Exact Inflation Targeting with Inflation Swaps

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Central banks can eliminate future inflation uncertainty by intervening in two inflation swap markets simultaneously, leaving nominal rates to float freely. The first swap pays the difference between gross inflation and its contract rate; the second pays gross inflation times the difference between gross inflation and its contract rate. Setting both contract rates to the central bank's short-run inflation target guarantees that inflation hits this target with probability one, via Jensen's inequality. It does this without relying on dubious terminal conditions on inflation. With a time-varying target, this enables the robust implementation of near-optimal monetary policy.

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Central banks would like to control future inflation. With the ability to pin future inflation at an arbitrary level, they could remove all inflation risk from the economy. If they allow the target to vary over the business cycle, then they could also implement fully optimal monetary policy in the continuous adjustment limit. Such perfect control of future inflation may sound impossible. However, this paper shows that the central bank can achieve it by intervening in appropriate

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inflation swap markets. Moreover, this strategy produces a globally unique equilibrium, without relying on ruling out certain equilibria on theoretical grounds, as is common in New Keynesian macroeconomics.

A central bank could also ensure it hits its target every period if they set nominal interest rates using either the "real rate rule" of Holden (2024) or using a traditional Taylor type rule (Woodford, 2003) with an extremely large coefficient on inflation. However, such rules only succeed in getting inflation to target if equilibria with explosive inflation are ruled out for some reason. Cochrane (2011) argues that there is no good reason for pruning away these "bad" equilibria, as transversality conditions only rule out explosions in real quantities, not in prices. Similarly, Holden (2024) argues that with geometric-coupon debt, the Fiscal Theory of the Price Level also relies on pruning away "bad" (bubbly) equilibria for unjustified reasons.

Even if one rejects the arguments of Cochrane (2011) for some reason, at best the standard rational expectations equilibrium selection mechanism looks implausible. For example, under the standard selection mechanism with passive fiscal policy (Leeper, 1991), it is completely irrelevant to determinacy whether the central bank follows the Taylor principle for the next 1000 years, as long as they can commit to following it from year 1001 onwards. While assuming agents are learning may prevent such paradoxes, we do not want price level determination to depend on the less than full rationality of agents.

By contrast, the "nominal/real swap targeting" approach outlined in this paper generates only one equilibrium. There are no other "bad" equilibria to be pruned away based on controversial asymptotic criteria. If central bankers have less than absolute belief in the standard equilibrium selection mechanism, then they should prefer implementing their inflation target via nominal and real inflation swaps.

I start by reviewing the conventional approach to equilibrium determination in New Keynesian models in Section I. I then introduce nominal and real inflation swaps in Section II, specify the targets for the contract rates on these swaps in

Section III, and demonstrate how they pin down future inflation with probability one in Section IV. I discuss why the central bank can set two prices in Section V, and discuss what out-of-equilibrium forces drive inflation to target in Section VI. Finally, I examine how these targets should be set in Section VII.

I. The conventional New Keynesian equilibrium determination mechanism

To fix ideas, let's start by briefly reviewing the standard New Keynesian equilibrium determination mechanism in a linearised setup.

Suppose the economy starts in period 0 and there is no uncertainty. And suppose the central bank sets nominal interest rates using the time-varying real rate rule (Holden, 2024) $i_t = r_t + \phi_t \pi_t$ for all t, where i_t is the nominal interest rate, r_t is the real interest rate, and π_t is inflation. Then by the Fisher equation ($i_t = r_t + \pi_{t+1}$), $\pi_t = \phi_{t-1}\pi_{t-1}$ for all $t \geq 1$, but π_0 is undetermined. There are a continuum of paths for inflation indexed by π_0, independent of the path of ϕ_t.

