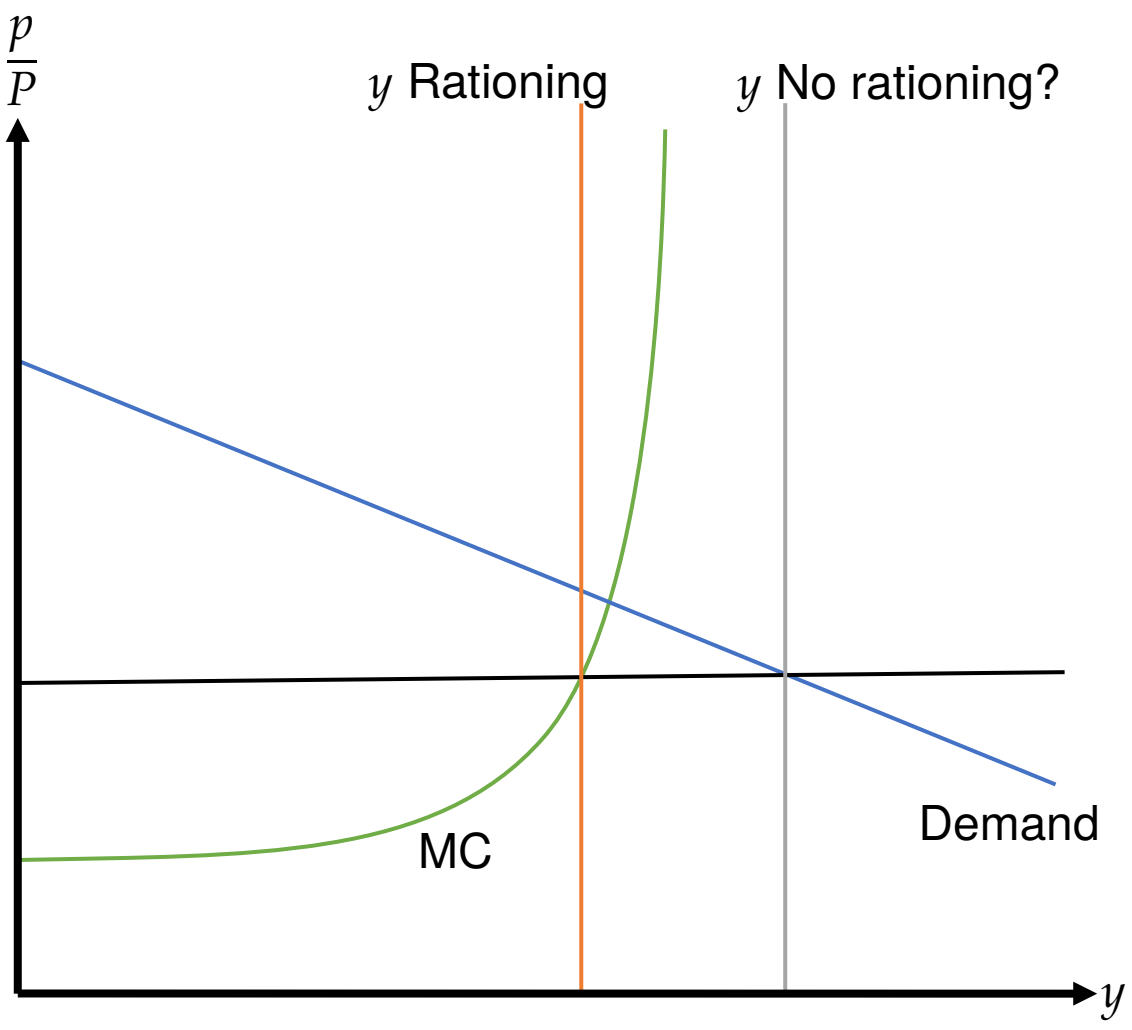
Strange properties of the Calvo model

- The Calvo model has some deeply strange properties (Holden, Marsal & Rabitsch 2024).
  - It implies a hard upper bound on steady-state inflation. With standard parameters, this is 5% to 10%.
  - Inflation above this level reduces the output *growth rate* not just the output level, due to ever growing price dispersion.
  - Under standard monetary rules, temporary high inflation can push the economy to this growing price dispersion path.

- These strange properties are tightly linked to the losses made by firms forced to sell at prices below marginal cost.

- When rationing is allowed, these strange properties disappear. Additional motivation for looking at it.

