Rationing Under Sticky Prices

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Paper and slides available at https://www.tholden.org/. PRELIMINARY!

Motivation

- Covid, the Suez Canal blockage and the war in Ukraine all led to widespread **stockouts** and **delivery delays**.
  - Many supermarket shelves were empty during the pandemic.
  - In 2022, many new cars were subject to delivery delays of at least a year.
- Stockouts and delivery delays are forms of **rationing**. They are ultimately a choice of the supplier.
  - MC is never infinite. If you're prepared to pay a high enough amount for a production input, you can always obtain it.
- If prices were flexible, they would have increased proportionally to the increase in MC $\Rightarrow$ no stockouts!
  - But with sticky prices, firms ration demand to avoid selling below MC.
- Rationing is **also common in normal times**.
  - Over 10% of all consumer goods in the US were out of stock pre-Covid (Cavallo & Kryvtsov 2023).

Sticky prices inevitably lead to rationing

- If a firm cannot adjust its nominal price, then its **real price will decline** over time at the rate of inflation.
- A lower real price implies **higher demand** for its good. Higher demand means **higher marginal costs**.
- Eventually, its marginal costs (rising) will be greater than its price (falling) if it continues to meet all demand.
- No firm wants to sell at a price below marginal cost. Instead, it should stop producing, **rationing demand**.
- Yet essentially all the prior sticky price literature (Calvo or menu cost) assumes that firms always meet all demand.

This paper

- What are the macroeconomic implications of allowing firms to ration?

- I allow for random rationing in a continuous time NK model with endogenous price rigidity, and find:

1. Rationing generates a convex Phillips curve.
2. Rationing massively reduces the welfare costs of inflation.
3. True output falls following “expansionary” monetary shocks (but measured output increases).

- Basic mechanism:

  High inflation $\Rightarrow$ low markups $\Rightarrow$ high rationing $\Rightarrow$ fewer varieties consumed $\Rightarrow$ downward pressure on output.

Empirical evidence for rationing

- Cavallo & Kryvtsov (2023) find that around 11% of all US consumer goods were out of stock (=rationing) in 2019.
- In 2022 (Jan-Aug), this number was around 23%. In line with my story: high inflation $\Rightarrow$ high rationing.
  - Cavallo & Kryvtsov (2023) stress causality in the opposite direction. (Stockouts lead to inflation.)

- **I'll show 1:** Quantities sold are concave in price age, in line with goods with old prices being rationed.

- **I'll show 2:** Rationing helps match the convexity of the Phillips curve (Forbes, Gagnon & Collins 2022).

- **I'll show 3:** Rationing helps match the fast response of prices to cleanly identified monetary policy shocks.
  - "Clean" monetary shock papers: Miranda-Agrippino & Ricco (2021), Bauer & Swanson (2023).

Average output over the life of a price: Setup

- Data: Dominick's Finer Foods (1989-1994).
- 21,474,126 observations after dropping for each product and store:
  - First/last price, any price $\neq$ cumulative max price, one week after each price change/missing, any price older than four years, one observation due to differencing.
- Specification, estimated via FGLS:

$$\frac{y_{i,j,k,l,t} - y_{i,j,k,l,t-1}}{\bar{y}_{i,j,k,l}} = \beta_{A(i,j,k,l,t)} + \gamma_{i,j,t} + \delta_{i,k,t} + \sigma^{(1)}_{i,j,A(i,j,k,l,t)} \sigma^{(2)}_{i,j,k} \sigma^{(3)}_{i,j,t} \sigma^{(4)}_{i,k,t} \varepsilon_{i,j,t}$$

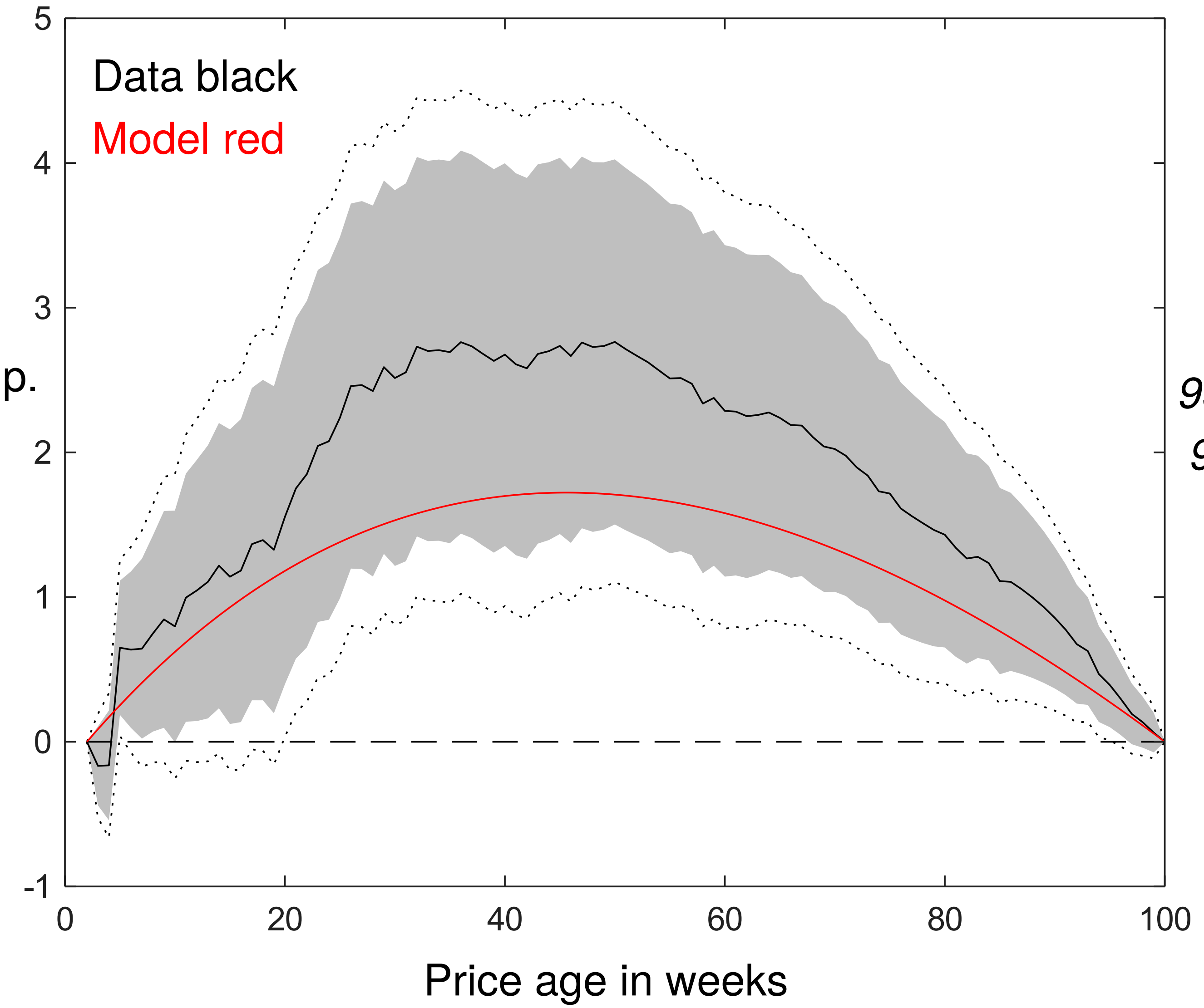
i indexes narrow categories (86)
k indexes products (10,166)
t indexes weeks (398)
j indexes stores (93)
l indexes prices (947,660)

- $A(i,j,k,l,t)$ is the "age" of the i,j,k,l price at t. $y_{i,j,k,l,t}$ gives units sold. $\bar{y}_{i,j,k,l}$ is average of $y_{i,j,k,l,t}$ over the life of the price.
- Standard errors 4-way clustered. Clusters indexed by: $(i,j,A(i,j,k,l,t))$, (i,j,k), (i,j,t), (i,k,t).

Average output over the life of a price: Results

Relative output by age p.p.

$$100 \sum_{a=3}^{\mathrm{AGE}} \beta_a$$



Normalized to 0 at 2 & 100.

95% confidence band shaded.

99% confidence band dotted.

The model

Setup

- The model is in continuous time, with no aggregate uncertainty, just MIT shocks.
- Assume firm price change opportunities arrive at rate $\lambda_t > 0$.
- The time t density of firms that last updated at τ is $\lambda_\tau e^{-\int_\tau^t \lambda_v \, \mathrm{d}v}$. Note $\int_{-\infty}^t \lambda_\tau e^{-\int_\tau^t \lambda_v \, \mathrm{d}v} \, \mathrm{d}\tau = 1$.
- Index firms (and products) by the time they last updated their price, τ, and by their demand shock ζ.
- Firm output: $y_{\zeta,\tau,t}$.
- $g(\zeta)$ is the PDF of the demand shock, which is independent across time and firms.
- For tractability, I assume $g(\zeta) = \theta\zeta^{\theta-1}$ where $\theta > 0$ (so $\zeta \sim \text{Beta}(\theta, 1)$). Mean ζ: $\frac{\theta}{\theta+1}$. Variance ζ: $\frac{\theta}{(\theta+1)^2(\theta+2)}$.