To produce determinacy needs another constraint. The New Keynesian literature imposes the additional assumption that $\pi_t \to 0$ as $t \to \infty$. It is this assumption that Cochrane (2011) argues to be unjustified. Given this assumption, though, if $|\prod_{t=0}^{\infty} \phi_t| \geq 1$, then $\pi_t = 0$ is the unique solution. Since an infinite product of numbers greater than 1 is infinite, this product condition makes clear that it is enough for the central bank to start following the Taylor principle (i.e. to set $\phi_t > 1$) at some arbitrarily distant time in the future for inflation to be determined today.[1] But it seems incredibly implausible that central bank decisions 1000 years from now should have an impact on outcomes today. This paper presents an approach to equilibrium determination that avoids this dependence on the future actions of the central bank, while also avoiding ad hoc equilibrium selection criteria like the one used in the New Keynesian literature.

[1]As long as for all t, $\phi_t \neq 0$.

II. Nominal and Real Inflation Swaps

I suppose that there are markets in two different varieties of inflation swaps in the economy. The first variety I refer to as a "nominal inflation swap". Liquid markets in such swaps already exist. A nominal inflation swap is a contract agreed in period t between two parties, A and B, in which party A promises to make a net payment of $\Pi_{t+1} - J_t$ to party B in period $t+1$, where J_t is the contract rate agreed at t, and where Π_{t+1} is gross inflation between periods t and $t+1$. No payments are made in period t.

The other variety of inflation swap I call a "real inflation swap". At present, there are no markets in this asset, but there is no reason why there should be any difficulty in creating such markets. Like a nominal inflation swap, a real inflation swap is a contract agreed in period t between two parties, A and B, with no payments in period t. However, with a real inflation swap, party A promises to make a net payment of $\Pi_{t+1}(\Pi_{t+1} - K_t)$ to party B in period $t+1$, where K_t is the contract rate agreed at t. K_t will in general differ from J_t. Just as the pay-out on a real bond is its gross rate times realised gross inflation, so too the real inflation swap's pay-out is an unadjusted return $(\Pi_{t+1} - K_t)$ times realised gross inflation. Indeed, the "fixed" leg of the real inflation swap looks just like a real bond.

Let $\Xi_{t+1} > 0$ be the economy's real stochastic discount factor between periods t and $t+1$, and let $\mathbb{E}_t$ denote expectations conditional on period t information. Then, in equilibrium with competitive swap markets, the two contract rates must satisfy:

$$0 = \mathbb{E}_t \left[\frac{\Xi_{t+1}}{\Pi_{t+1}} (\Pi_{t+1} - J_t) \right], \tag{1}$$

and:

$$0 = \mathbb{E}_t \left[\Xi_{t+1} (\Pi_{t+1} - K_t) \right]. \tag{2}$$

These pricing equations can be rewritten in terms of the gross return on nominal bonds, I_t, and the gross return on real bonds, R_t, using the facts that: $1 = I_t \mathbb{E}_t \left[\frac{\Xi_{t+1}}{\Pi_{t+1}}\right]$ and $1 = R_t \mathbb{E}_t \left[\Xi_{t+1}\right]$. Hence:

$$I_t = R_t J_t, \tag{3}$$

and:

$$R_t \mathbb{E}_t \left[\Xi_{t+1} \Pi_{t+1}\right] = K_t. \tag{4}$$

III. Nominal/Real Swap Targeting

Suppose that the central bank intervenes in both the nominal and real inflation swap markets, but does not intervene in nominal debt markets, meaning that nominal rates are left to float freely. This means that the central bank sets some target J_t^* for the nominal inflation swap contract rate, and some target K_t^* for the real inflation swap contract rate. They stand ready to accept any contract offering them $\Pi_{t+1} - J_t^* + \varepsilon$ or $\Pi_{t+1}(\Pi_{t+1} - K_t^* + \varepsilon)$ next period, for any $\varepsilon > 0$, and they offer market participants unlimited contracts paying $\Pi_{t+1} - J_t^* - \varepsilon$ or $\Pi_{t+1}(\Pi_{t+1} - K_t^* - \varepsilon)$ next period, for any $\varepsilon > 0$. To transition to this rule in practice, the central bank would gradually increase the width of their nominal rate corridor while tightening their swap market spread corridors (i.e. the minimum ε).