Sticky prices with near vertical marginal costs



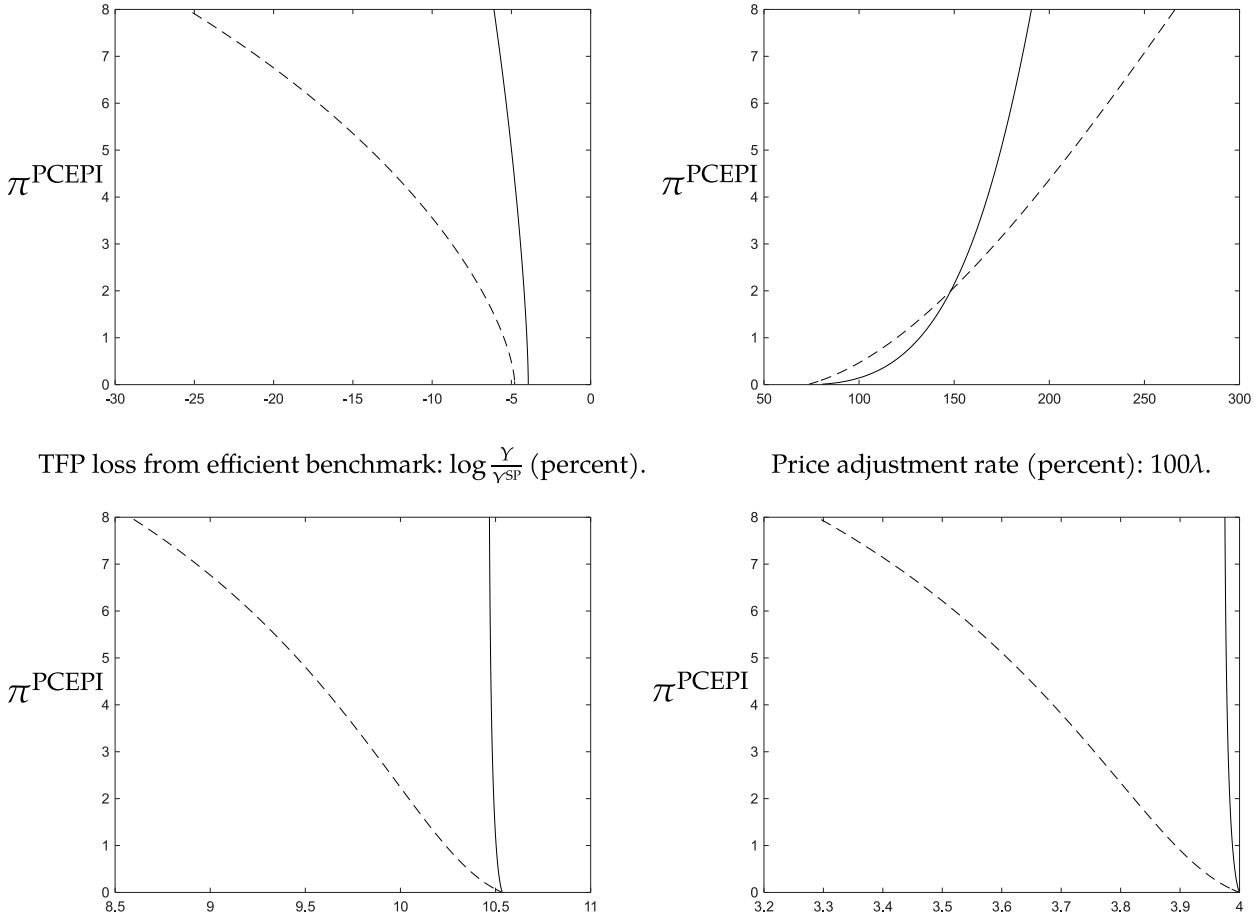
- Holding macro quantities fixed, there is no way to have equilibrium in this (micro) market without rationing.
- Total cost to produce "y No rationing" is infinite.
- So, what happens?
- As we climb the green line, an ever-increasing share of the economy's productive capacity goes to this market.
- $\Rightarrow$ Aggregate output falls.
- $\Rightarrow$ Lower demand for this good at any price.
- Macro quantities move to clear this micro market!
- Is this really plausible???
- Rationing seems more reasonable.

The quasi flexible and fully flexible price cases

- The limit as $\lambda_t \to \infty$ is not fully flexible prices, as for any λ_t, firms face all possible ζ before changing price.
- Instead, the limit is quasi flexible prices, which maximize $o_{\tau,t} := \int_0^1 o_{\zeta,\tau,t} g(\zeta)\,\mathrm{d}\zeta$.
- If $\frac{\epsilon}{\epsilon-1}\frac{\theta+\epsilon}{\theta+\frac{\epsilon}{1-\alpha}} \leq 1$ then even quasi-flex-price firms ration with positive probability (for all t), meaning $\bar{\zeta}_{\tau,t} < 1$.
  - This condition will hold in my calibration. It would be violated if α was very small, or θ was very large.