Rationing and aggregation

- $\psi \in [0,1]$ denotes a purchaser-good-time-specific shock controlling if a given purchaser can buy a given good.
- $y_{\psi,\zeta,\tau,t}$: sales to buyer with shock ψ, at t, of good produced by firm that updated price at τ, with demand shock ζ.
- Specialize to ψ uniform, $y_{\psi,\zeta,\tau,t} = \begin{cases} y^*_{\zeta,\tau,t}, & \psi \le \bar{\psi}_{\zeta,\tau,t} \\ 0, & \psi > \bar{\psi}_{\zeta,\tau,t} \end{cases}$. $y^*_{\zeta,\tau,t}$ sales when not rationed.
- $\bar{\psi}_{\zeta,\tau,t}$ is the probability a consumer is not rationed at the firm. Adjusts to ensure: $y_{\zeta,\tau,t} = \int_0^1 y_{\psi,\zeta,\tau,t} \,\mathrm{d}\psi = y^*_{\zeta,\tau,t} \bar{\psi}_{\zeta,\tau,t}$.
- The aggregate good is produced from intermediates by a perfectly competitive industry with technology:

$$Y_t = D^{-\frac{\epsilon}{\epsilon-1}} \left[\int_{-\infty}^{t} \lambda_\tau e^{-\int_\tau^t \lambda_v \,\mathrm{d}v} \int_0^1 \zeta g(\zeta) \int_0^1 y_{\psi,\zeta,\tau,t}^{\frac{\epsilon-1}{\epsilon}} \,\mathrm{d}\psi \,\mathrm{d}\zeta \,\mathrm{d}\tau \right]^{\frac{\epsilon}{\epsilon-1}}, \qquad D = \frac{\theta}{\theta+1}$$

Firm production

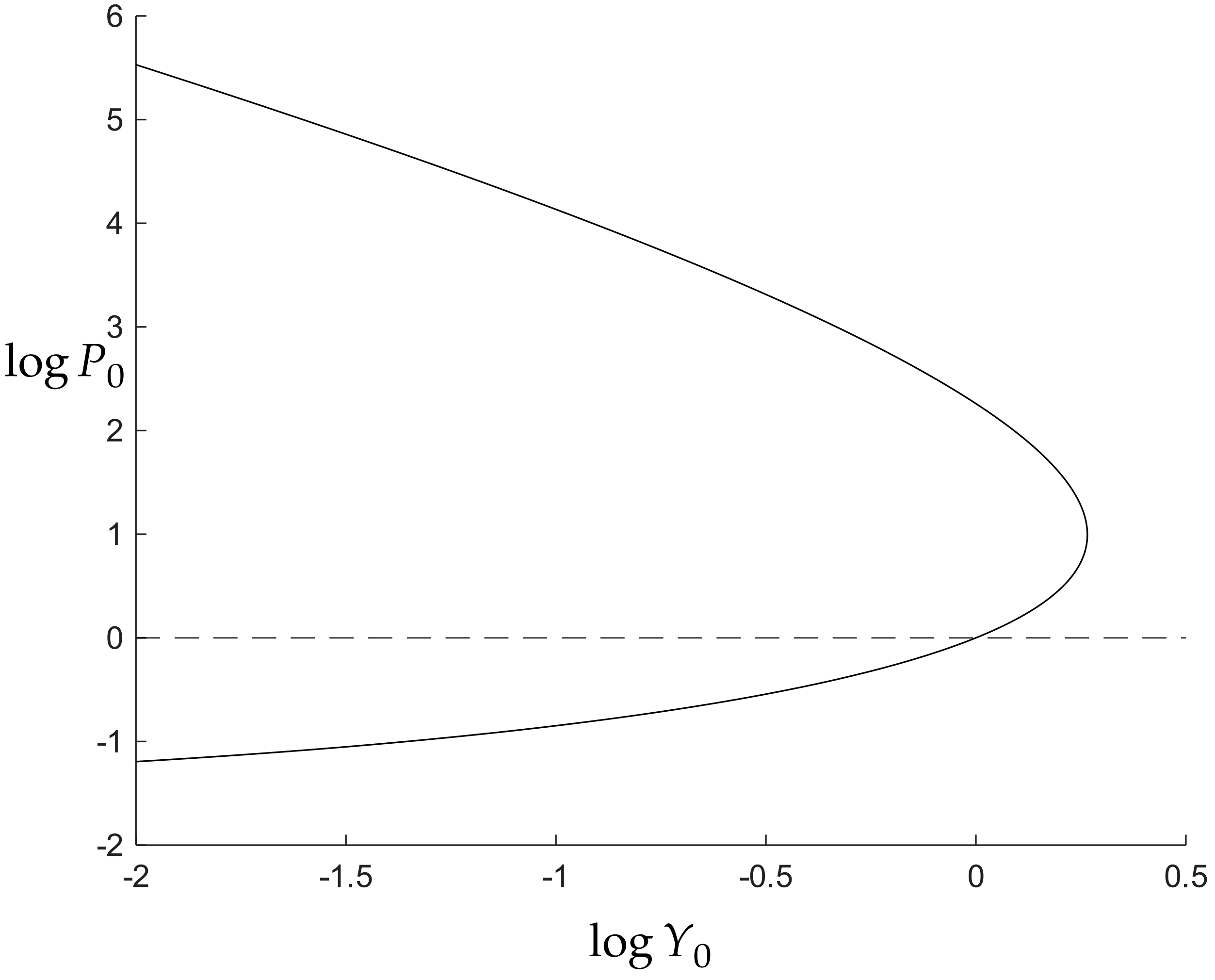
- The FOC of the aggregators imply demand must satisfy: $y_{\zeta,\tau,t} \leq y^*_{\zeta,\tau,t} := \left(\frac{D}{\zeta}\frac{p_\tau}{P_t}\right)^{-\epsilon} Y_t$.

- Firms produce using the production technology: $y_{\zeta,\tau,t} = \left(A_t l_{\zeta,\tau,t}\right)^{1-\alpha}$. Real wage is W_t.

- Firm flow real production profits are guaranteed to be positive for small enough $l_{\zeta,\tau,t}$.

- Optimal production: There is a quantity $\bar{\zeta}_{\tau,t} := D\left(\frac{p_\tau}{P_t}\right)^{1+\frac{1-\alpha}{\epsilon\alpha}} \left(\frac{1-\alpha}{W_t/A_t}\right)^{\frac{1-\alpha}{\epsilon\alpha}} Y_t^{-\frac{1}{\epsilon}} > 0$ such that:
    - If $\zeta < \bar{\zeta}_{\tau,t}$, there is no rationing, so: $y_{\zeta,\tau,t} = \left(\frac{D}{\zeta}\frac{p_\tau}{P_t}\right)^{-\epsilon} Y_t$.
    - If $\zeta > \bar{\zeta}_{\tau,t}$, there is rationing, so: $y_{\zeta,\tau,t} = \left(\frac{p_\tau}{P_t}\frac{1-\alpha}{W_t/A_t}\right)^{\frac{1-\alpha}{\alpha}}$.
    - $\bar{\psi}_{\zeta,\tau,t} = \min\left\{1, \left(\frac{\bar{\zeta}_{\tau,t}}{\zeta}\right)^{\epsilon}\right\}$. High $\bar{\zeta}_{\tau,t}$, less likely to be rationed.

Plotting the short-run Phillips curve (true price/output)



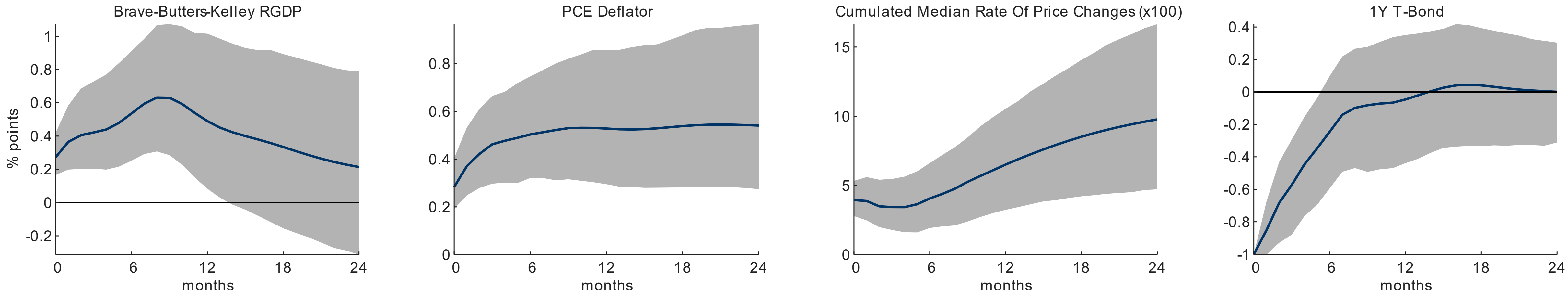
- Assume: $P_t = \exp(\pi t)$ for $t < 0$.
- And: $P_t = P_0 \exp(\pi t)$ for $t \geq 0$.
- So, prices jump at time 0.
- Graphs plot possible (Y_0, P_0) (%, relative to s.s.).

- Solid line is short-run PC allowing rationing.
- Dashed line is short-run PC without rationing.