In particular, I assume they set $J_t^* = K_t^* = \Pi_{t+1|t}^*$, where $\Pi_{t+1|t}^*$ is their short-run target for gross inflation in period $t + 1$. This is not their long-run target (of 2% say), but a short-run time-varying target determined in period t based on current economic conditions. I call this approach "nominal/real swap targeting".

Given the central bank's market interventions, the actual market-clearing contract rates must actually satisfy $J_t = K_t = \Pi_{t+1|t}^*$. The only way this could fail would be if there were no market equilibrium consistent with $J_t = K_t = \Pi_{t+1|t}^*$, but this is clearly not the case, since $\Pi_{t+1} = \Pi^*_{t+1|t} = J_t = K_t$ is consistent with equations (1) and (2).

Finally, note that with $J_t = \Pi^*_{t+1|t}$, equation (3) implies:

$$I_t = R_t \Pi^*_{t+1|t}, \tag{5}$$

so nominal interest rates are pinned down by a Fisher-like relationship.

IV. Uniqueness

With $J_t = K_t = \Pi^*_{t+1|t}$, equations (1) and (2) imply that:

$$\Pi^*_{t+1|t} = \left(\mathbb{E}_t\left[\frac{\Xi_{t+1}}{\Pi_{t+1}}\right]\right)^{-1} \mathbb{E}_t[\Xi_{t+1}], \tag{6}$$

and:

$$\Pi^*_{t+1|t} = \frac{\mathbb{E}_t[\Xi_{t+1}\Pi_{t+1}]}{\mathbb{E}_t[\Xi_{t+1}]}. \tag{7}$$

As already mentioned, one solution to these two equations is $\Pi_{t+1} = \Pi^*_{t+1|t}$, meaning that inflation hits its target every period. I now prove that this is unique.

Note that the mapping $z \mapsto z^{-1}$ is strictly convex for positive z, so by Jensen's inequality, and the fact that $\Xi_{t+1}/\mathbb{E}_t[\Xi_{t+1}]$ defines a probability measure (the "real forward measure"—the real version of the risk neutral measure):

$$\left(\mathbb{E}_t\left[\frac{\Xi_{t+1}}{\mathbb{E}_t[\Xi_{t+1}]}\Pi_{t+1}\right]\right)^{-1} \le \mathbb{E}_t\left[\frac{\Xi_{t+1}}{\mathbb{E}_t[\Xi_{t+1}]}\Pi_{t+1}^{-1}\right], \tag{8}$$

with equality if and only if there exists some $\Phi_t > 0$ (known at t) such that:

$$\mathbb{E}_t\left[\frac{\Xi_{t+1}}{\mathbb{E}_t[\Xi_{t+1}]}\mathbf{1}_{\{\Pi_{t+1}=\Phi_t\}}\right] = 1, \tag{9}$$

where $\mathbf{1}$ is the indicator function.

But, inverting equations (6) and (7), we find that in fact:

$$\mathbb{E}_t\left[\frac{\Xi_{t+1}}{\mathbb{E}_t[\Xi_{t+1}]}\Pi_{t+1}^{-1}\right]=(\Pi^*_{t+1|t})^{-1}=\left(\mathbb{E}_t\left[\frac{\Xi_{t+1}}{\mathbb{E}_t[\Xi_{t+1}]}\Pi_{t+1}\right]\right)^{-1}. \tag{10}$$

Thus, Jensen's inequality holds with equality, and so there is indeed some $\Phi_t > 0$ satisfying (9). Suppose for a contradiction that despite this, $\Pr_t(\Pi_{t+1} = \Phi_t) < 1$, where $\Pr_t$ denotes probability conditional on period t information. In that case, $\Pr_t(\Pi_{t+1} \neq \Phi_t) > 0$. Because $\Xi_{t+1} > 0$, $\mathbb{E}_t[\Xi_{t+1} \mid \Pi_{t+1} \neq \Phi_t]$ is well-defined and positive. But then:

