Exact Inflation Targeting with Inflation Swaps

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SUPER PRELIMINARY!

Equilibrium selection in monetary economics

- Convention in New Keynesian macro: Select the equilibrium in which inflation is eventually stable.
  - Not justified: See Cochrane (2011). No transversality condition rules out non-bounded inflation.
- Convention in the FTPL literature: Select the equilibrium in which government debt prices are eventually stable.
  - Not justified: See Holden (2023). With multi-period debt, active fiscal only implies a lower bound on the price level.
- Both the NK and FTPL equilibrium selection mechanisms seem implausible.
  - Under NK selection: Sufficient that monetary policy eventually turns active (even after 1000 years). Active now is irrelevant.
  - Under FTPL selection: Sufficient that fiscal policy eventually turns active (even after 1000 years). Active now is irrelevant.
- Can CBs/governments determine inflation without relying on asymptotic promises?

Recap: Filtering equilibria

- Assume no uncertainty. Start in period 0. Take the Fisher equation and monetary rule (real rate rule (Holden 2023)):

$$i_t = r_t + \pi_{t+1}, \qquad i_t = r_t + \phi_t \pi_t.$$

- So: $\pi_t = \phi_{t-1}\pi_{t-1}$ for $t \geq 1$, where π_0 can take any value. Infinitely many solutions independent of path of ϕ_t.

- NK selection: Assume $\pi_t \to 0$ as $t \to \infty$. Then if $\left|\prod_{t=0}^{\infty} \phi_t\right| \geq 1$, then $\pi_t \equiv 0$ is the unique solution.
  - Exogenously filtering out all those other asymptotically explosive equilibria.
  - Not justified by transversality, at least under flexible prices (Cochrane 2011).
- With multi-period debt, FTPL relies on a similar ad hoc pruning of equilibria.

- Central banks/governments try to do two things: set the monetary rule, and somehow impose the terminal condition.
- This paper: Instead, do two things now!

Nominal and real inflation swaps

- Standard “nominal” inflation swaps (already traded):
  - Contract agreed at t between parties A and B.
  - Party A promises to make a net payment of $\Pi_{t+1} - J_t$ to party B in period $t+1$.
  - Π_{t+1} is realized gross inflation.
  - J_t is the negotiated contract rate on these nominal inflation swaps.

- “Real” inflation swaps (not currently traded):
  - As before, but party A promises to make a net payment of $\Pi_{t+1}(\Pi_{t+1} - K_t)$ to party B in period $t+1$.
  - K_t is the contract rate on these real inflation swaps.

- Asset pricing. Let Ξ_{t+1} be the real SDF between t and $t+1$. Then with competitive swap pricing:

$$0 = \mathbb{E}_t \frac{\Xi_{t+1}}{\Pi_{t+1}} (\Pi_{t+1} - J_t), \qquad 0 = \mathbb{E}_t \Xi_{t+1} (\Pi_{t+1} - K_t)$$

N/R Swap Targeting

- Suppose the central bank intervenes in both the nominal swap market, and the real swap market.

- They set $J_t = K_t = \Pi^*_{t+1|t}$.
  - $\Pi^*_{t+1|t}$ is the CB's target for gross inflation in period $t+1$. A time-varying short-run inflation target. Not a varying long-run target!
  - This means they: accept any contract offering them $\Pi_{t+1} - \Pi^*_{t+1|t} + \varepsilon$ or $\Pi_{t+1}\left(\Pi_{t+1} - \Pi^*_{t+1|t} + \varepsilon\right)$ in period $t+1$, for any $\varepsilon > 0$.
  - And they: offer market participants unlimited contracts paying $\Pi_{t+1} - \Pi^*_{t+1|t} - \varepsilon$ or $\Pi_{t+1}\left(\Pi_{t+1} - \Pi^*_{t+1|t} - \varepsilon\right)$ in period $t+1$, for any $\varepsilon > 0$.

- Nominal rates are left to float freely.
  - To switch to this rule, the CB would gradually decrease ε, while increasing the width of their nominal rate target.

Implications

- The swap pricing equations imply:

$$\left[\mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \Pi_{t+1}^{-1}\right]^{-1} = \Pi_{t+1|t}^* = \mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \Pi_{t+1}.$$

- But by Jensen's inequality:

$$\left[\mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \Pi_{t+1}\right]^{-1} \le \mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \Pi_{t+1}^{-1}$$

  - With equality if and only if there exists some $\Phi_t > 0$ (known at t) such that $\mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \mathbb{1}[\Pi_{t+1} = \Phi_t] = 1$.
  - (Because $z \mapsto z^{-1}$ is strictly convex for positive z, and $\frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}}$ defines a (risk neutral) probability measure.)
  - But $\mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \Pi_{t+1}^{-1} = \left(\Pi_{t+1|t}^*\right)^{-1} = \mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \Pi_{t+1}$ so there is indeed equality!
  - Thus: inflation is at target with probability one under risk neutral measure. With $\Xi_{t+1} > 0$ this implies $\Pr_t\left(\Pi_{t+1} = \Pi_{t+1|t}^*\right) = 1$.

- The CB hits its target with probability one! No asymptotic condition is needed.

How can the central bank set two prices?

- Standard argument: There is one source of nominal indeterminacy (the price level), so the CB can set one price.