- Independent of price setting! From aggregation.

- Full calibration will be given shortly.

The short-run Phillips curve in the data



- Following Figure 3 of Miranda-Agrippino & Ricco (2021) (95% bands) but with:
  - Brave-Butters-Kelley RGDP (Brave, Cole & Kelley 2019; Brave, Butters & Kelley 2019) not IP,
  - with PCEPI not CPI,
  - and with the cumulation of the median rate of price changes, excluding sales (Montag & Villar 2025).
- Note jump of price level! Scepticism about monthly date leads me to target responses three-months out.
- Note: not directly comparable to previous model result as model's equivalent of PCEPI differs from true price index.

Price change opportunity arrival rate choice

- If long-run inflation were higher, then prices would be changed more frequently.
  - Aggregate state dependence is necessary for reasonable comparative static results.
  - I broadly follow Blanco et al. (2024) in modelling an endogenous rate of price change opportunities.

- All firms are owned by conglomerates. Each conglomerate owns countably many firms.
- Each conglomerate chooses the price adjustment rate λ_t for the firms it owns (the same rate for all firms).
  - The conglomerate maximizes its firms' total profit, minus a cost of $\frac{\kappa_1}{1+\kappa_2}(\max\{0, \lambda_t - \underline{\lambda}\})^{1+\kappa_2}$ labour units.
- The conglomerate cannot control which particular firms update at any point in time, only the total quantity.
  - Surprisingly consistent with price micro data, which finds hazard rates are flat in price age (Klenow & Malin 2010).

- Optimal:
  - MC of increasing $\lambda_t =$ expected profits of firm with new price (over price life) $-$ expected profits of current firms (over price life).

Households and monetary policy

- In period τ, households maximize: $\int_\tau^\infty e^{-\int_\tau^t \rho_v \,\mathrm{d}v} \left[\log Y_t - \Psi_t \frac{1}{1+\nu} \left(L_t + \frac{\kappa_1}{1+\kappa_2} (\lambda_t - \underline{\lambda})^{1+\kappa_2}\right)^{1+\nu}\right] \mathrm{d}t$.

- They face the budget constraint: $Y_t + \frac{\dot{B}_t^{(i)}}{P_t} + \dot{B}_t^{(r)} = W_t \left(L_t + \frac{\kappa_1}{1+\kappa_2} (\lambda_t - \underline{\lambda})^{1+\kappa_2}\right) + i_t \frac{B_t^{(i)}}{P_t} + r_t B_t^{(r)} + \mathrm{T}_t$.

    - $B_t^{(i)}$ nominal bonds. $B_t^{(r)}$ real bonds. Y_t output = consumption, at price P_t. W_t wage. L_t labour. T_t profits from owning firms.

- FOCs imply $\Psi_t \left(L_t + \frac{\kappa_1}{1+\kappa_2} (\lambda_t - \underline{\lambda})^{1+\kappa_2}\right)^{\nu} = \frac{W_t}{Y_t}$, $r_t = \rho_t + \frac{\dot{Y}_t}{Y_t}$, $i_t = r_t + \pi_t$, where $\pi_t = \frac{\dot{P}_t}{P_t}$.

- Monetary policy sets $i_t = r_t + \pi_t^* + \phi(\pi_t - \pi_t^*)$ with $\phi > 1$ and π_t^* an exogenous target (Holden 2024).

- From Fisher equation, $r_t + \pi_t = i_t = r_t + \pi_t^* + \phi(\pi_t - \pi_t^*)$, so $\pi_t = \pi_t^*$ for all t. Inflation is effectively exogenous.

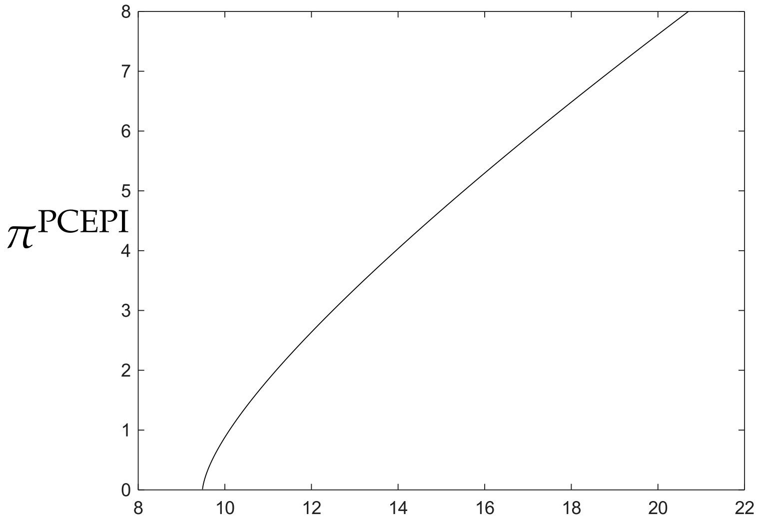
Parameterization / calibration

- $\rho = 2\%$. $\pi^{\text{PCEPI}} := 2\%$ unless otherwise stated. With rationing, requires $\pi^* := 2.04\%$, without $\pi^* := 2.00\%$.
- $\epsilon := 10$, $\nu := 2$, Smets & Wouters (2007). $\alpha := \frac{3}{5}$, Abraham et al. (2024).
- $\theta := 27$. Matching 11% stockouts in 2019 from Cavallo & Kryvtsov (2023) to $1 - \bar{\psi}$. ζ mean 0.96. ζ s.d. 0.03.
- $\underline{\lambda} := 0.73$, minimum annual median price adjustment rate in the Montag & Villar (2025) data.

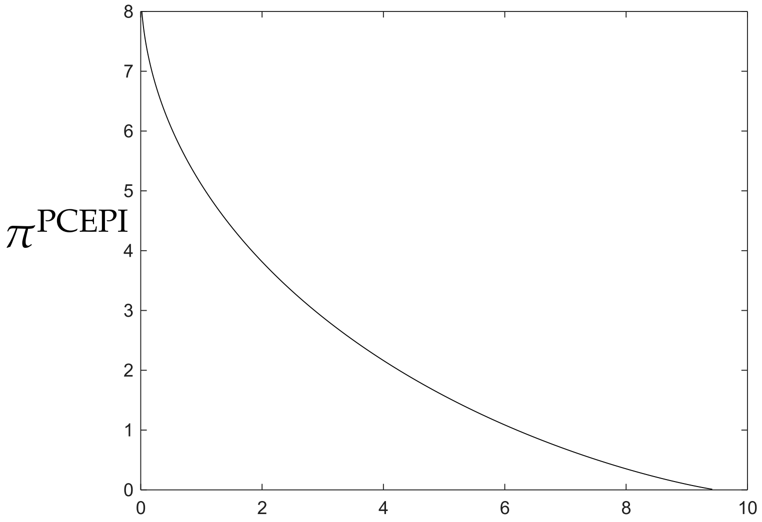
- With rationing, $\kappa_1 = 0.016$ and $\kappa_2 = 3.75$, without rationing $\kappa_1 = 0.105$ and $\kappa_2 = 2.06$. Imply $\lambda = 1.48$.
  - Matching time series mean of the median rate of price adjustment from the Montag & Villar (2025) data, $\lambda = 1.48$.
  - And matching $\int_0^{\frac{1}{4}}(\lambda_t - \lambda)\,\mathrm{d}t \Big/ \int_0^{\frac{1}{4}}\left(\pi_t^{\text{PCEPI}} - \pi^{\text{PCEPI}}\right)\mathrm{d}t$ following a monetary policy shock to 8.2 as estimated from Figure 1.
  - With rationing: 0.1% of labour is used for price adjustment. Without: 2.0% of labour is used for price adjustment.
  - Rationing reduces the price adjustment frictions needed to match the data!

Results

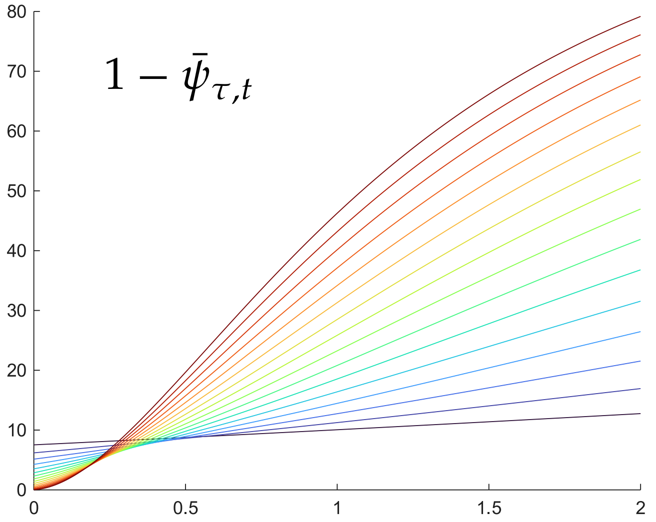
Stockouts and rationing as a function of inflation



Average stockout level, $1 - \bar{\psi}$ (percent).



Stockout rate at firms with new prices, $1 - \bar{\psi}_{t,t}$ (percent).