$$\begin{aligned}
0 &= \mathbb{E}_t\left[\frac{\Xi_{t+1}}{\mathbb{E}_t[\Xi_{t+1}]}\right]-\mathbb{E}_t\left[\frac{\Xi_{t+1}}{\mathbb{E}_t[\Xi_{t+1}]}\mathbf{1}_{\{\Pi_{t+1}=\Phi_t\}}\right]\\
&= \mathbb{E}_t\left[\frac{\Xi_{t+1}}{\mathbb{E}_t[\Xi_{t+1}]}\mathbf{1}_{\{\Pi_{t+1}\neq\Phi_t\}}\right]\\
&= \mathbb{E}_t\left[\frac{\Xi_{t+1}}{\mathbb{E}_t[\Xi_{t+1}]}\,\middle|\,\Pi_{t+1}\neq\Phi_t\right]\Pr_t(\Pi_{t+1}\neq\Phi_t)>0, \qquad (11)
\end{aligned}$$

giving a contradiction. Hence, $\Pr_t(\Pi_{t+1} = \Phi_t) = 1$, and so from equation (7), in fact $\Pr_t(\Pi_{t+1} = \Pi^*_{t+1|t}) = 1$. Thus, the central bank succeeds in hitting its target every period, with probability one.

V. How can the central bank set two prices?

There is one degree of nominal indeterminacy in an economy in which the central bank does not intervene in markets, the aggregate price level. Thus, it is natural to assume that a central bank can only set one price, to remove this one degree of freedom. You might then worry that by fixing two distinct prices (J_t and K_t), a central bank is attempting something impossible, which must backfire in some way. This fear is unwarranted.

Firstly, note that log-linearizing the two asset pricing conditions yields:

$$0 = \mathbb{E}_t \pi_{t+1} - j_t, \tag{12}$$

$$0 = \mathbb{E}_t \pi_{t+1} - k_t, \tag{13}$$

where lower case variables denote log deviations. These two conditions are identical. Setting $j_t = k_t = \pi^*_{t+1|t}$ is perfectly consistent with both equations simultaneously.

Next, consider what would happen if the central bank just set $j_t = \pi^*_{t+1|t}$ and abandoned the real inflation swap. By the Fisher equation, this is equivalent to the central bank setting nominal rates equal to $i_t = r_t + \pi^*_{t+1|t}$. This is a "peg"-type monetary rule with zero response to realized inflation. This produces global indeterminacy, as the equilibrium restriction $\mathbb{E}_t \pi_{t+1} = \pi^*_{t+1|t}$ places no restrictions on the current price level π_t. Thus, while setting $j_t = \pi^*_{t+1|t}$ sets one price, doing so does not resolve the nominal price level indeterminacy. Controlling the second price selects the single, desired equilibrium, removing the remaining indeterminacy.

In fact, post financial crisis, the idea that a central bank can set two prices should not seem so strange. Many central banks have been setting both a short rate (near the zero lower bound) and effectively setting a rate on longer assets via quantitative easing. One reason this may have been possible is that there are multiple equilibria at the ZLB (see e.g. Holden, 2023), and by setting longer rates the central bank is selecting from those equilibria.

Furthermore, in the conventional approach to price level determination, central bankers are already trying to do two things at once, even without quantitative easing. They both have to set rates today via an active rule, and to somehow enforce the terminal condition ruling out explosions (anchoring expectations). The nominal/real inflation swap targeting approach just has the central bank make two concrete actions today instead.

VI. What ensures inflation materialises at target?

We have shown that for all t, $\Pr_{t-1}(\Pi_t = \Pi^*_{t|t-1}) = 1$. But when period t arrives, why should Π_t actually equal $\Pi^*_{t|t-1}$? What stops firms from deviating and setting a different price level? We will examine this in a simple representative agent model.

