Exact Inflation Targeting with Inflation Swaps

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Abstract: Central banks can remove all uncertainty from future inflation by simultaneously intervening in two different inflation swap markets, leaving nominal rates to float freely. The first swap type returns gross inflation minus the contract rate. The second, returns gross inflation times gross inflation minus the contract rate. If the central bank sets both contract rates equal to the inflation target, then a Jensen's inequality argument implies that inflation is at target with probability one. The central bank is able to fix two prices simultaneously as fixing just one would produce indeterminacy. Fixing both pins down inflation without invoking any dubious terminal conditions on inflation. With a time-varying target, this enables the central bank to robustly implement near-optimal policy.

Keywords: inflation targeting, inflation swaps, monetary policy, determinacy

JEL codes: E52, E43, E31

Central banks would like to control future inflation. With the ability to pin future inflation at an arbitrary level, they could remove all inflation risk from the economy. If they allow the target to vary over the business cycle, then they could also implement fully optimal monetary policy in the continuous adjustment limit. Such perfect control of future inflation may sound impossible. However, this paper shows that the central

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bank can achieve it by intervening in appropriate inflation swap markets. Moreover, this strategy produces a unique equilibrium, without relying on ruling out certain equilibria on theoretical grounds, as is common in New Keynesian macroeconomics.

We suppose that there are markets in two different varieties of inflation swaps in the economy. The first variety, we refer to as a "nominal inflation swap". Liquid markets in such swaps already exist. A nominal inflation swap is a contract agreed in period t between two parties, A and B, in which party A promises to make a net payment of $\Pi_{t+1} - J_t$ to party B in period $t+1$, where J_t is the contract rate agreed at t, and where Π_{t+1} is gross inflation between periods t and $t+1$. No payments are made in period t.

The other variety of inflation swap we call a "real inflation swap". At present, there are no markets in this asset, but there is no reason there should be any difficulty in creating such markets. Like a nominal inflation swap, a real inflation swap is a contract agreed in period t between two parties, A and B, with no payments in period t. However, with a real inflation swap, party A promises to make a net payment of $\Pi_{t+1}(\Pi_{t+1} - K_t)$ to party B in period $t+1$, where K_t is the contract rate agreed at t. K_t will in general differ from J_t. If they were equal, then the real inflation swap's pay-out would be Π_{t+1} times that of the nominal inflation swap, thus adjusting it for inflation.

Let $\Xi_{t+1} > 0$ be the economy's real stochastic discount factor between periods t and $t+1$, and let $\mathbb{E}_t$ denote expectations conditional on period t information. Then in equilibrium with competitive swap markets, the two contract rates must satisfy:

$$0 = \mathbb{E}_t \frac{\Xi_{t+1}}{\Pi_{t+1}} (\Pi_{t+1} - J_t), \tag{1}$$

and:

$$0 = \mathbb{E}_t \Xi_{t+1} (\Pi_{t+1} - K_t). \tag{2}$$

We suppose that the central bank intervenes in both the nominal and real inflation swap markets, but does not intervene in nominal debt markets. In particular, we assume they set $J_t = K_t = \Pi^*_{t+1|t}$, where $\Pi^*_{t+1|t}$ is their target for gross inflation in period $t+1$. This means that they accept any contract offering them $\Pi_{t+1} - \Pi^*_{t+1|t} + \varepsilon$

or $\Pi_{t+1}\left(\Pi_{t+1} - \Pi^*_{t+1|t} + \varepsilon\right)$ in period $t+1$, for any $\varepsilon > 0$, and that they offer market participants unlimited contracts paying $\Pi_{t+1} - \Pi^*_{t+1|t} - \varepsilon$ or $\Pi_{t+1}\left(\Pi_{t+1} - \Pi^*_{t+1|t} - \varepsilon\right)$ in period $t+1$, for any $\varepsilon > 0$. We call this approach "N/R swap targeting" in the below.

With $J_t = K_t = \Pi^*_{t+1|t}$, equations (1) and (2) imply that:

$$\Pi^*_{t+1|t} = \left[\mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \Pi^{-1}_{t+1}\right]^{-1}, \tag{3}$$

and:

$$\Pi^*_{t+1|t} = \mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \Pi_{t+1}. \tag{4}$$

One solution to these two equations is $\Pi_{t+1} = \Pi^*_{t+1|t}$, meaning that inflation hits its target every period. We now just have to prove that this is unique.

Note that the mapping $z \mapsto z^{-1}$ is strictly convex for positive z, so by Jensen's inequality, and the fact that $\frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}}$ defines a probability measure:

$$\left[\mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \Pi_{t+1}\right]^{-1} \le \mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \Pi^{-1}_{t+1},$$

with equality if and only if there exists some $\Phi_t > 0$ (known at t) such that:

$$\mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \mathbb{1}[\Pi_{t+1} = \Phi_t] = 1, \tag{5}$$

where $\mathbb{1}$ is the indicator function. But, by equations (3) and (4), in fact:

$$\left[\mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \Pi_{t+1}\right]^{-1} = \left(\Pi^*_{t+1|t}\right)^{-1} = \mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \Pi^{-1}_{t+1},$$

so there is indeed equality. Thus, there is some $\Phi_t > 0$ satisfying (5). Suppose for a contradiction that despite this, $\Pr_t(\Pi_{t+1} = \Phi_t) < 1$, (where $\Pr_t$ denotes probability conditional on period t information). In that case, $\Pr_t(\Pi_{t+1} \ne \Phi_t) > 0$, so $\mathbb{E}_t\left[\frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \middle| \Pi_{t+1} \ne \Phi_t\right]$ is well-defined and positive, as $\Xi_{t+1} > 0$. But then:

$$\begin{aligned} 0 &= \mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} - \mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \mathbb{1}[\Pi_{t+1} = \Phi_t] = \mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \mathbb{1}[\Pi_{t+1} \ne \Phi_t] \\ &= \mathbb{E}_t \left[\frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \middle| \Pi_{t+1} \ne \Phi_t\right] \Pr_t(\Pi_{t+1} \ne \Phi_t) > 0, \end{aligned}$$

giving a contradiction. Hence, $\Pr_t(\Pi_{t+1} = \Phi_t) = 1$, and so from equation (4), in fact $\Pr_t\left(\Pi_{t+1} = \Pi^*_{t+1|t}\right) = 1$. Thus, the central bank succeeds in hitting its target every period, with probability one.

A central bank could also ensure it hits its target every period if they set nominal interest rates using either the "real rate rule" of Holden (2022) or using a traditional Taylor type rule (Woodford 2003) with an infinite coefficient on inflation. However, such rules only succeed in getting inflation to target if equilibria with explosive inflation are ruled out for some reason. Cochrane (2011) argues that there is no good reason for pruning away these "bad" equilibria. By contrast, the "N/R swap targeting" approach to monetary policy outlined above generates only one equilibrium. There are no other "bad" equilibria to be pruned away based on controversial criteria. If central bankers have less than absolute belief in the New Keynesian equilibrium selection mechanism, then they should prefer implementing their inflation target via nominal and real inflation swaps.

The rest of this paper further examines inflation control via nominal and real inflation swaps. TODO

1 TODO

2 Conclusion

TODO

References

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Online Appendix to: “Exact Inflation Targeting with Inflation Swaps”

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Appendix A TODO

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