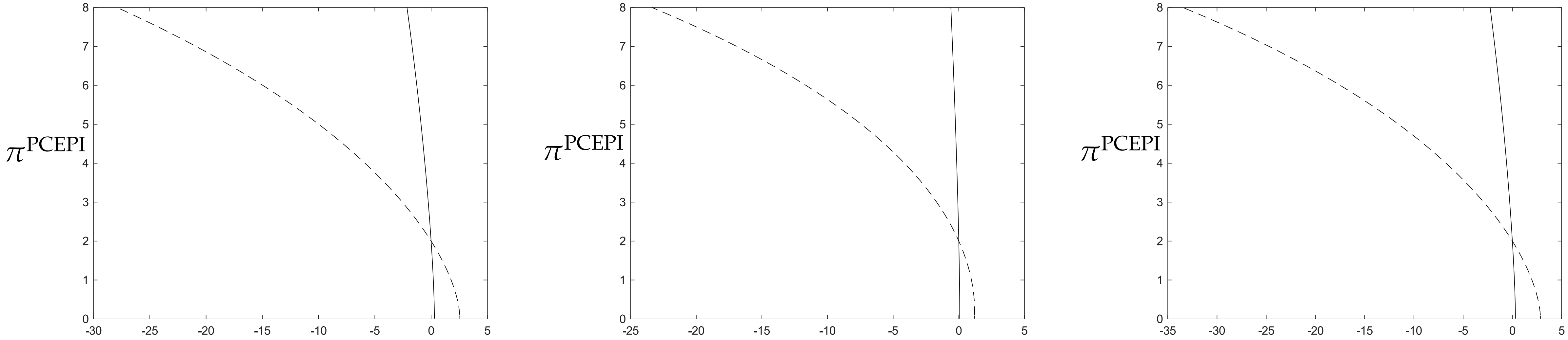
Stockout levels (percent) as a function of price age (years), with varying steady state inflation levels. Dark blue corresponds to 0.5% inflation. Dark red corresponds to 8% inflation.

Figure 6: Stockouts and rationing as a function of PCEPI inflation (percent).

When inflation is high rationing is high. High inflation quickly erodes mark-ups.

Firms with new prices ration less with high inflation as they set high initial mark-ups.

Output and welfare as a function of inflation



Relative output: $\log Y$ (percent).

Relative production labour supply: $\log L$ (percent).

Relative welfare:

$$100\left(\log Y - \Psi \frac{1}{1+\nu}\left(L + \frac{\kappa_1}{1+\kappa_2}\left(\lambda_t - \underline{\lambda}\right)_t^{1+\kappa_2}\right)^{1+\nu}\right)$$

Figure 7: Output and welfare as a function of inflation (percent).

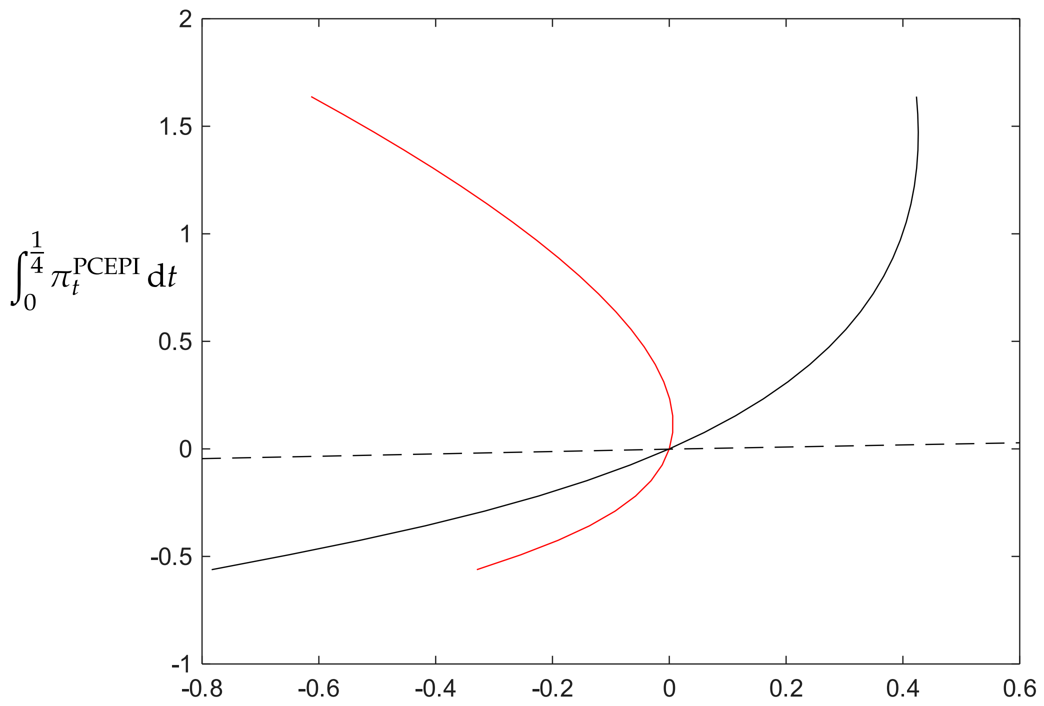
Black solid lines are the model with rationing. Black dashed lines are the model without rationing.

All plots are normalized to hit 0% on the horizontal axis when $\pi^{\text{PCEPI}} = 2\%$.

Welfare costs of inflation are much lower under rationing.

Firms with old prices are no longer making losses on every unit sold. Less labour is used in price adjustment.

The three-month Phillips curve



Black solid line: measured cumulated real GDP, $\log \int_0^{\frac{1}{4}} \frac{P_t Y_t}{P_t^{\text{PCEPI}}}\, \mathrm{d}t$, with rationing.

Red solid line: cumulated true output, $\log \int_0^{\frac{1}{4}} Y_t\, \mathrm{d}t$, with rationing.

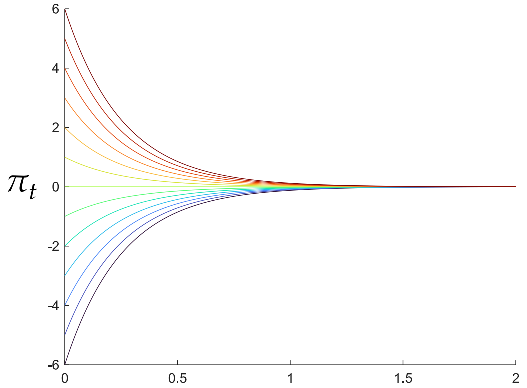
Black dashed line: measured cumulated real GDP, $\log \int_0^{\frac{1}{4}} \frac{P_t Y_t}{P_t^{\text{PCEPI}}}\, \mathrm{d}t$, without rationing.

Figure 9: The three-month Phillips curve with (solid lines) and without (dashed lines) rationing.

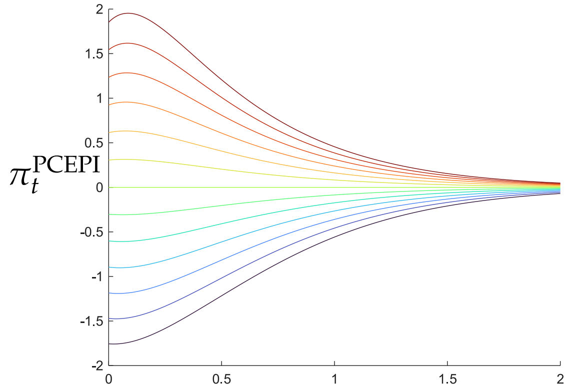
All variables in percent. All variables are relative to the no-shock counterfactual.

- The black line (measured output with rationing) matches the PC slope of 1.2 derived from applying the same calculations to the VAR IRFs I showed previously.
- Note that expansionary monetary policy reduces true output but increases measured output.
- The PCEPI deflator does not capture gains from variety, so it misses the losses coming from rationing reducing the measure of consumed varieties.

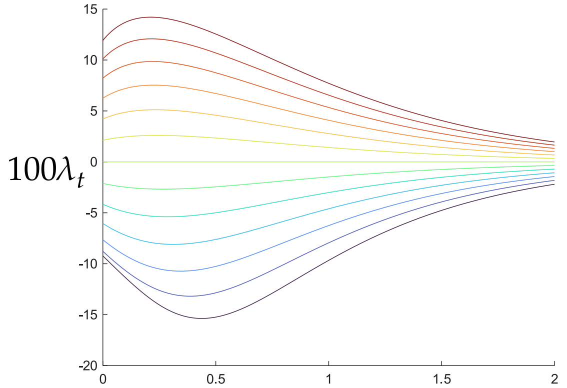
IRFs to π shocks (persistence matched to VAR)



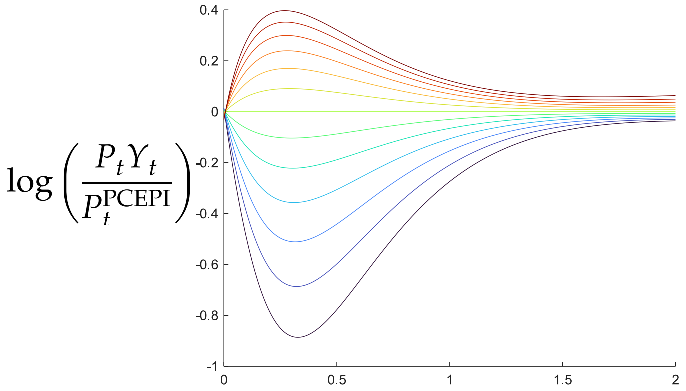
Driving inflation shocks (percent) as a function of time (years).