Suppose in an economy that the representative household receives 1 unit of the final good per period. We look at decisions in period t, but we assume that the nominal/real inflation swap targeting regime was also in place before period t. In period t, the representative household chooses a path for their consumption c_s for $s \geq t$ to maximize:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s),$$

subject to the budget constraint

$$P_s c_s + B_s + \mathcal{T}_s = P_s + I_{s-1} B_{s-1} + F_{s-1}(\Pi_s - \Pi^*_{s|s-1}) + G_{s-1} \Pi_s (\Pi_s - \Pi^*_{s|s-1}),$$

for all $s \geq t$, where P_s is the price level, with $\Pi_s = P_s / P_{s-1}$, B_s is the quantity of nominal bonds the household purchases at s, F_s and G_s are the quantities of nominal and real inflation swaps (respectively) they "purchase" that period (buying inflation risk), and $\mathcal{T}_s$ is the net amount of taxes they pay in period s. The representative household's real stochastic discount factor is then given by: $\Xi_s = \beta \frac{u'(c_s)}{u'(c_{s-1})}$, and the arguments above apply to this economy, implying $\Pr_s(\Pi_{s+1} = \Pi^*_{s+1|s}) = 1$ for all s (including for $s < t$). We want to know whether it must be the case that $\Pi_t = \Pi^*_{t|t-1}$.

For simplicity, we suppose that the central bank sets $\Pi^*_{s+1|s} = 1$ for all s. In this case, the household's transversality condition just requires $\lim_{s \to \infty} \beta^s B_s = 0$.

We consider a game between the government and the private sector (including the representative household), and for simplicity we assume all decisions are made at the start of period t. The government sets a path for taxes; the private sector sets a path for prices and debt. Crucially, we assume that the government acts as a Stackelberg leader at the start of the game, so the government cannot condition their actions on the period $s \geq t$ price level or the period $s \geq t$ level of nominal debt.

We assume the government would like to ensure $B_s = B_s^*$ for some process B_s^* with entire path known at t, and with $B_{t-1}^* = B_{t-1}$. Since they act before households, they must act under the assumption that the household "plays" the equilibrium action and sets $\Pi_s = \Pi_{s|s-1}^*$. Thus, from imposing market clearing ($c_s = 1$), equilibrium debt ($B_s = B_s^*$) and equilibrium inflation ($\Pi_s = \Pi_{s|s-1}^*$) in the household budget constraint, they must set:

$$\mathcal{T}_s = I_{s-1}B_{s-1}^* - B_s^*.$$

Substituting this back into the household budget constraint implies:

$$P_s(c_s - 1) + (B_s - B_s^*) = I_{s-1}(B_{s-1} - B_{s-1}^*) + (F_{s-1} + G_{s-1}\Pi_s)(\Pi_s - 1).$$

Note that we are being careful not to impose goods market clearing here, as we want to see under what conditions goods markets can clear.

Since there is no uncertainty after period t, transversality implies the optimum features perfect consumption smoothing with real interest rate $R_s = \beta^{-1}$ for all $s \geq t$ (by the Euler equation), meaning $I_s = \beta^{-1}$ by (5) as $\Pi_{s+1|s}^* = 1$ for all s. Hence, at t, the household will choose $c_s = c^*$ for all $s \geq t$, where:

$$c^* = 1 + \frac{1-\beta}{P_t}\left[(F_{t-1} + G_{t-1}\Pi_t)(\Pi_t - 1)\right]. \tag{14}$$

Now, let us finally impose market clearing, so $c^* = 1$. This implies that:

$$(F_{t-1} + G_{t-1}\Pi_t)(\Pi_t - 1) = 0.$$

For $\Pi_t = 1$ to be the only solution, it must be the case that $F_{t-1} + G_{t-1}\Pi_t \neq 0$ for any $\Pi_t > 0$. The central bank can ensure this by either acting as an "inflation buyer" of both nominal and real inflation swaps (so F_{t-1} and G_{t-1} are negative), or by acting as an "inflation seller" for both types of swaps (so F_{t-1} and G_{t-1} are positive).[2] Note that since in equilibrium, $\Pi_{s+1} = \Pi^*_{s+1|s}$ for all s, households are indifferent about their positions in nominal and real inflation swaps, ensuring that the central bank's choice of these positions is not constrained by their need to set the contract rates.