- Note 1: Log-linearizing asset pricing conditions gives: $0 = \mathbb{E}_t \pi_{t+1} - j_t$, $0 = \mathbb{E}_t \pi_{t+1} - k_t$. (Lower case log deviations.)
  - Identical! So, setting $j_t = k_t = \pi^*_{t+1|t}$ is consistent with both equations.
- Note 2: Suppose the CB just set $j_t = \pi^*_{t+1|t}$. By Fisher equation, this is equivalent to CB setting $i_t = r_t + \pi^*_{t+1|t}$.
  - This monetary rule is indeterminate. Zero response to inflation. It means $\mathbb{E}_t \pi_{t+1} = \pi^*_{t+1|t}$, which leaves π_t free.
  - Although in setting j_t the CB has set a price, they have not changed the original nominal indeterminacy.
- Note 3: Setting multiple prices perhaps seems less weird in a ZLB/QE context. For example:
  - Suppose there are multiple equilibria at the ZLB, one with a long ZLB stay, and one with a short ZLB stay.
  - The long ZLB stay equilibria has high government bond prices. By setting a low government bond price, can select good eq.

Choosing the target

- The time-varying short-run target $\Pi^*_{t+1|t}$ must satisfy a few conditions:
  - The target for period $t+1$ inflation must be announced in period t.
  - The economy must be determinate if the asset pricing and targeting equations are replaced with the equation $\Pi_t = \Pi^*_{t-1|t}$.
  - The target should be consistent with the ZLB on one period bonds. $I_t = [\mathbb{E}_t \Xi_{t+1}]^{-1} \Pi^*_{t+1|t} = R_t \Pi^*_{t+1|t} \geq 1$.
  - Can instead use a modified target of $\breve{\Pi}^*_{t+1|t} = \max\{R_t^{-1}, \Pi^*_{t+1|t}\}$ following Holden (2023).

- Being determined one period in advance changes optimal policy calculations. For example:
  - Suppose the economy has the Phillips curve: $\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + \kappa \omega_t$, and the CB wants to minimise $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda x_t^2]$.
  - With the target set one period in advance, CB minimises $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\pi_{t|t-1}^2 + \lambda x_t^2]$ subject to $\pi_{t|t-1} = \beta \pi_{t+1|t} + \kappa x_t + \kappa \omega_t$.
  - Solution: $\pi^*_{t+1|t} = -\kappa^{-1} \lambda \mathbb{E}_t (x_{t+1} - x_t)$. (Commitment=Discretion here.)

How does setting swap prices set inflation?

- General equilibrium magic???

- Given the NK terminal condition, there are other approaches to hitting a target every period.
  - E.g.: Follow a real rate rule (Holden 2023), or a standard Taylor rule with $\phi \approx \infty$.
  - These approaches also seem like "equilibrium magic".

- Even if how future inflation is pinned down is clear, what stops price setters / markets from deviating today?
  - One case in which no deviation is intuitive follows.

A more intuitive special case

- Consider an OLG economy with two life stages. Young earn $y_t = a_t l_t$ units of consumption. Old earn nothing.

- Household born at t maximizes $\mathbb{E}_t[\log c_{0,t} - l_t + \beta \log c_{1,t+1}]$ such that:

$$P_t c_{0,t} + B_t + \mathrm{T}_t = P_t a_t l_t, \qquad P_t c_{1,t} = I_{t-1} B_{t-1} + F_{t-1}\left(\Pi_t - \Pi^*_{t|t-1}\right) + G_{t-1}\Pi_t\left(\Pi_t - \Pi^*_{t|t-1}\right)$$

- Optimum: $c_{0,t} = a_t$, $b_t = \beta a_t$, $l_t = 1 + \beta + \frac{\tau_t}{a_t}$, $c_{1,t} = \beta a_t + \tau_t$. (Lower case t-dated is divided by P_t.)

- Government budget constraint: $b_t + \tau_t = \frac{I_{t-1}}{\Pi_t} b_{t-1} + \frac{f_{t-1}}{\Pi_t}\left(\Pi_t - \Pi^*_{t|t-1}\right) + g_{t-1}\left(\Pi_t - \Pi^*_{t|t-1}\right)$.

- In desired equilibrium, $\Pi_t = \Pi^*_{t|t-1}$, so $\tau_t = I_{t-1} \frac{\beta}{\Pi^*_{t|t-1}} a_{t-1} - \beta a_t$. Suppose government sets this τ_t irrespective of Π_t.
  - Then market clearing implies: $0 = \left[\Pi^*_{t|t-1}(f_{t-1} + g_{t-1}\Pi_t) - \beta I_{t-1} a_{t-1}\right]\left(\Pi_t - \Pi^*_{t|t-1}\right)$.
  - If g_{t-1} is non-positive and f_{t-1} is not too positive, then $\Pi_t = \Pi^*_{t|t-1}$ is the unique solution.
  - Unexpected payoff is inflationary. Want this to happen when $\Pi_t < \Pi^*_{t|t-1}$.

Conclusion

- The central bank can hit a given time-varying inflation target with probability one.
- Real inflation swap markets do not currently exist but could be easily created (by CB or private sector).
- No need to rely on equilibrium selection based on dubious asymptotic assumptions.
- To move to N/R swap targeting CB would gradually increase interest rate corridor while tightening spread corridors.
- Is this too good to be true? Why?

Extra slides

References

Cochrane, John H. 2011. 'Determinacy and Identification with Taylor Rules'. *Journal of Political Economy* 119 (3): 565–615.

Holden, Tom D. 2023. *Robust Real Rate Rules*. Kiel, Hamburg: ZBW - Leibniz Information Centre for Economics.