Catch-up cycles.

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Abstract: *We build a DSGE model of technological catch-up in countries behind the global frontier. We go on to estimate it on Spanish and U.S. data, with the U.S. acting as the technological leader, and Spain as the follower. The model generates highly persistent movements in productivity in the emerging economy from un-persistent shocks to the technological leader, helping to explain Spanish data. Unlike the prior literature, our model would generate permanent productivity gaps between the global technological leader and other economies even were the leader's technological progress to halt, helping to explain why the great recession has not been accompanied by convergence in productivity. This is due to the diminishing returns to technological transfer in our model. It is further exacerbated by competition between firms in the following economy, which reduce each firm's share of production profits. We draw tentative conclusions on policies to enhance productivity in countries behind the global frontier.*

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1. Introduction

Aguiar & Gopinath (2007) find that emerging economies are driven by highly persistent movements in productivity. In this paper, we present a theory of the origins of such productivity movements in emerging economies, and in non-frontier countries more generally. In particular, we build a model in which firms in the less productive economy must perform a costly process of technology transfer to improve their productivity level. We go on to estimate this model on Spanish and U.S. data, with the U.S. acting as the technological leader.

Crucially, we model technology transfer as subject to decreasing returns. In particular, we suppose that it would take an infinite amount of input to reach the frontier productivity level in a single period. This reflects the idiosyncratic differences across industries and countries that make it impossible to perfectly copy a productivity process from one industry in one country, to another industry in another country. It also reflects the difficulty in parallelizing technological transfer. Many tasks in R&D, broadly construed, are inherently serial. For example, creating an app for a banking service first requires the development of servers with which the app will communicate. Thus, beyond a certain level further input is entirely ineffective.

The consequence of the strongly decreasing returns to technology transfer is that countries not performing frontier research will always remain behind the frontier productivity level. Indeed, this would remain true even were growth in the frontier nation to stop completely, since as the productivity gap falls, the returns to technology transfer soon falls below its cost. This is in marked contrast to models which merely generate delays in the adoption of technologies in lagging countries, such as that of Comin et al. (2014). In such models, it is always optimal for the lagging country to eventually adopt all of the technologies of the leader, even if the leader has ceased innovation. While we do not expect technological progress to stop in the U.S., this is still a highly relevant thought experiment, since it speaks to the model's ability to explain the divergence in outcomes between the U.S. and Europe during the great recession. In a model that just generated delays in adoption, a reduction in U.S. TFP growth would be associated with convergence between the U.S. and Europe. In our model however, a reduction in U.S. TFP growth will lead to a large recession in Europe.

Our model of technology transfer is based on that contained in Holden (2016a), a model of research in a frontier country. However, in that paper technology transfer has no effect on the dynamics of the simulated model. This is because in industries producing patent protected products the costs of technology transfer are asymptotically dominated by those of frontier R&D, while in industries producing non-patent-protected products, the entry costs are so low asymptotically that all incentives for technology transfer are destroyed. Consequently, by examining the dynamics of technology transfer in non-frontier countries, this paper can also give insight into the non-asymptotic behaviour of industries producing non-patent-protected products in frontier countries.

The internationally tradeable final good in our model is produced from the differentiated industry goods of a continuum of industries. These industries each contain finitely many firms, each of which is producing a slightly differentiated product, thanks to branding for example. Firms enter the period before they start production, paying a fixed cost of managerial labour, and undertaking costly technology transfer to increase their next period productivity. We assume that there is free transfer of technologies within an industry, thus individual firms will not have any state variables. This means we can treat each firm as only existing for two periods, as in the model of Holden (2016a). The input to the technology transfer process will be the output of a "technology transfer services" sector, producing using an aggregate of sector specific capital and sector specific labour. Likewise, the input to the firms' production will be the output of a "production services" sector, again producing using an aggregate of sector specific capital and sector specific labour. The model is closed with households who can hold foreign bonds, following a standard small open economy set-up.

From calibrating and estimating our model of technology transfer on data from Spain and the U.S., we find that Spanish outcomes are highly influenced by shocks to TFP in the U.S.. Indeed, U.S. TFP shocks emerge as the largest driver of labour supply movements in Spain, speaking to the strength of the international transmission mechanism here. This is despite our model having minimal links between countries beyond the technology transfer mechanism: net exports are effectively a residual in our model. We also find an important role for shocks to managerial labour supply, with increases in managerial labour associated with large declines in productivity over the medium term. This is because the increase in managerial labour leads to greater firm entry, reducing each firm's share of production profits, and thus depressing technological transfer incentives. Indeed, our estimated model implies that Spain's productivity would be higher if there were fewer firms producing each product, thus "more competition", crudely understood, is not necessarily the solution.

What would help productivity in the lagging country according to our model is a reduction in the mark-ups that firms can charge even when they have many competitors in the same industry. These mark-ups might reflect market power coming from branding, advertising, locally segmented markets, consumer habits or anti-competitive behaviour. A reduction in this distortion would increase the level of relative productivity in the lagging country far more than would be suggested by the mark-up wedge on its own.

The rest of the paper is structured as follows. In Section 2 we describe the model, and its theoretical behaviour. Section 3 describes our calibration and estimation procedure, then Section 4 presents our numerical results. We conclude in Section 5, giving also some tentative policy suggestions.

2. The model

We now present our model. We start by describing its various components, before turning to some limited theoretical results on its steady-state and growth rate.