Intuitively, with $F_{t-1} + G_{t-1}\Pi_t > 0$, so the household is an "inflation buyer", then if $\Pi_t > 1$, the swap payouts create windfall wealth for the household. And the government cannot tax this away since its tax path is set in advance. The household would like to consume at least some of this windfall, but since the household is already consuming its full endowment, this creates excess demand that is inconsistent with equilibrium. A similar argument applies if $\Pi_t < 1$ and/or if $F_{t-1} + G_{t-1}\Pi_t < 0$.

One might wonder if this is just the Fiscal Theory of the Price Level (FTPL) (see e.g. Cochrane, 2023). After all, the household transversality condition played a key role here in deriving (14), just as it does in the FTPL literature. But without non-zero holdings of nominal and real inflation swaps, the argument of this section does not imply a unique initial price level, unlike under the FTPL. (If $F_{t-1} = G_{t-1} = 0$, then (14) implies $c^* = 1$ even without imposing market clearing, leaving Π_t undetermined.) The difference is that whereas the FTPL literature assumes that the government commits to a path for real surpluses, here the government is only committing to a path for nominal surpluses. The government's intertemporal budget constraint cannot pin down the price level if both sides are nominal.

[2]If F_{t-1} and G_{t-1} have the same sign, then for any $\Pi_t > 0$, $F_{t-1} + G_{t-1}\Pi_t \neq 0$, whereas if F_{t-1} and G_{t-1} have opposite signs, then $\Pi_t = -F_{t-1}/G_{t-1} > 0$ is another valid solution.

VII. Choosing the target and optimal policy

Finally, we consider how the central bank should set their short-run inflation target, $\Pi^*_{t+1|t}$, announced in period t. The central bank faces two constraints on their process for setting this target.

The first is that if one were to replace the asset pricing conditions with $\Pi_t = \Pi^*_{t|t-1}$, the resulting model must remain determinate. This means the central bank cannot use rules like $\Pi^*_{t+1|t} := \mathbb{E}_t\Pi_{t+1}$ which would reintroduce indeterminacy, but it does not prevent the central bank from using the kind of instrument rules based on current observables that they would actually like to use.

The second is that the target must remain consistent with the zero lower bound on one-period bonds since $1 \le I_t = (\mathbb{E}_t\Xi_{t+1})^{-1}\Pi^*_{t+1|t} = R_t\Pi^*_{t+1|t}$, where I_t is the gross nominal rate and R_t is the gross real rate. As long as the central bank can observe real rates (via TIPS—treasury inflation protected securities—for example, see Holden (2024)), they can ensure this by setting $\Pi^*_{t+1|t} = \max(R_t^{-1}, \hat{\Pi}^*_{t+1|t})$, where $\hat{\Pi}^*_{t+1|t}$ is the central bank's desired inflation target in the absence of the zero lower bound.

How should a central bank optimally set $\Pi^*_{t+1|t}$ (or $\hat{\Pi}^*_{t+1|t}$)? Setting the target one period in advance alters the standard optimal policy problem.

Suppose the central bank chooses policy to minimize the standard quadratic loss function:

$$\mathcal{L} := \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t(\pi^2_{t|t-1} + \lambda x_t^2),$$

where x_t is the output gap and $\pi_{t|t-1} := \log \Pi^*_{t|t-1} = \log \Pi_t$ is (log) inflation, which is determined one period in advance. And suppose they optimize subject to the New Keynesian Phillips Curve $\pi_{t|t-1} = \beta\pi_{t+1|t} + \kappa x_t + \kappa\nu_t$. Because the constraint is purely backward-looking, commitment and discretion are equivalent, with solution:

$$\pi_{t+1|t} = -\lambda\kappa^{-1}\mathbb{E}_t[x_{t+1} - x_t].$$

Thus, the target for next period's inflation should respond negatively to expected growth in the output gap. When mark-up shocks are sufficiently persistent, the benefit of the one-period commitment is sufficiently great to outweigh the cost of having the output gap absorb the shock's entire initial impact, and the loss can be lower than in the standard discretion solution (without predetermined inflation). While the losses under full commitment are always lower than under the predetermined inflation regime, the losses converge to the same value in the continuous time limit (i.e. as β and the shock persistence tend to 1). Hence, if the central bank acts frequently, the nominal/real inflation swap regime is near optimal.[3]

VIII. Conclusion

The central bank can hit a given time-varying short-run inflation target with probability one using nominal/real swap targeting, without relying on dubious equilibrium selection mechanisms. While real inflation swap markets do not currently exist, they could be easily seeded by the central bank itself.