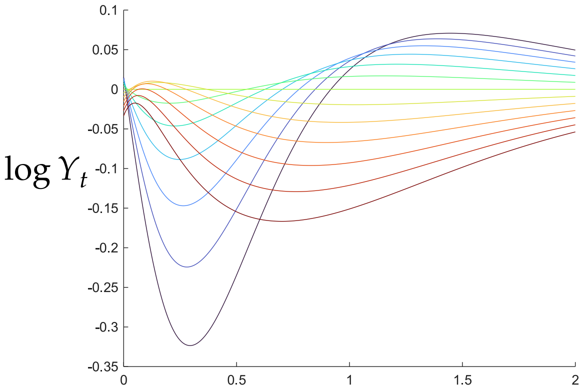
PCEPI inflation (percent) as a function of time (years).



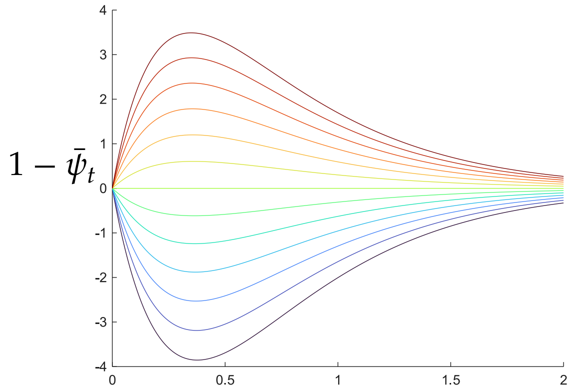
Price adjustment rate (× 100) as a function of time (years).



Measured real GDP response (percent) as a function of time (years).



True output response (percent) as a function of time (years).

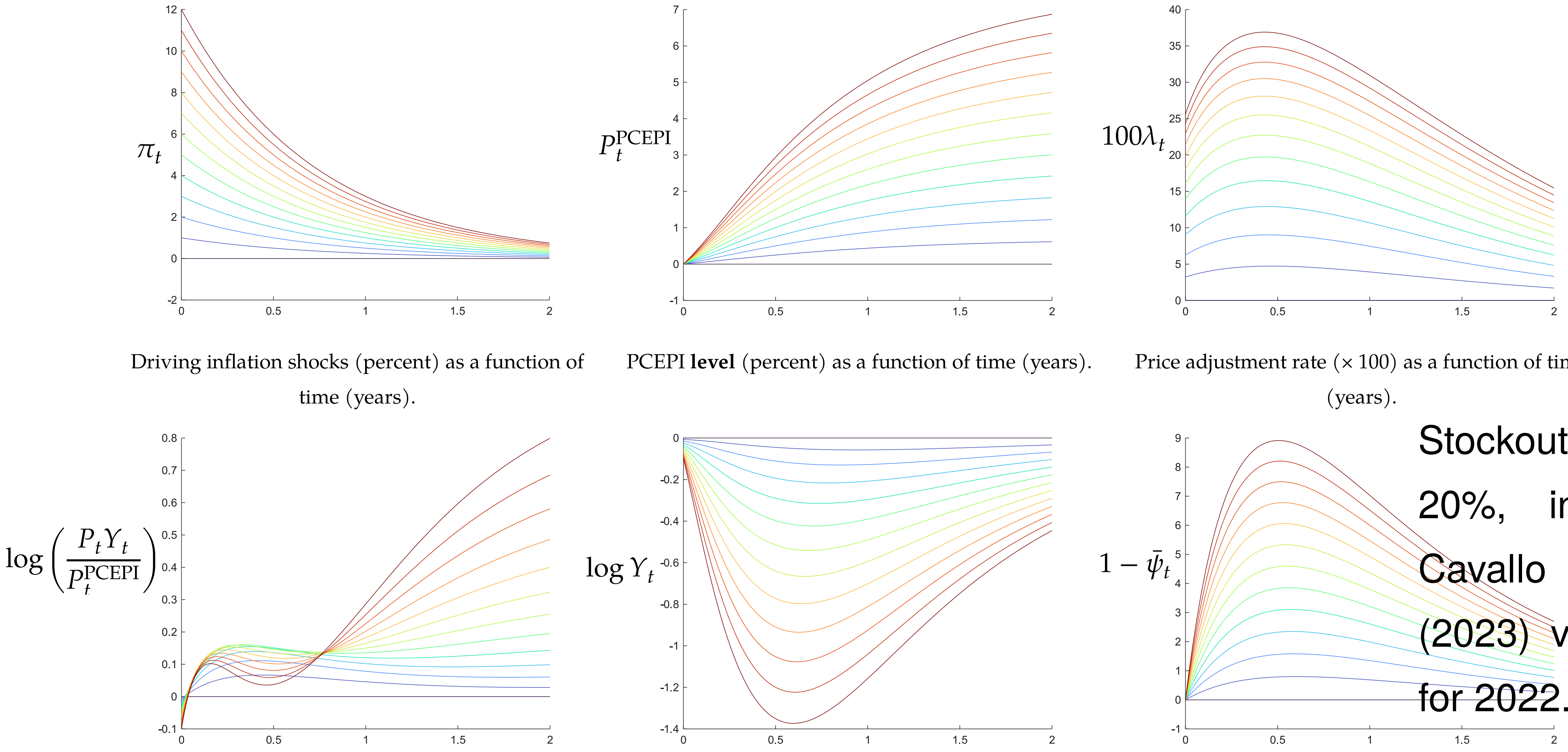


Stockout share response (percentage points) as a function of time (years).

Figure 10: Impulse responses to monetary shocks, with rationing.

Colours are consistent across subplots. All responses are relative to the no-shock counterfactual.

IRFs to π shocks (matching 7% post-Covid inflation)



Driving inflation shocks (percent) as a function of time (years).

PCEPI **level** (percent) as a function of time (years).

Price adjustment rate ($\times$ 100) as a function of time (years).

Measured real GDP response (percent) as a function of time (years).

True output response (percent) as a function of time (years).

Stockout share response (percentage points) as a function of time (years).

Figure 11: Impulse responses to more persistent monetary shocks, with rationing.

Colours are consistent across subplots. All responses are relative to the no-shock counterfactual.

Stockout rates hit 20%, in line with Cavallo & Kryvtsov (2023) value of 23% for 2022.

Conclusion

- The standard assumption that firms always satisfy all demand is **not innocuous**.
- Allowing rationing produces a model that **fits the data** better and performs more reasonably in extreme conditions.

- Allowing rationing drastically **reduces the welfare costs** of steady state inflation.
- But when rationing is allowed, **true output declines** following “expansionary” monetary shocks.
- Monetary policy may be **less useful for stabilisation** than we previously thought.

- Extensions in final paper: quantity-capped rationing, consumer distaste for rationed varieties, firm specific capital, partially fixed intermediaries, long-run growth.
- I am interested to hear thoughts on other essential extensions, or crucial empirical results to establish.

Extra slides

Does rationing matter in practice?

“Mark-ups are 10%, inflation is 2%, prices are updated at least once per year, real prices will not hit marginal cost.”

- But: firm demand: $y \propto \left(\frac{p}{P}\right)^{-\epsilon}$ ($\epsilon \approx 10$) and marginal costs: $mc \propto y^{\frac{\alpha}{1-\alpha}}$ ($\alpha \approx \frac{3}{5}$), so $mc \propto \left(\frac{p}{P}\right)^{-\epsilon\frac{\alpha}{1-\alpha}} \approx \left(\frac{p}{P}\right)^{-15}$.
  - In the short run, some labour and intermediate inputs are fixed ($\approx \frac{2}{5}$ at annual freq. (Abraham et al. 2024)) $\Rightarrow \alpha \approx \frac{3}{5}$.
- So: A 2% fall in real prices increases marginal costs by 30%. Good-bye mark-ups! Hello rationing!

- This calculation understates firms’ reasons to ration:
  - Firms face high frequency demand fluctuations. Mark-ups are much lower at times of high demand.
  - Inflation can be much higher than 2%. It was around 7% post-Covid!
  - Demand is growing over time due to aggregate income growth. A 2% increase in aggregate demand increases MC by 3%.
  - Marginal costs are also rising over time if not all capital depreciation can be fixed quickly.

Random rationing vs sales-capped rationing

- Two potential models of rationing:
  - Sales-capped rationing: The firm caps the quantity it sells to any given consumer.
  - Random rationing: Some consumers get lucky and buy their entire order. Others go home with nothing.
- While we saw some sales caps during Covid, random rationing seems more natural.
  - It also fits the aggregate data better. (Sales-capped rationing generates an excessively steep Phillips curve.)
  - While with tight consumer storage constraints, if goods are semi-durable and consumers shop frequently, the result of random rationing can look a lot like quantity-capped rationing, without such constraints the result is random rationing again.
- With random rationing, changes in rationing change measure of varieties consumed. Consumers love variety!