- A hypothetical fully flexible price firm would choose its price to maximize $o_{\zeta,\tau,t}$.
- Optimal choice is: $\left(\frac{p_{\zeta,\tau,t}}{P_t}\right)^{1+\epsilon\frac{\alpha}{1-\alpha}} = \frac{\epsilon}{\epsilon-1}\left(\zeta\,\frac{\theta+1}{\theta}\right)^{\epsilon\frac{\alpha}{1-\alpha}} \frac{\widehat{W}_t}{1-\alpha}\,\Upsilon_t^{\frac{\alpha}{1-\alpha}}$.
- Note that this is increasing in ζ, while the price of a sticky or quasi-flex-price firm is not increasing in ζ.
- Rationing reduces quantities for high ζ, like in the fully flex price case!

Other consequences of varying inflation



TFP loss from efficient benchmark: $\log \frac{Y}{Y^{\text{SP}}}$ (percent).

Price adjustment rate (percent): 100λ.

Aggregate mark-ups: $\log \frac{(1-\alpha)Y}{WL}$ (percent).

Excess firm profits shares: $\frac{O}{Y} - \alpha$ (percent).

Figure 8: Other consequences of varying inflation (percent).

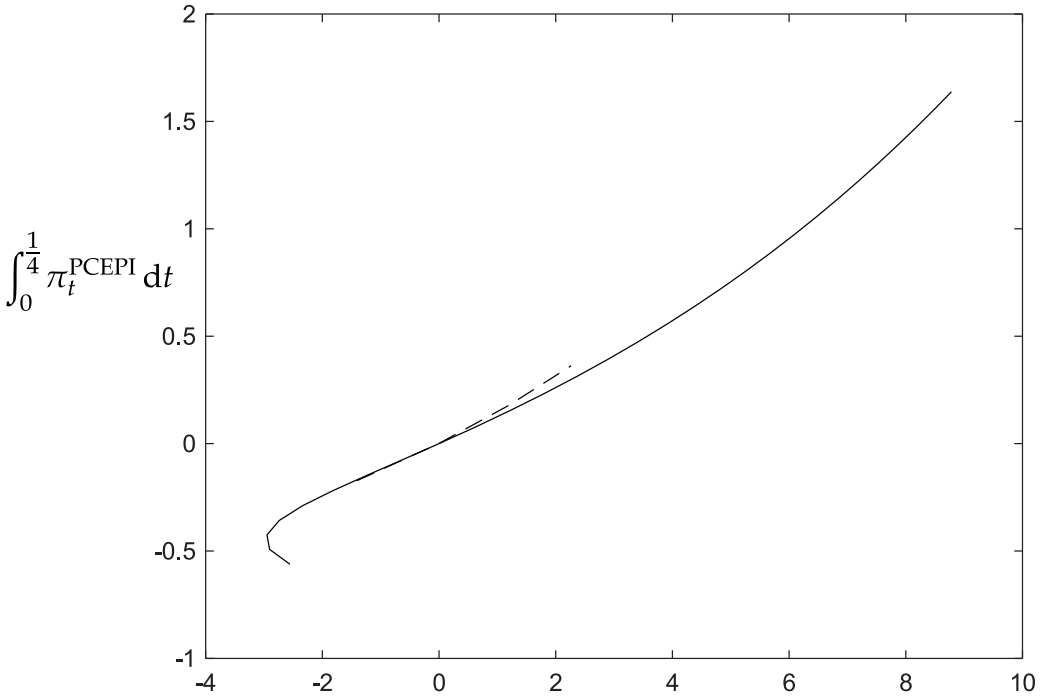
Black solid lines are the model with rationing. Black dashed lines are the model without rationing.

The three-month adjustment rate Phillips curve

- The slope of these lines was a calibration target. (It matches the slope derived from applying the same calculations to the VAR IRFs I showed previously.)

- Small contractionary shocks reduce price adjustment rates, as real prices will be eroded by trend inflation anyway.

- Following large contractionary shocks though, price adjustment rates increase as firms benefit from cutting prices.



Black solid line: cumulated price adjustment rate, $100 \int_0^{\frac{1}{4}} \lambda_t \, \mathrm{d}t$, with rationing.
Black dashed line: cumulated price adjustment rate, $100 \int_0^{\frac{1}{4}} \lambda_t \, \mathrm{d}t$, without rationing.

Figure 9: The three-month Phillips curve with (solid lines) and without (dashed lines) rationing.
All variables in percent. All variables are relative to the no-shock counterfactual.

References

Abraham, Filip, Yannick Bormans, Jozef Konings & Werner Roeger. 2024. ‘Price-Cost Margins, Fixed Costs and Excess Profits’. *The Economic Journal*: ueae037.

Alessandria, George, Joseph P. Kaboski & Virgiliu Midrigan. 2010. ‘Inventories, Lumpy Trade, and Large Devaluations’. *American Economic Review* 100 (5): 2304–2339.