2.1. Productive services

A perfectly competitive sector supplies productive services S_t at a price of $P_{S,t}$. The firms in this sector have access to the Cobb-Douglas production technology:

$$S_t = K_{S,t-1}^{\alpha_S}(Z_{S,t}L_{S,t})^{1-\alpha_S},$$

where $K_{S,t-1}$ is the amount of capital rented in the sector, and $L_{S,t}$ is the amount of labour hired in the sector. Firms choose inputs to maximise their profits, taking as given the rental rate of capital, $\mathcal{R}_{S,t}$, and the wage, $W_{S,t}$, in the sector. This implies that:

$$\alpha_S P_{S,t} S_t = \mathcal{R}_{S,t} K_{S,t-1},$$
$$(1-\alpha_S) P_{S,t} S_t = W_{S,t} L_{S,t}.$$

2.2. Technology transfer services

Likewise, technology transfer services T_t are produced by a perfectly competitive sector with access to the Cobb-Douglas production technology:

$$T_t = \left(\frac{K_{T,t-1}}{A_t^*}\right)^{\alpha_T} (Z_{T,t}L_{T,t})^{1-\alpha_T},$$

where $K_{T,t-1}$ is the amount of capital rented in the sector, $L_{T,t}$ is the amount of labour hired in the sector, and A_t^* is the international frontier productivity level. Capital in this sector should be interpreted as including both advanced physical capital, as well as human capital. Here capital is divided by frontier productivity to capture the fact that transferring more complicated technologies requires more advanced machines and knowledge. It is also necessary to ensure stationarity in this model, though it could be avoided in a richer model with entry of new products. Firms choose inputs to maximise their profits, taking as given the rental rate of capital, $\mathcal{R}_{T,t}$, and the wage, $W_{T,t}$, in the sector. Denoting the price of technology transfer services by $P_{T,t}$, this implies that:

$$\alpha_T P_{T,t} T_t = \mathcal{R}_{T,t} K_{T,t-1},$$
$$(1-\alpha_T) P_{T,t} T_t = W_{T,t} L_{T,t}.$$

2.3. Aggregators

The final good Y_t is produced by a perfectly competitive industry from the aggregated output $Y_t(i)$ of each industry $i \in [0,1]$, using the following CES technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda}} di\right]^{1+\lambda}$$

where $\frac{1+\lambda}{\lambda}$ is the elasticity of substitution between goods. We normalize the price of the final good to 1, and we assume that it is freely tradeable internationally. We will assume that other countries are producing a similar good that is a perfect substitute for this one.

Each industry aggregate good $Y_t(i)$ is produced by a perfectly competitive industry from the intermediate goods $Y_t(i,j)$ for $j \in \{1, \dots, J_{t-1}(i)\}$,[2] using the technology:

$$Y_t(i) = J_{t-1}(i) \left[\frac{1}{J_{t-1}(i)} \sum_{j=1}^{J_{t-1}(i)} Y_t(i,j)^{\frac{1}{1+\eta\lambda}} \right]^{1+\eta\lambda}$$

where $\eta \in (0,1)$ controls the degree of differentiation between firms, relative to that between industries. Here, the exponent on the number of firms ($J_{t-1}(i)$) has been chosen to remove any preference for variety within an industry. We assume that the industry aggregate from industry i has price $P_t(i)$ and that the intermediate good from firm j in industry i has price $P_t(i,j)$. Profit maximisation then implies that:

$$Y_t(i) = P_t(i)^{-\frac{1+\lambda}{\lambda}} Y_t,$$

and:

$$Y_t(i,j) = \frac{Y_t(i)}{J_{t-1}(i)} \left(\frac{P_t(i,j)}{P_t(i)} \right)^{-\frac{1+\eta\lambda}{\eta\lambda}}.$$

So:

$$P_t(i) = \left[\frac{1}{J_{t-1}(i)} \sum_{j=1}^{J_{t-1}(i)} P_t(i,j)^{-\frac{1}{\eta\lambda}} \right]^{-\eta\lambda},$$

and:

$$1 = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda}} \right]^{-\lambda}.$$

2.4. Intermediate firms

Pricing

Firm j in industry i has access to the linear production technology $Y_t(i,j) = A_t(i,j) S_t(i,j)$ for production in period t, where $A_t(i,j)$ is their productivity and $S_t(i,j)$ is their input of production services. As in Jaimovich (2007), strategic profit maximisation then implies that in an equilibrium symmetric across firms in the same industry, the price of the good sold by firm j in industry i is given by $P_t(i,j) = P_t(i) = \left(1 + \mu_{t-1}(i)\right) \frac{P_{S,t}}{A_t(i,j)} = \left(1 + \mu_{t-1}(i)\right) \frac{P_{S,t}}{A_t(i)}$, where $\mu_t(i) := \lambda \frac{\eta J_t(i)}{J_t(i) - (1-\eta)} \in (\eta\lambda, \lambda]$ is the industry i mark-up in period $t+1$ and $A_t(i) = A_t(i,j)$ is the productivity shared by all firms in industry i in symmetric equilibrium. From aggregating

[2] The $t-1$ subscript reflects the fact that firms enter one period before production.

across industries, we then have that $P_{S,t} = \frac{A_t}{1+\mu_{t-1}}$ where $\frac{1}{1+\mu_t} = \left[\int_0^1 \left(\frac{1}{1+\mu_t(i)}\right)^{\frac{1}{\lambda}} di\right]^{\lambda}$ determines the aggregate mark-up μ_{t-1} and where:

$$A_t := \frac{\left[\int_0^1 \left(\frac{A_t(i)}{1+\mu_{t-1}(i)}\right)^{\frac{1}{\lambda}} di\right]^{\lambda}}{\left[\int_0^1 \left(\frac{1}{1+\mu_{t-1}(i)}\right)^{\frac{1}{\lambda}} di\right]^{\lambda}}$$

is a measure of the aggregate productivity level.[3] The equilibrium we solve for will also be symmetric across industries, implying that in fact $\mu_t = \mu_t(i)$ and $A_t = A_t(i)$ for all i.