Price adjustment

- Why don't firms just change prices, rather than rationing?
  - Firms can benefit from tolerating $P<MC$ if they anticipate lower MC (mean reversion?) or lower menu costs in future.
  - Modern menu cost models rely heavily on random menu costs and/or free price change opportunities.
  - By revealed preference, firms that can ration make higher profits than firms that cannot. Under rationing, profits always >0.
  - Since profits are higher when rationing is allowed, lower menu (etc.) costs are needed to justify the observed price stickiness.

- I will take a tractable approach to state-dependent pricing broadly following Blanco et al. (2024).
  - Firms will be owned by conglomerates. Conglomerates choose the rate of price adjustment, not which firms adjust.
  - Provides aggregate state dependence.
  - Matches flat adjustment hazard rate found by Klenow & Kryvtsov (2008), Nakamura & Steinsson (2008), Klenow & Malin (2010).

Prior literature

- Early:
    - Drèze (1975), Barro (1977), Svensson (1984), Corsetti & Pesenti (2005) (restrict shocks to ensure no rationing).
- Stockouts in inventory models:
    - Alessandria, Kaboski & Midrigan (2010), Kryvtsov & Midrigan (2013), Bils (2016).
    - In these papers, firms always meet demand if they have stock available, even if marginal value of that stock > price.
- NK rationing models (all with sales-capped rationing):
    - Under sticky wages: Huo & Ríos-Rull (2020), Gerke et al. (2023): Infinite dimensional state, numerical.
    - Under sticky prices: Hahn (2022): Only steady state results. No dynamics. No idiosyncratic shocks.
- Other related work:
    - Continuous time NK models: Posch, Rubio-Ramírez & Fernández-Villaverde (2011), (2018)
    - Endogenous price adjustment frequency: Blanco et al. (2024).

State variables and the short-run Phillips curve

- All of the model's state variables are of the form: $X_{j,t} := \int_{-\infty}^{t} \lambda_\tau e^{-\int_\tau^t \lambda_v \, dv} p_\tau^{\chi_j} \, d\tau$, so: $\dot{X}_{j,t} = \lambda_t \left[p_t^{\chi_j} - X_{j,t} \right]$ (for $j \in \mathbb{Z}$).

- The definition of total labour demand is: $L_t := \int_{-\infty}^{t} \lambda_\tau e^{-\int_\tau^t \lambda_v \, dv} \int_0^1 l_{\zeta,\tau,t} \, g(\zeta) \, \mathrm{d}\zeta \, \mathrm{d}\tau$.
- This implies an equilibrium condition relating L_t, A_t, $\widehat{W}_t$, Y_t, P_t, $X_{1,t}$ & $X_{2,t}$, with $\chi_1 := \theta + \frac{1}{\alpha} + \frac{\theta}{\epsilon} \frac{1-\alpha}{\alpha}$, $\chi_2 := \frac{1}{\alpha}$.
- The definition of aggregate output implies an equilibrium condition relating $\widehat{W}_t$, Y_t, P_t, $X_{1,t}$ & $X_{2,t}$.
- Combined with the household labour FOC, these two equations give a short-run Phillips curve, holding states fixed.

- Without rationing, the equivalent first equation relates L_t, A_t, $\widehat{W}_t$, Y_t, P_t & $X_{-1,t}$ with $\chi_{-1} := -\frac{\epsilon}{1-\alpha}$.
- And the second just relates P_t & $X_{-2,t}$ with $\chi_{-2} := -(\epsilon - 1)$.
  - Thus, if $X_{-2,t}$ is fixed, P_t is fixed. The short-run Phillips curve is horizontal in the NK model without rationing!

Other aggregates

- Average probability that a buyer from a particular firm receives order: $\bar{\psi}_{\tau,t} := \int_0^1 \bar{\psi}_{\zeta,\tau,t} g(\zeta)\, d\zeta = \frac{\theta \bar{\zeta}_{\tau,t}^{\epsilon} - \epsilon \bar{\zeta}_{\tau,t}^{\theta}}{\theta - \epsilon}$.

- Average probability of receiving order across all firms: $\bar{\psi}_t := \int_{-\infty}^{t} \lambda_\tau e^{-\int_\tau^t \lambda_v \,\mathrm{d}v} \bar{\psi}_{\tau,t} \,\mathrm{d}\tau$.
    - Equal weighted for comparability with Cavallo & Kryvtsov (2023) evidence.

- Model PCEPI, following BLS imputation procedure (unobserved price changes assumed equal average change):
    - $\frac{1}{\Delta}\left(\log P_t^{\text{PCEPI}} - \log P_{t-\Delta}^{\text{PCEPI}}\right) = \frac{1}{\Delta}\int_{-\infty}^{t-\Delta} \lambda_\tau e^{-\int_\tau^{t-\Delta} \lambda_v \,\mathrm{d}v} \left[\bar{\psi}_{\tau,t-\Delta}\lambda_t \Delta \left[\bar{\psi}_{t,t} \log \frac{p_t}{p_\tau} + (1-\bar{\psi}_{t,t}) \log \frac{P_t^{\text{PCEPI}}}{P_{t-\Delta}^{\text{PCEPI}}}\right] + \bar{\psi}_{\tau,t-\Delta}(1-\lambda_t \Delta)\left[\bar{\psi}_{\tau,t} 0 + (1 - \bar{\psi}_{\tau,t}) \log \frac{P_t^{\text{PCEPI}}}{P_{t-\Delta}^{\text{PCEPI}}}\right] + (1-\bar{\psi}_{\tau,t-\Delta}) \log \frac{P_t^{\text{PCEPI}}}{P_{t-\Delta}^{\text{PCEPI}}}\right] \mathrm{d}\tau$
    - So: $\pi_t^{\text{PCEPI}} := \frac{\mathrm{d}\log P_t^{\text{PCEPI}}}{\mathrm{d}t} = \lambda_t \bar{\psi}_{t,t} \frac{\int_{-\infty}^{t} \lambda_\tau e^{-\int_\tau^t \lambda_v \,\mathrm{d}v} \bar{\psi}_{\tau,t}^2 \frac{1}{\bar{\psi}_{\tau,t}} \log \frac{p_t}{p_\tau} \,\mathrm{d}\tau}{\int_{-\infty}^{t} \lambda_\tau e^{-\int_\tau^t \lambda_v \,\mathrm{d}v} \bar{\psi}_{\tau,t}^2 \,\mathrm{d}\tau}$.

Instability without rationing

- I stationarize $X_{j,t}$ by defining $\widehat{X}_{j,t} := \frac{X_{j,t}}{P_t^{\chi_j}}$. And I define: $\hat{p}_t := \frac{p_t}{P_t}$. Then: $\dot{\widehat{X}}_{j,t} = \lambda_t \hat{p}_t^{\chi_j} - (\lambda_t + \chi_j \pi_t)\widehat{X}_{j,t}$.

- So: $\lambda_t + \chi_j \pi_t$ determines the stability of $\widehat{X}_{j,t}$. It is stable if $\lambda_t + \chi_j \pi_t > 0$.

- For the model with rationing, $\chi_1 = \theta + \frac{1}{\alpha} + \frac{\theta}{\epsilon}\frac{1-\alpha}{\alpha} > 0$ and $\chi_2 = \frac{1}{\alpha} > 0$. Stability guaranteed!

- For the model without rationing, $\chi_{-1} = -\frac{\epsilon}{1-\alpha} < 0$ and $\chi_{-2} = -(\epsilon - 1) < 0$.

- If ϵ, α or π_t are large enough, then $\lambda_t + \chi_{-1}\pi_t < 0$ or $\lambda_t + \chi_{-2}\pi_t < 0$. Potential instability!