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[3]See the appendix for both results.

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Appendix

Assume a permanent cost-push shock such that $\nu_t = \nu$ for all $t \geq 0$. By continuity, results in this extreme case will also be informative about persistences below (but close to) one.

The central bank minimizes the intertemporal loss function $\mathcal{L} = \sum_{t=0}^{\infty} \beta^t(\pi_t^2 + \lambda x_t^2)$, subject to the New Keynesian Phillips Curve $\pi_t = \beta\pi_{t+1} + \kappa x_t + \kappa\nu_t$. I compare the total loss under discretion ($\mathcal{L}_D$), commitment ($\mathcal{L}_C$) and predetermined inflation ($\mathcal{L}_P$).

Under discretion, the central bank optimizes period-by-period. The standard first-order condition yields the targeting rule $\pi_t = -\lambda\kappa^{-1}x_t$. Because the shock is permanent and there are no endogenous state variables, the system immediately jumps to a constant steady state with:

$$\pi_t = \pi_D := \frac{\lambda\kappa}{\lambda(1-\beta)+\kappa^2}\nu, \quad x_t = x_D := -\frac{\kappa^2}{\lambda(1-\beta)+\kappa^2}\nu,$$

thus the total loss is:

$$\mathcal{L}_D = \frac{\kappa^2(\lambda+\kappa^2)}{(\lambda(1-\beta)+\kappa^2)^2}\frac{\lambda}{1-\beta}\nu^2.$$

With commitment, the central bank optimizes over the entire path of inflation and output gaps. The first-order conditions yield the targeting rule $\pi_0 = -\lambda\kappa^{-1}x_0$ and $\pi_t = -\lambda\kappa^{-1}(x_t - x_{t-1})$ for $t > 0$. Rather than immediately jumping to a new steady state, the system gradually adjusts to the permanent shock with:

$$\pi_t = \frac{\lambda}{\kappa}(1-\delta)\nu\delta^t, \quad x_t = -\nu(1-\delta^{t+1}),$$

where $\delta \in (0,1)$ is the stable root of $\beta\lambda\delta^2 - (\lambda(1+\beta)+\kappa^2)\delta + \lambda = 0$. Using the identity $\kappa^2\delta = \lambda(1-\delta)(1-\beta\delta)$, the total loss is:

$$\mathcal{L}_C = \frac{1-\delta}{1-\beta\delta}\frac{\lambda}{1-\beta}\nu^2.$$

In the predetermined inflation regime (as when using nominal/real swap targeting), $\pi_t = \pi_{t|t-1}$ is chosen at $t-1$. I assume the initial condition $\pi_{0|-1} = 0$, meaning inflation was at steady state before the shock hit. As in the text, the optimal targeting rule is $\pi_{t+1|t} = -\lambda\kappa^{-1}[x_{t+1} - x_t]$. Again, this results in the system jumping to a new constant steady state with:

$$\pi_t = \pi_P := 0, \quad x_t = x_P := -\nu,$$

thus the total loss is:

$$\mathcal{L}_P = \frac{\lambda}{1-\beta}\nu^2.$$

Therefore, the predetermined inflation regime features a lower central bank loss than discretion if and only if $\kappa^2(\lambda + \kappa^2) > (\lambda(1-\beta) + \kappa^2)^2$. With $\lambda > 0$, this in turn holds if and only if $(2\beta - 1)\kappa^2 > (1-\beta)^2\lambda$. As $\beta \to 1$, the left hand side tends to $\kappa^2 > 0$, while the right hand side tends to 0. Thus for β sufficiently close to 1 the predetermined inflation regime features a lower central bank loss than discretion.

The loss under commitment is always lower than that with predetermined inflation (since $\beta, \delta \in (0,1)$), but as $\beta \to 1$, the two losses converge.