Entry and technology transfer

Following Jaimovich (2007), we assume that the number of firms in an industry is pinned down by the zero profit condition that equates pre-production costs to production period profits. Firms raise equity from households in order to cover these upfront costs, which have two sources in this model.

Firstly, firms must pay a fixed operating cost of Ψ_t units of management labour. This covers things such as bureaucracy, human resources, facility maintenance, training, advertising, shop set-up, capital installation/creation and general management.

Secondly, firms will expand labour effort on transferring technologies from the global frontier.[4] We assume that the global frontier corresponds to a productivity level of A_t^*, where A_t^* will evolve exogenously in this model. We assume that there is free in-industry transfer of technologies, even without exerting any transfer effort, thus firms in industry i may start their technology transfer effort from a base productivity level of $A_t^*(i) := \max_{j\in\{1,\dots,J_{t-1}(i)\}} A_t(i,j)$ in period t. By undertaking technology transfer, a firm may raise their productivity from $A_t^*(i)$ towards A_t^*. In particular, much as in Holden (2016a), we assume that the productivity of firm $j \in \{1,\dots,J_t(i)\}$ in period $t+1$ is given by:

$$A_{t+1}(i,j) = \left[A_t^*(i)^\tau + \left(A_t^{*\tau} - A_t^*(i)^\tau\right)\frac{\Omega_{t+1}T_t(i,j)}{1+\Omega_{t+1}T_t(i,j)}\right]^{\frac{1}{\tau}}, \quad (1)$$

where $\tau > 0$ controls whether the catch-up amount is a proportion of the technology difference in levels ($\tau = 1$), log-levels ($\tau = 0$) or anything in between or beyond, $T_t(i,j)$ is the input of technology transfer services, and Ω_{t+1} is a transfer productivity shock, capturing how successful the firm is in technology transfer.

This specification captures the key idea that the further a firm is behind the frontier, the more productive will be technology transfer. It also ensures that it would take an infinite amount of technology transfer input in order to reach the frontier productivity level within one period. This reflects both the difficulty in parallelizing technology transfer, and the fact

[3] Due to the non-linear aggregation, it will not generically be the case that aggregate output is aggregate input or productive services times A_t. However, the aggregation chosen here is the unique one under which aggregate mark-ups are known one period in advance, as industry mark-ups are.

[4] In Holden (2016a), this was termed "appropriation".

that no production process in one industry and country can be a perfect fit for production in another industry and country.

Technology transfer decisions

Firms are owned by households and so they choose technology transfer to maximize their profits, which are given by:

$$\mathbb{E}_t\left[\Xi_{t+1}\left(P_{t+1}(i,j)-\frac{P_{S,t+1}}{A_{t+1}(i,j)}\right)Y_{t+1}(i,j)\right]-\Psi_t W_{M,t}-T_t(i,j)P_{T,t},$$

where Ξ_{t+1} is the household's stochastic discount factor from period t to $t+1$. Furthermore, free entry implies that in equilibrium, this quantity must be zero.

To derive the first order conditions for technology transfer, first note that in their production period, in the general (possibly non-symmetric) case, the firm's optimal price satisfies:

$$P_{t+1}(i,j)=\left[1+\frac{\eta\lambda}{1-(1-\eta)\frac{1}{J_t(i)}\left(\frac{P_{t+1}(i,j)}{P_{t+1}(i)}\right)^{-\frac{1}{\eta\lambda}}}\right]\frac{P_{S,t+1}}{A_{t+1}(i,j)}.$$

Since we are looking for a symmetric equilibrium, it is sufficient to approximate this locally around $P_{t+1}(i)=P_{t+1}(i,j)$ in order to calculate firms' technology transfer incentives. Taking a first order Taylor approximation of $\log P_{t+1}(i,j)$ in $\log\frac{P_{t+1}(i,j)}{P_{t+1}(i)}$ around this point gives us that:

$$P_{t+1}(i,j)\approx\left(1+\mu_t(i)\right)\left(\frac{P_{t+1}(i,j)}{P_{t+1}(i)}\right)^{-\omega_t(i)}\frac{P_{S,t+1}}{A_{t+1}(i,j)}$$

where $\omega_t(i):=\frac{J_t(i)(1-\eta)}{\left(J_t(i)-(1-\eta)\right)^2\left(1+\mu_t(i)\right)}$ captures the strength of these incentives to deviate from setting the same mark-up as all other firms in their industry, with $\omega_t(i)\to 0$ as $J_t(i)\to\infty$. Therefore $P_{t+1}(i)\approx\left(1+\mu_t(i)\right)\frac{P_{S,t+1}}{A_{t+1}(i)}$ and $P_{t+1}(i,j)\approx\left(1+\mu_t(i)\right)\left(\frac{A_{t+1}(i,j)}{A_{t+1}(i)}\right)^{\frac{\omega_t(i)}{1+\omega_t(i)}}\frac{P_{S,t+1}}{A_{t+1}(i,j)}$ where:

$$A_{t+1}(i):=\left[\frac{1}{J_t(i)}\sum_{j=1}^{J_t(i)}A_{t+1}(i,j)^{\frac{1}{\eta\lambda(1+\omega_t(i))}}\right]^{\eta\lambda(1+\omega_t(i))}.$$