New prices

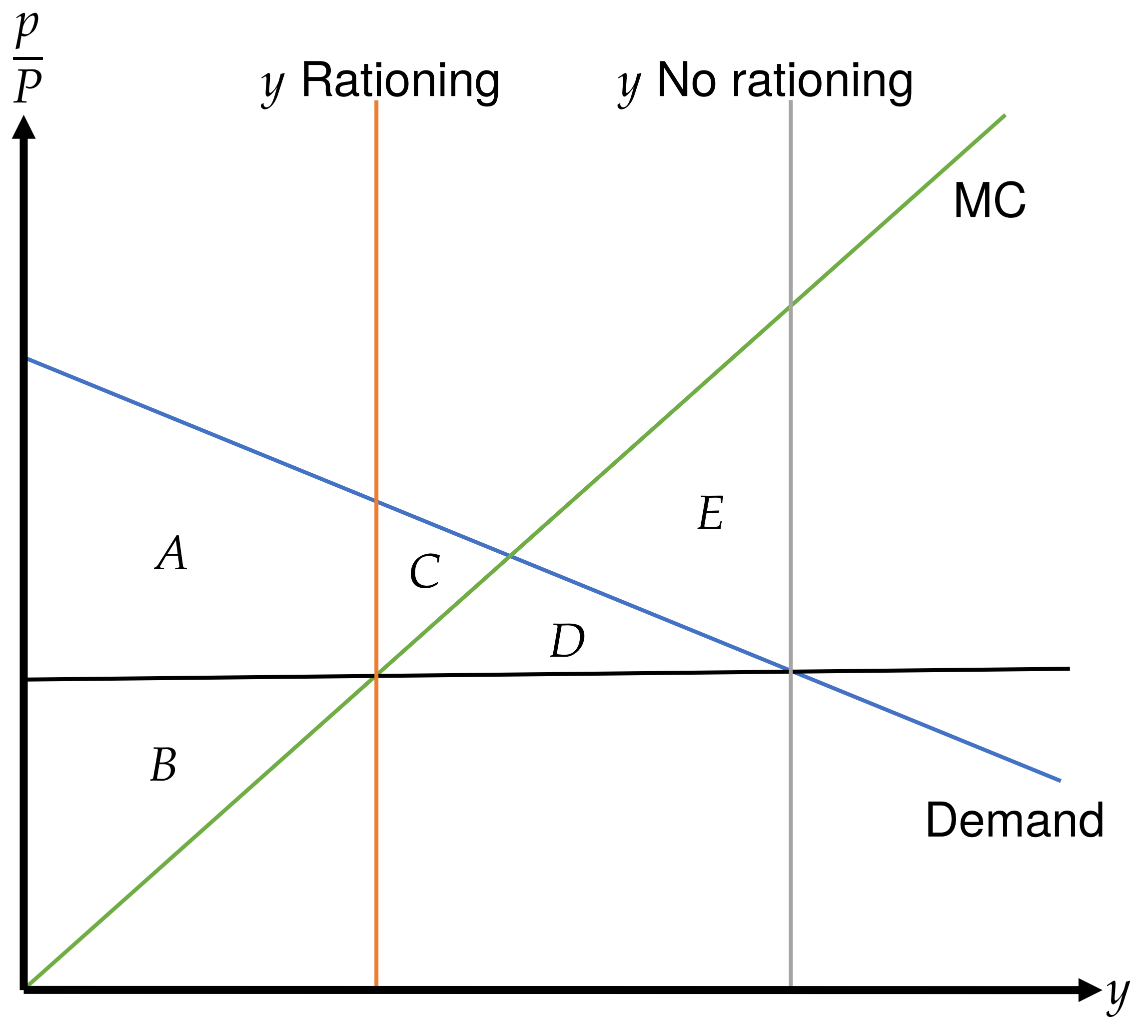
- For $j \in \mathbb{N}$, define: $z_{j,\tau} := \int_\tau^\infty e^{-\int_\tau^t (\lambda_v + r_v)\,\mathrm{d}v} D^{\omega_{j,1}} \widehat{W}_t^{\omega_{j,2}} Y_t^{\omega_{j,3}} P_t^{\omega_{j,4}}\,\mathrm{d}t$, so $\dot{z}_{j,\tau} = -D^{\omega_{j,1}} \widehat{W}_t^{\omega_{j,2}} Y_\tau^{\omega_{j,3}} P_\tau^{\omega_{j,4}} + (\lambda_\tau + r_\tau) z_{j,\tau}$.

- Allowing rationing, updating firms optimally set: $p_\tau^{\theta + \frac{\theta}{\epsilon}\frac{1-\alpha}{\alpha}} \propto \frac{z_{2,\tau}}{z_{1,\tau}}$.
  - Where: $\omega_{1,1} := \theta$, $\omega_{1,2} := -\frac{\theta+\epsilon}{\epsilon}\frac{1-\alpha}{\alpha}$, $\omega_{1,3} := -\frac{\theta}{\epsilon}$, $\omega_{1,4} := -\chi_1$, $\omega_{2,1} := 0$, $\omega_{2,2} := -\frac{1-\alpha}{\alpha}$, $\omega_{2,3} := 0$, $\omega_{2,4} := -\chi_2$.

- Without rationing, updating firms optimally set: $p_\tau^{1+\epsilon\frac{\alpha}{1-\alpha}} \propto \frac{z_{-2,\tau}}{z_{-1,\tau}}$.
  - Where: $\omega_{-1,1} := 0$, $\omega_{-1,2} := -\epsilon$, $\omega_{-1,3} := 1$, $\omega_{-1,4} := -\chi_{-2}$, $\omega_{-2,1} := -\frac{\epsilon}{1-\alpha}$, $\omega_{-2,2} := 1$, $\omega_{-2,3} := \frac{1}{1-\alpha}$, $\omega_{-2,4} := -\chi_{-1}$.

- I stationarize by defining: $\hat{z}_{j,t} := \frac{z_{j,t}}{P_t^{\omega_{j,4}}}$. Again, with rationing the $\hat{z}_{j,t}$ are (backwards) stable, but not without.

The microeconomics of rationing vs excess production



- Without rationing: CS is $A + C + D$. PS is $B - D - E$.
- Without rationing: Welfare is $A + B + C - E$.
- With rationing: CS is A. PS is B. Welfare is $A + B$.
- Welfare is higher with rationing when $E > C$.
- Plausible as demand ($\propto y^{-\frac{1}{\epsilon}}$) is flatter than MC ($\propto y^{\frac{\alpha}{1-\alpha}}$).
- The economy with rationing should be less distorted!

The macroeconomics of rationing vs excess production

- If too much is produced by some firms (with old prices), other firms face higher marginal costs, so produce less.
- Demand is shifted from undistorted firms (with new prices) to distorted ones (with old prices).
- Bad!

- If demand is rationed for some goods (with old prices), other firms face lower marginal costs, so produce more.
- Demand is shifted from distorted firms (with old prices) to undistorted ones (with new prices).
- Good!

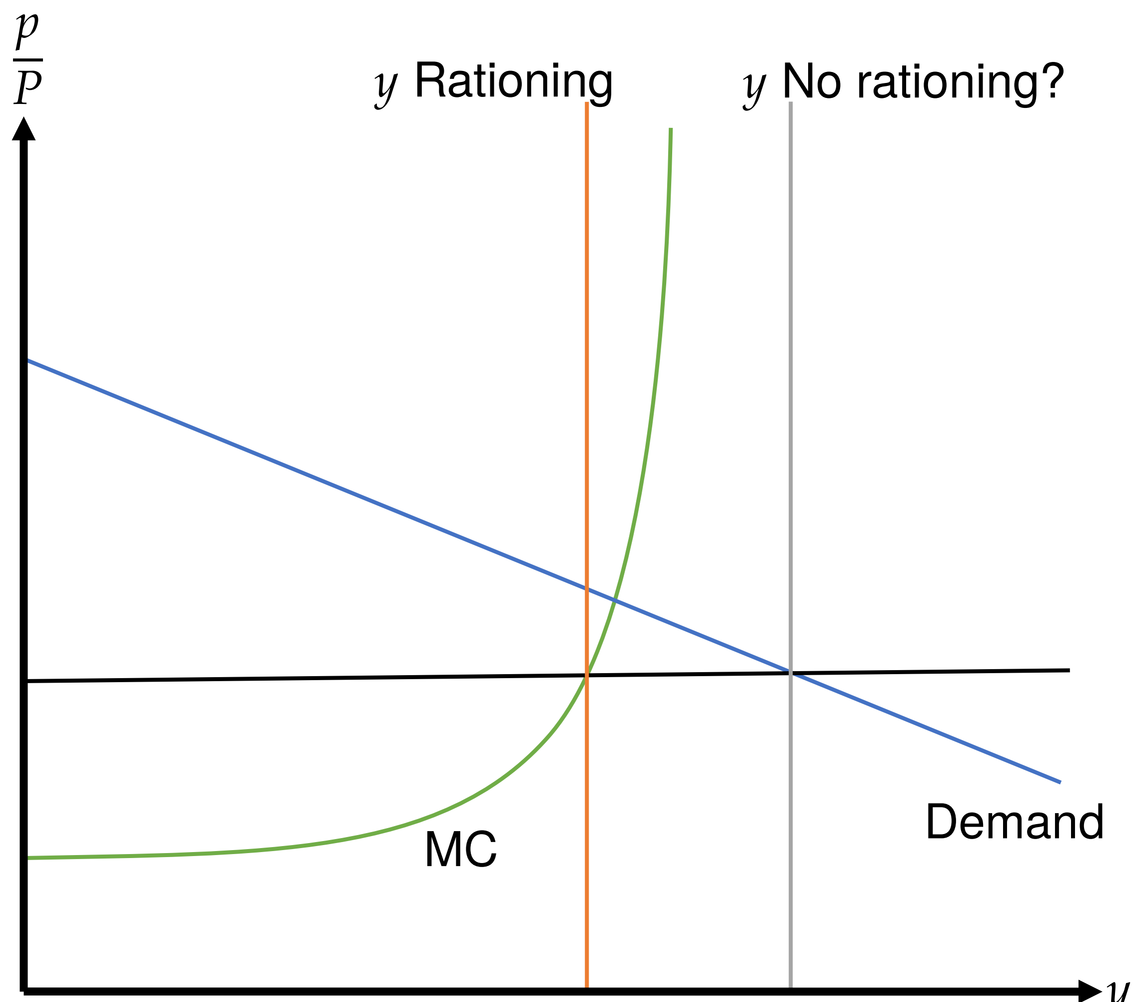
Strange properties of the Calvo model

- The Calvo model has some deeply strange properties (Holden, Marsal & Rabitsch 2024).
  - It implies a hard upper bound on steady-state inflation. With standard parameters, this is 5% to 10%.
  - Inflation above this level reduces the output *growth rate* not just the output level, due to ever growing price dispersion.
  - Under standard monetary rules, temporary high inflation can push the economy to this growing price dispersion path.