Barro, Robert J. & Herschel I. Grossman. 1971. ‘A General Disequilibrium Model of Income and Employment’. *The American Economic Review* 61 (1): 82–93.

Bils, Mark. 2016. ‘Deducing Markups from Stockout Behavior’. *Research in Economics* 70 (2): 320–331.

Blanco, Andrés, Corina Boar, Callum J. Jones & Virgiliu Midrigan. 2024. ‘The Inflation Accelerator’. Working Paper. Working Paper Series. National Bureau of Economic Research.

Brave, Scott A., R. Andrew Butters & David Kelley. 2019. ‘A New “Big Data” Index of U.S. Economic Activity’. *Economic Perspectives* (1): 1–30.

Brave, Scott A., Ross Cole & David Kelley. 2019. ‘A “Big Data” View of the U.S. Economy: Introducing the Brave-Butters-Kelley Indexes’. *Chicago Fed Letter*.

Caldara, Dario, Matteo Iacoviello & David Yu. 2025. ‘Measuring Shortages since 1900’.

Cavallo, Alberto & Oleksiy Kryvtsov. 2023. ‘What Can Stockouts Tell Us about Inflation? Evidence from Online Micro Data’. *Journal of International Economics* 146. NBER International Seminar on Macroeconomics 2022: 103769.

Corsetti, Giancarlo & Paolo Pesenti. 2005. ‘International Dimensions of Optimal Monetary Policy’. *Journal of Monetary Economics* 52 (2): 281–305.

Drèze, Jacques H. 1975. ‘Existence of an Exchange Equilibrium under Price Rigidities’. *International Economic Review* 16 (2): 301–320.

Forbes, Kristin J., Joseph E. Gagnon & Christopher G. Collins. 2022. ‘Low Inflation Bends the Phillips Curve around the World’. *Economia* 45 (89): 52–72.

Gerke, Rafael, Sebastian Giesen, Matija Lozej & Joost Röttger. 2023. ‘On Household Labour Supply in Sticky-Wage HANK Models’. SSRN Scholarly Paper. Rochester, NY.

Hahn, Volker. 2022. ‘Price Dispersion and the Costs of Inflation’. *Journal of Money, Credit and Banking* 54 (2–3): 459–491.

Holden, Tom D. 2024. *Robust Real Rate Rules*. Working Paper. Kiel, Hamburg: ZBW – Leibniz Information Centre for Economics.

Holden, Tom D., Ales Marsal & Katrin Rabitsch. 2024. ‘From Linear to Nonlinear: Rethinking Inflation Dynamics in the Calvo Pricing Mechanism’.

Huo, Zhen & José-Víctor Ríos-Rull. 2020. ‘Sticky Wage Models and Labor Supply Constraints’. *American Economic Journal: Macroeconomics* 12 (3): 284–318.

Kharroubi, Enisse & Frank Smets. 2024. ‘Monetary Policy with Profit-Driven Inflation’. *BIS Working Papers*. BIS Working Papers.

Klenow, Peter J. & Oleksiy Kryvtsov. 2008. ‘State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?*’. *The Quarterly Journal of Economics* 123 (3): 863–904.

Klenow, Peter J. & Benjamin A. Malin. 2010. ‘Microeconomic Evidence on Price-Setting☆’. In *Handbook of Monetary Economics*, edited by Benjamin M. Friedman & Michael Woodford, 3:231–284. Elsevier.

Kryvtsov, Oleksiy & Virgiliu Midrigan. 2013. ‘Inventories, Markups, and Real Rigidities in Menu Cost Models’. *The Review of Economic Studies* 80 (1): 249–276.

Liu, Yang. 2025. ‘Inflation and Competition in an Old Keynesian Model’. Boston College.

Michaillat, Pascal & Emmanuel Saez. 2015. ‘Aggregate Demand, Idle Time, and Unemployment *’. *The Quarterly Journal of Economics* 130 (2): 507–569.

Miranda-Agrippino, Silvia & Giovanni Ricco. 2021. ‘The Transmission of Monetary Policy Shocks’. *American Economic Journal: Macroeconomics* 13 (3): 74–107.

Montag, Hugh & Daniel Villar. 2025. ‘Post-Pandemic Price Flexibility in the U.S.: Evidence and Implications for Price Setting Models’.

Nakamura, Emi & Jón Steinsson. 2008. ‘Five Facts about Prices: A Reevaluation of Menu Cost Models*’. *The Quarterly Journal of Economics* 123 (4): 1415–1464.

Smets, Frank & Rafael Wouters. 2007. ‘Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach’. *American Economic Review* 97 (3): 586–606.

Svensson, Lars EO. 1984. *Sticky Goods Prices, Flexible Asset Prices, and Optimum Monetary Policy*. IIES.