Therefore, up to a first order approximation around the symmetric solution, expected profits are given by:

$$\frac{1}{J_t(i)}\left(\frac{1+\mu_t}{1+\mu_t(i)}\right)^{\frac{1}{\lambda}}\mathbb{E}_t\Xi_{t+1}Y_{t+1}\left[\left(\frac{A_{t+1}(i,j)}{A_{t+1}(i)}\right)^{\frac{\omega_t(i)}{1+\omega_t(i)}}-\frac{1}{1+\mu_t(i)}\right]\left(\frac{A_{t+1}(i,j)}{A_{t+1}(i)}\right)^{\frac{1-\eta\lambda\omega_t(i)}{\eta\lambda(1+\omega_t(i))}}\left(\frac{A_{t+1}(i)}{A_{t+1}}\right)^{\frac{1}{\lambda}}$$
$$-\Psi_t W_{M,t}-T_t(i,j)P_{T,t}.$$

Note that if $J_t(i)>\frac{2\sqrt{2}(3-\sqrt{2})}{1+2\sqrt{2}}\approx 1.17$, then $1-\eta\lambda\omega_t(i)>0$ (by tedious algebra), so providing there are at least two firms in the industry, this expression is guaranteed to be increasing and concave in $A_{t+1}(i,j)$.

Let $m_t(i,j)P_{T,t}$be the Lagrange multiplier on technology transfer's positivity constraint. Then in a symmetric equilibrium, the first order condition implies:

$$\frac{1}{J_t(i)}\frac{\mu_t(i)}{1+\mu_t(i)}\left(\frac{1+\mu_t}{1+\mu_t(i)}\right)^{\frac{1}{\lambda}}\mathbb{E}_t\Xi_{t+1}Y_{t+1}\left(\frac{A_{t+1}(i)}{A_{t+1}}\right)^{\frac{1}{\lambda}}\frac{d_t(i)}{\mu_t(i)}\frac{1}{\tau}\frac{\Omega_{t+1}(A_t^{*\tau}-A_t^*(i)^\tau)}{A_{t+1}(i)^\tau\big(1+\Omega_{t+1}T_t(i)\big)^2}$$
$$=P_{T,t}\big(1-m_t(i)\big).$$

where:

$$d_t(i):=1-\frac{\omega_t(i)}{1+\omega_t(i)}\frac{\big(\lambda-\mu_t(i)\big)\big(\mu_t(i)-\eta\lambda\big)}{\lambda(1-\eta)\mu_t(i)}<1$$

and where we have dropped j indices on variables which are the same across the industry. We also have that:

$$\frac{\big(\lambda-\mu_t(i)\big)\big(\mu_t(i)-\eta\lambda\big)}{\lambda(1-\eta)\mu_t(i)}\le\frac{(1-\sqrt{\eta})(\sqrt{\eta}-\eta)}{(1-\eta)\sqrt{\eta}}=\frac{1-\sqrt{\eta}}{1+\sqrt{\eta}}<1$$

so $d_t(i)>0$, as $\omega_t(i)<\frac{1}{\eta\lambda}$. In fact, we may derive tighter bounds on $d_t(i)$. Tedious algebra (available on request) gives that as $J_t(i)\ge 2$, $d_t(i)>\frac{2}{3}$, with equality in the limit as $\eta\to 0$ and $J_t(i)\to 2$.

Also, note that in symmetric equilibrium, the free entry condition implies:

$$\frac{1}{J_t(i)}\frac{\mu_t(i)}{1+\mu_t(i)}\left(\frac{1+\mu_t}{1+\mu_t(i)}\right)^{\frac{1}{\lambda}}\mathbb{E}_t\Xi_{t+1}Y_{t+1}\left(\frac{A_{t+1}(i)}{A_{t+1}}\right)^{\frac{1}{\lambda}}=\Psi_tW_{M,t}+T_t(i)P_{T,t}.$$

Using this condition, we may simplify the first order condition for technology transfer in the special case in which $\operatorname{var}_t\Omega_{t+1}=0$, since then Ω_{t+1} and $A_{t+1}(i)$ are actually known at t, so:

$$1-m_t(i)=\left(\frac{\Psi_tW_{M,t}}{P_{T,t}}+T_t(i)\right)\frac{d_t(i)}{\mu_t(i)}\frac{1}{\tau}\frac{\Omega_{t+1}(A_t^{*\tau}-A_t^*(i)^\tau)}{A_{t+1}(i)^\tau\big(1+\Omega_{t+1}T_t(i)\big)^2}\ge 0,$$

i.e.:

$$\left(\frac{A_t^*(i)}{A_t^*}\right)^\tau\left[\frac{n_t(i)}{1-m_t(i)}-1\right]$$
$$=2\left(1-\frac{1}{2}\left[1+\frac{1}{1-m_t(i)}\frac{d_t(i)}{\mu_t(i)}\frac{1}{\tau}\right]\left[1-\left(\frac{A_t^*(i)}{A_t^*}\right)^\tau\right]\right)\Omega_{t+1}T_t(i)+\big(\Omega_{t+1}T_t(i)\big)^2,$$

where:

$$n_t(i):=\Omega_{t+1}\frac{\Psi_tW_{M,t}}{P_{T,t}}\frac{d_t(i)}{\mu_t(i)}\frac{1}{\tau}\left[\left(\frac{A_t^*}{A_t^*(i)}\right)^\tau-1\right]>0.$$