- These strange properties are tightly linked to the losses made by firms forced to sell at prices below marginal cost.

- When rationing is allowed, these strange properties disappear. Additional motivation for looking at it.

Sticky prices with near vertical marginal costs



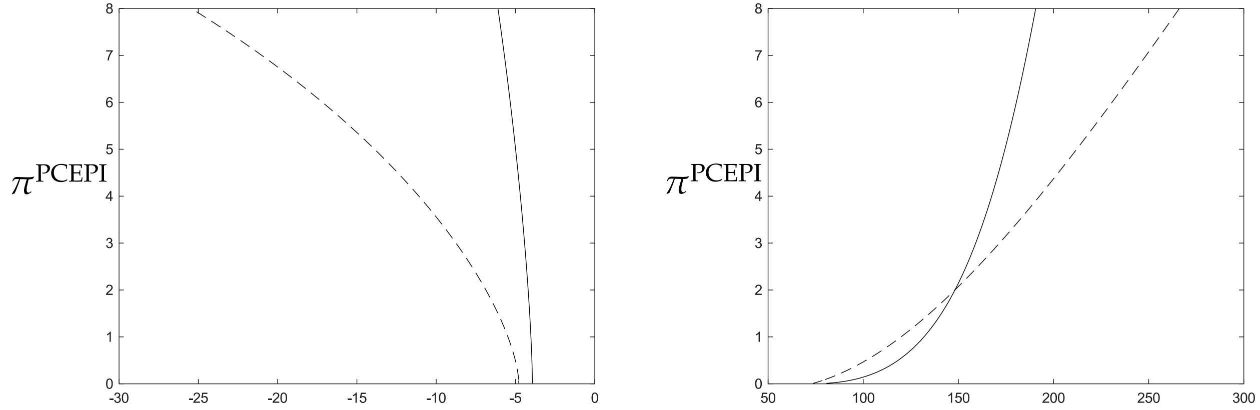
- Holding macro quantities fixed, there is no way to have equilibrium in this (micro) market without rationing.
- Total cost to produce "y No rationing" is infinite.
- So, what happens?
- As we climb the green line, an ever-increasing share of the economy's productive capacity goes to this market.
- $\Rightarrow$ Aggregate output falls.
- $\Rightarrow$ Lower demand for this good at any price.
- Macro quantities move to clear this micro market!
- Is this really plausible???
- Rationing seems more reasonable.

The quasi flexible and fully flexible price cases

- The limit as $\lambda_t \to \infty$ is not fully flexible prices, as for any λ_t, firms face all possible ζ before changing price.
- Instead, the limit is quasi flexible prices, which maximize $o_{\tau,t} := \int_0^1 o_{\zeta,\tau,t} g(\zeta)\,\mathrm{d}\zeta$.
- If $\frac{\epsilon}{\epsilon-1}\frac{\theta+\epsilon}{\theta+\frac{\epsilon}{1-\alpha}} \leq 1$ then even quasi-flex-price firms ration with positive probability (for all t), meaning $\bar{\zeta}_{\tau,t} < 1$.
  - This condition will hold in my calibration. It would be violated if α was very small, or θ was very large.

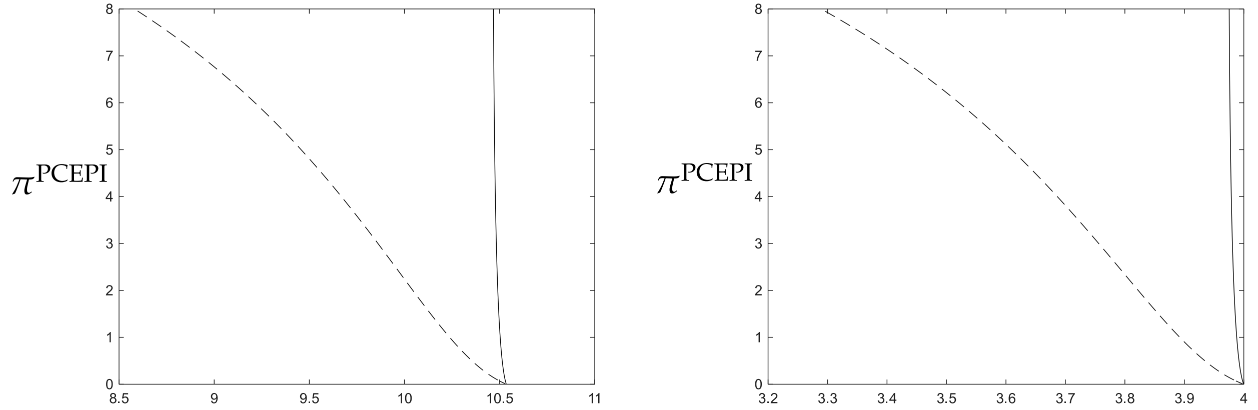
- A hypothetical fully flexible price firm would choose its price to maximize $o_{\zeta,\tau,t}$.
- Optimal choice is: $\left(\frac{p_{\zeta,\tau,t}}{P_t}\right)^{1+\epsilon\frac{\alpha}{1-\alpha}} = \frac{\epsilon}{\epsilon-1}\left(\zeta\,\frac{\theta+1}{\theta}\right)^{\epsilon\frac{\alpha}{1-\alpha}} \frac{\widehat{W}_t}{1-\alpha} Y_t^{\frac{\alpha}{1-\alpha}}$.
- Note that this is increasing in ζ, while the price of a sticky or quasi-flex-price firm is not increasing in ζ.
- Rationing reduces quantities for high ζ, like in the fully flex price case!

Other consequences of varying inflation



TFP loss from efficient benchmark: $\log \frac{Y}{Y^{\text{SP}}}$ (percent).

Price adjustment rate (percent): 100λ.



Aggregate mark-ups: $\log \frac{(1-\alpha)Y}{WL}$ (percent).

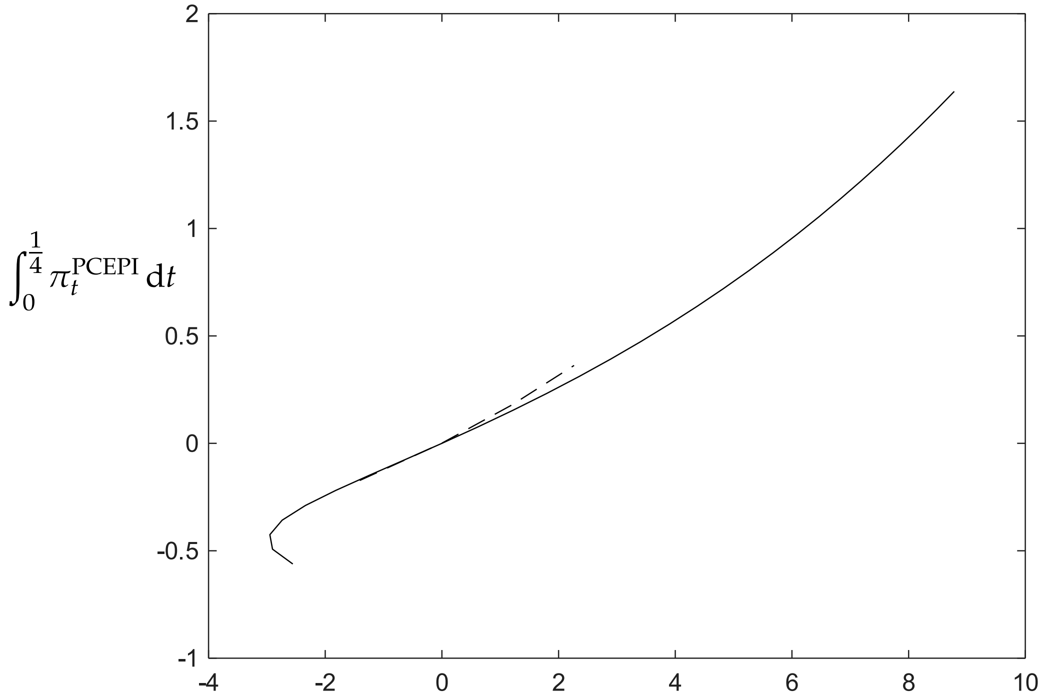
Excess firm profits shares: $\frac{O}{Y} - \alpha$ (percent).

Figure 8: Other consequences of varying inflation (percent).

Black solid lines are the model with rationing. Black dashed lines are the model without rationing.

The three-month adjustment rate Phillips curve

- The slope of these lines was a calibration target. (It matches the slope derived from applying the same calculations to the VAR IRFs I showed previously.)
- Small contractionary shocks reduce price adjustment rates, as real prices will be eroded by trend inflation anyway.
- Following large contractionary shocks though, price adjustment rates increase as firms benefit from cutting prices.



Black solid line: cumulated price adjustment rate, $100 \int_0^{\frac{1}{4}} \lambda_t \, \mathrm{d}t$, with rationing.
Black dashed line: cumulated price adjustment rate, $100 \int_0^{\frac{1}{4}} \lambda_t \, \mathrm{d}t$, without rationing.

Figure 9: The three-month Phillips curve with (solid lines) and without (dashed lines) rationing.

All variables in percent. All variables are relative to the no-shock counterfactual.

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