Now, note that as $T_t(i)\to\infty$, $A_{t+1}(i,j)\to A_t^*$, so net profits tend to $-\infty$, as production period profits are finite even with $A_{t+1}(i,j)=A_t^*$. Thus, if it exists, the higher solution to this quadratic equation must be a local maximum, and, again if it exists, the lower solution must be a local minimum. Consequently, either $T_t(i)=0$, or $T_t(i)$ is the higher solution to this quadratic equation, i.e.:

$$\Omega_{t+1}T_t(i)=-\ell_t(i)+\sqrt{\ell_t(i)^2+\left(\frac{A_t^*(i)}{A_t^*}\right)^\tau(n_t(i)-1)}.$$

where:

$$\mathscr{f}_t(i) := 1 - \frac{1}{2}\left[1 + \frac{d_t(i)}{\mu_t(i)}\frac{1}{\tau}\right]\left[1 - \left(\frac{A_t^*(i)}{A_t^*}\right)^\tau\right].$$

Hence, for:

$$T_t(i) = \begin{cases} \frac{1}{\Omega_{t+1}} \max\left\{0, -\mathscr{f}_t(i) + \sqrt{\mathscr{f}_t(i)^2 + \left(\frac{A_t^*(i)}{A_t^*}\right)^\tau (n_t(i) - 1)}\right\}, & \mathscr{f}_t(i)^2 + \left(\frac{A_t^*(i)}{A_t^*}\right)^\tau (n_t(i) - 1) \geq 0 \\ 0, & \mathscr{f}_t(i)^2 + \left(\frac{A_t^*(i)}{A_t^*}\right)^\tau (n_t(i) - 1) < 0 \end{cases}$$

to be the global optimum, it is sufficient that the quadratic's lower solution is weakly negative whenever $\mathscr{f}_t(i)^2 + \left(\frac{A_t^*(i)}{A_t^*}\right)^\tau (n_t(i) - 1) \geq 0$, i.e.:

$$-\mathscr{f}_t(i) - \sqrt{\mathscr{f}_t(i)^2 + \left(\frac{A_t^*(i)}{A_t^*}\right)^\tau (n_t(i) - 1)} \leq 0,$$

so:

$$-\mathscr{f}_t(i) \leq \sqrt{\mathscr{f}_t(i)^2 + \left(\frac{A_t^*(i)}{A_t^*}\right)^\tau (n_t(i) - 1)}$$

implying $\mathscr{f}_t(i) \geq 0$ or $\mathscr{f}_t(i) < 0$ and $1 \leq n_t(i)$. This may be ensured, for example, by making entry sufficiently costly, i.e. by making Ψ_t large, in which case this will be the unique symmetric equilibrium. Unfortunately, we cannot rule out the possibility of asymmetric equilibria more generally.[5] However, since the coordination requirements of asymmetric equilibria render them somewhat implausible, we restrict ourselves to the unique equilibrium in which all firms within an industry choose the same levels of technology transfer.

Our solution for technology transfer means that its level is increasing in the entry cost a firm must pay prior to production, but decreasing in mark-ups, just as in Holden (2016a). They also mean that the strategic distortions caused by there being a small number of firms within an industry tend to reduce levels of technology transfer. Entry costs matter for technology transfer levels because when entry costs are high, entry into the industry is lower, meaning that each firm receives a greater slice of production-period profits, and so has correspondingly

[5] The equilibrium concept we use is that of pure-strategy subgame-perfect *local* Nash equilibria (SP*L*NE) (i.e. only profitable local deviations are ruled out). We have no reason to believe the equilibrium we find is not in fact a subgame-perfect Nash equilibria (SPNE). Indeed, if there is a pure-strategy symmetric SPNE then it will be identical to the unique pure-strategy symmetric SP*L*NE that we find. Furthermore, our numerical investigations suggest that at least in steady-state, at our calibrated parameters, the equilibrium we describe is indeed an SPNE. (Code available on request.) However, due to the analytic intractability of the second stage pricing game when productivities are asymmetric, we cannot guarantee that it remains an equilibrium away from the steady-state, or for other possible calibrations. However, SPLNE's are independently plausible since they only require firms to know the demand curve they face in the local vicinity of an equilibrium, which reduces the riskiness of the experimentation they must perform to find this demand curve (Bonanno 1988). It is arguable that the coordination required to sustain asymmetric equilibria and the computational demands of mixed strategy equilibria render either of these less plausible than our SPLNE.

amplified incentives to catch up to the global frontier. Due to free entry, transfer incentives are independent of the level of demand, except insomuch as demand affects mark-ups or the level of strategic distortion. This is because when demand is high there is greater entry, so each firm still faces roughly the same demand.

To see why mark-up increases decrease catch-up incentives, it is clearest if one considers exogenous changes in the elasticity of substitution. When products are close substitutes, then by performing technology transfer (and reducing prices) a firm may significantly expand its market-share, something that will not happen when the firm's good is a poor substitute for its rivals. When $d_t(i) \approx 1$ (i.e. there are a lot of firms in the industry) firms act as if they faced an exogenous elasticity of substitution of $\frac{1+\mu_t(i)}{\mu_t(i)}$, and so when mark-ups are high they will want to perform little research. When $d_t(i)$ is small (i.e. there are only a few firms[6]) then firms' behaviour is distorted by strategic considerations. Each firm realises that if they perform extra catch-up innovation today then their competitors will accept lower mark-ups the next period. This reduces the extent to which such innovation allows market-share expansion, depressing incentives.

Our solution for technology transfer also implies that firms in industries with sufficiently high productivity, relative to the frontier, will perform no transfer at all. To see this, note that $T_t(i) = 0$ when $n_t(i) \leq 1$, i.e. when:

$$\left(\frac{A_t^*}{A_t^*(i)}\right)^\tau \leq 1 + \frac{P_{T,t}\mu_t(i)\tau}{\Omega_{t+1}\Psi_t W_{M,t} d_t(i)}.$$

Since the right-hand side is strictly greater than 1, if $A_t^*(i)$ is high enough, no transfer will be performed. This is driven by the fact that the marginal returns to technology transfer remain finite as the quantity performed goes to zero. Thus, even with no technology transfer being performed, the marginal costs can outweigh the marginal benefits. When entry costs, Ψ_t, are low, the right-hand side is larger, so it is more likely that no technological transfer will occur. Consequently, our model has the surprising implication that creating barriers to entry can enhance productivity in countries performing technological transfer.

2.5. Capital holding companies

Without loss of generality, we assume that the capital used in sector $\mathcal{S} \in \{S, T\}$ is owned by a representative capital holding company. The capital stock in sector $\mathcal{S} \in \{S, T\}$ evolves according to:

$$K_{\mathcal{S},t} = (1 - \delta_{\mathcal{S}})K_{\mathcal{S},t-1} + \left[1 - \Phi_{\mathcal{S}}\left(\frac{I_{\mathcal{S},t}}{I_{\mathcal{S},t-1}}\right)\right] I_{\mathcal{S},t},$$

[6] The minimum value of $d_t(i)$ occurs when there is more than one firm in the industry. If there is a single firm in an industry, then, as you would expect, very little research will be performed (because the firm's only incentive to cut prices comes from competition from other industries, competition which is very weak, since those industries are producing poor substitutes to its own good). However, this drop in research incentives works entirely through the mark-up channel, and $d_t(i) \to 1$ as $J_t(i) \to 1$. One intuition for this is that there can be no strategic behaviour when there is only a single firm.

where $I_{\mathcal{S},t}$ is investment in sector $\mathcal{S}$, $\delta_{\mathcal{S}}$ is the depreciation rate of capital in that sector, and $\Phi_{\mathcal{S}}$ reflects Christiano, Eichenbaum, and Evans (2005) style investment adjustment costs, with $\Phi_{\mathcal{S}}(1) = \Phi'_{\mathcal{S}}(1) = 0$ and $\Phi''_{\mathcal{S}}(1) > 0$. Capital is rented out at a rate $\mathcal{R}_{\mathcal{S},t}$ per unit in sector $\mathcal{S}$ in period t and cannot be transferred across sectors. Including investment adjustment costs ensures that it is hard to move capital across sectors by disinvesting in one sector and reinvesting in the other.

The representative capital holding company in sector $\mathcal{S}$ chooses period t investment to maximise their profits:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \left[\prod_{k=1}^{s} \Xi_{t+k} \right] (\mathcal{R}_{\mathcal{S},t+s} K_{\mathcal{S},t+s-1} - I_{\mathcal{S},t+s})$$

subject to law of motion for capital. Writing $Q_{x,t}$ for the Lagrange multiplier on this law of motion, this leads to the first order condition for $K_{\mathcal{S},t}$:

$$1 = \mathbb{E}_t \Xi_{t+1} \frac{\mathcal{R}_{\mathcal{S},t+1} + Q_{\mathcal{S},t+1}(1-\delta_{\mathcal{S}})}{Q_{\mathcal{S},t}},$$

and first order condition for $I_{\mathcal{S},t}$:

$$1 = Q_{\mathcal{S},t} \left(1 - \Phi_{\mathcal{S}} \left(\frac{I_{\mathcal{S},t}}{I_{\mathcal{S},t-1}} \right) - \Phi' \left(\frac{I_{\mathcal{S},t}}{I_{\mathcal{S},t-1}} \right) \frac{I_{\mathcal{S},t}}{I_{\mathcal{S},t-1}} \right) + \mathbb{E}_t \Xi_{t+1} Q_{\mathcal{S},t+1} \Phi'_{\mathcal{S}} \left(\frac{I_{\mathcal{S},t+1}}{I_{\mathcal{S},t}} \right) \left(\frac{I_{\mathcal{S},t+1}}{I_{\mathcal{S},t}} \right)^2.$$

2.6. Households

We complete the model with a representative household who maximises the following objective in period t:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \left[\prod_{k=1}^{s} \beta_{t+k-1} \right] \frac{U_{t+s}^{1-\varsigma} - 1}{1-\varsigma},$$

where:

$$U_t := C_t - X_{t-1} \left(\Lambda_{1,t} + \Lambda_{S,t} \frac{L_{S,t}^{1+\nu_S}}{1+\nu_S} + \Lambda_{T,t} \frac{L_{T,t}^{1+\nu_T}}{1+\nu_T} + \Lambda_{M,t} \frac{L_{M,t}^{1+\nu_M}}{1+\nu_M} \right),$$

with:

$$X_t = C_t^{1-\gamma} X_{t-1}^{\gamma}.$$

Here, C_t is consumption, X_t is a habit stock, $\Lambda_{1,t}$ is a stochastic process controlling the level of habits, $\Lambda_{S,t}$ is a stochastic process controlling the amount of production services labour ($L_{S,t}$) that is supplied, $\Lambda_{T,t}$ is a stochastic process controlling the amount of technology transfer services labour ($L_{T,t}$) that is supplied, and $\Lambda_{M,t}$ is a stochastic process controlling the amount of management labour ($L_{T,t}$) that is supplied. We use these Jaimovich & Rebelo (2009) style preferences to enhance the comovement generated by the model, particularly in response to the endogenous news about future productivity implied by our catch-up technology. The three types of labour enter separately to capture the fact that the labour used in management or in technology transfer will require special skills, limiting the substitutability between the three types.

The representative household faces the budget constraint:

$$C_t + P_{\$,t}B_t^* + \frac{\theta}{2}\frac{(P_{\$,t}B_t^*)^2}{A_t^*} + B_t$$
$$= W_{S,t}L_{S,t} + W_{T,t}L_{T,t} + W_{M,t}L_{M,t} + P_{\$,t}R_{t-1}^*B_{t-1}^* + R_{t-1}B_{t-1} + \Pi_t,$$

where B_t^* is holdings of nominal U.S. debt in dollars, $P_{\$,t}$ is the exogenous price of one dollar in units of the consumption good, R_{t-1}^* is the U.S. nominal interest rate from period $t-1$ to t, B_t is holdings of domestic (zero net supply) real bounds, R_{t-1} is the domestic real interest rate, and Π_t are net transfers from owning the firms in the model. As is standard in the small open economy literature, we include a cost of portfolio holdings to ensure assets remain stationary.

Writing $\kappa_{B,t}$ for the Lagrange multiplier on the budget constraint, and $\kappa_{X,t}$ for the Lagrange multiplier on the law of motion of the habit stock, the first order conditions imply:

$$\kappa_{B,t} := U_t^{-\varsigma} - (1-\gamma)\kappa_{X,t}\frac{X_t}{C_t},$$
$$\kappa_{X,t} = \beta_t \mathbb{E}_t\left[\gamma\kappa_{X,t+1}\frac{X_{t+1}}{X_t} + U_{t+1}^{-\varsigma}\left(\Lambda_{1,t+1} + \Lambda_{S,t+1}\frac{L_{S,t+1}^{1+\nu_S}}{1+\nu_S} + \Lambda_{T,t+1}\frac{L_{T,t+1}^{1+\nu_T}}{1+\nu_T} + \Lambda_{M,t+1}\frac{L_{M,t+1}^{1+\nu_M}}{1+\nu_M}\right)\right],$$
$$P_{\$,t}\kappa_{B,t}\left(1 + \theta\frac{P_{\$,t}B_t^*}{A_t^*}\right) = \beta_t R_t^* \mathbb{E}_t P_{\$,t+1}\kappa_{B,t+1},$$
$$\kappa_{B,t} = \beta_t R_t \mathbb{E}_t \kappa_{B,t+1},$$
$$U_t^{-\varsigma}X_{t-1}\Lambda_{S,t}L_{S,t}^{\nu_S} = \kappa_{B,t}W_{S,t},$$
$$U_t^{-\varsigma}X_{t-1}\Lambda_{T,t}L_{T,t}^{\nu_T} = \kappa_{B,t}W_{T,t},$$
$$U_t^{-\varsigma}X_{t-1}\Lambda_{M,t}L_{M,t}^{\nu_M} = \kappa_{B,t}W_{M,t}.$$

Furthermore, the household's stochastic discount factor is given by:

$$\Xi_{t+1} := \beta_t\frac{\kappa_{B,t+1}}{\kappa_{B,t}}.$$

2.7. Market clearing

The model is closed with the following market clearing conditions:

$$Y_t = C_t + I_{S,t} + I_{T,t} + \frac{\theta}{2}\frac{(P_{\$,t}B_t^*)^2}{A_t^*} + E_t,$$
$$B_t = 0,$$
$$E_t = P_{\$,t}B_t - P_{\$,t}R_{t-1}^*B_{t-1},$$
$$S_t = \int_0^1\left[\sum_{j=1}^{J_{t-1}(i)}\frac{Y_t(i,j)}{A_t(i,j)}\right]di = \frac{Y_t}{A_t},$$
$$T_t = \int_0^1\left[\sum_{j=1}^{J_t(i)}T_t(i,j)\right]di = J_t(\cdot)T_t(\cdot),$$
$$L_{M,t} = \int_0^1\left[\sum_{j=1}^{J_t(i)}\Psi_t\right]di = J_t(\cdot)\Psi_t,$$

where E_t is net exports, and where we have replaced i arguments by dots where quantities are constant over industries.

We give the full set of equilibrium conditions in Appendix A.

2.8. Stochastic processes

We suppose that the model is driven by the following stochastic processes:[7]

$$
\begin{aligned}
A_t^* &= G_{A^*,t} A_{t-1}^*, \\
P_{\$,t} &= G_{P_\$,t} P_{\$,t-1}, \\
\log G_{A^*,t} &= \left(1-\rho_{G_{A^*}}\right) \log G_{A^*} + \rho_{G_{A^*}} \log G_{A^*,t-1} + \sigma_{G_{A^*}} \varepsilon_{G_{A^*},t}, \\
\log G_{P_\$,t} &= \left(1-\rho_{G_{P_\$}}\right) \log G_{P_\$} + \rho_{G_{P_\$}} \log G_{P_\$,t-1} + \sigma_{G_{P_\$}} \varepsilon_{G_{P_\$},t}, \\
\log \Psi_t &= (1-\rho_\Psi) \log \Psi + \rho_\Psi \log \Psi_{t-1} + \sigma_\Psi \varepsilon_{\Psi,t}, \\
\log \Omega_t &= (1-\rho_\Omega) \log \Omega + \rho_\Omega \log \Omega_{t-1} + \sigma_\Omega \varepsilon_{\Omega,t}, \\
\log \Lambda_{1,t} &= \left(1-\rho_{\Lambda_1}\right) \log \Lambda_1 + \rho_{\Lambda_1} \log \Lambda_{1,t-1} + \sigma_{\Lambda_1} \varepsilon_{\Lambda_1,t}, \\
\log \Lambda_{S,t} &= \left(1-\rho_{\Lambda_S}\right) \log \Lambda_S + \rho_{\Lambda_S} \log \Lambda_{S,t-1} + \sigma_{\Lambda_S} \varepsilon_{\Lambda_S,t}, \\
\log \Lambda_{T,t} &= \left(1-\rho_{\Lambda_T}\right) \log \Lambda_T + \rho_{\Lambda_T} \log \Lambda_{T,t-1} + \sigma_{\Lambda_T} \varepsilon_{\Lambda_T,t}, \\
\log \Lambda_{M,t} &= \left(1-\rho_{\Lambda_M}\right) \log \Lambda_M + \rho_{\Lambda_M} \log \Lambda_{M,t-1} + \sigma_{\Lambda_M} \varepsilon_{\Lambda_M,t}, \\
\log Z_{S,t} &= \left(1-\rho_{Z_S}\right) \log Z_S + \rho_{Z_S} \log Z_{S,t-1} + \sigma_{Z_S} \varepsilon_{Z_S,t}, \\
\log Z_{T,t} &= \left(1-\rho_{Z_T}\right) \log Z_T + \rho_{Z_T} \log Z_{T,t-1} + \sigma_{Z_T} \varepsilon_{Z_T,t}, \\
\log R_t^* &= (1-\rho_{R^*}) \log R^* + \rho_{R^*} \log R_{t-1}^* + \cdots + \sigma_{R^*} \varepsilon_{R^*,t}, \\
\log \beta_t &= \left(1-\rho_\beta\right) \operatorname{logit} \beta + \rho_\beta \log \beta_{t-1} + \sigma_\beta \varepsilon_{\beta,t}.
\end{aligned}
$$

We set R^*, so that $B_t = 0$ in steady-state.[8] Furthermore, without loss of generality we set Z_S and Z_T such that $S_t = T_t = 1$ in steady-state.

2.9. The steady-state and growth rate

To facilitate solving for the steady-state, we re-parameterise the model in terms of the steady-states of:

$$
\frac{A_t}{A_t^*}, \qquad \frac{U_t}{C_t}, \qquad L_{M,t}, \qquad \frac{W_{T,t}}{W_{S,t}}, \qquad R_t, \qquad J_t(\cdot),
$$

replacing the original parameters:

$$
\Psi, \qquad \Omega, \qquad \Lambda_1, \qquad \Lambda_S, \qquad \Lambda_T, \qquad \beta.
$$

We also impose the restriction that the steady-states of $L_{S,t}$, $L_{T,t}$ and $L_{M,t}$ sum to 1, enabling us to also solve for Λ_M. This restriction is without loss of generality, since it just amounts to choosing the units of labour.

With these substitutions and restrictions, it turns out that model has a unique steady-state, with a closed form solution.[9] For some values of the parameters, this will correspond to the correct (higher) solution of the firm's technology transfer decision, but for others, for example when $\frac{A_t}{A_t^*}$ is very low, it will correspond to the non-optimal (lower) solution. Thus, there is a minimum possible level of relative productivity that can be explained by the model, given

[7] The process for $\log R_t^*$ will have additional lags. See Section 3.3 for details.

[8] I.e. $R^* := \frac{G_{A^*}^{\frac{\varsigma}{1-\alpha_S}}}{\beta G_{P_\$}}$.

[9] Please see the code here: https://github.com/tholden/CatchUpCycles/blob/master/CatchUpCycles.mod for full details.

other parameters. We verify that the steady-state we find corresponds to the correct solution to the transfer decision.

In steady-state, output and its components have stochastic trend ${A_t^*}^{\frac{1}{1-\alpha_S}}$, S_t has stochastic trend ${A_t^*}^{\frac{\alpha_S}{1-\alpha_S}}$, and T_t is stationary. Foreign assets, B_t^*, have stochastic trend $\frac{{A_t^*}^{\frac{1}{1-\alpha_S}}}{P_{\$,t}}$.

3. Calibration and estimation

3.1. Fixed parameters and steady-states

To ease the computational burden of calibration and estimation, we fix some parameters and steady-states at standard values from the literature. Following the estimates of Smets & Wouters (2003) for the Euro-area, the coefficient of relative risk aversion, ς, is set to 1.4, the parameter governing the inverse Frisch labour supply parameter for productive services, ν_S, is set to 2.5, the steady-state of $\frac{U_t}{C_t}$ is set to 0.4 (equivalent to a habit parameter of 0.6), the investment adjustment cost parameters, $\Phi_S''(1)$ and $\Phi_T''(1)$, are set to 3.5, [10] and the depreciation rate for production capital, δ_S, is set to 0.025.[11] We set the persistence of the habit stock, γ, to 0.999, as in Jaimovich and Rebelo (2009), implying that the habit stock's evolution is very smooth. We set the steady-state of R_t to 1.03 in line with the estimates for the United States of Smets & Wouters (2007). Using U.S. data to establish the real interest rate is appropriate given that we are calibrating to a steady-state in which $B_t^* = 0$.

For the model's new parameters, it is harder to use results from the prior literature. Some are calibrated, and others are estimated, as we discuss below, but still others are fixed. In particular, we set the capital share in technology transfer, α_T, to 0.5, the depreciation rate of the the capital used in technology transfer, δ_T to 0.25, [12] and the relative differentiation within an industry, η to 0.5. In all cases, increasing these values further would have brought us closer to our calibration objectives (to be defined), but we viewed these values as reasonable upper limits. A high capital share in technology transfer may be justified by noting that understanding and implementing new technologies often requires special equipment and computational resources. High depreciation rates in the capital used for technological transfer seem natural given the short half-lives of electronic goods, and the fact that old tools can become irrelevant as the technologies one is trying to transfer change. High η implies that mark-ups would remain fairly high even if there were large numbers of firms in an industry. This can proxy for a variety of frictions giving market-power to firms, such as advertising,

---

[10] Obviously Φ_T does not appear in the Smets & Wouters (2003) model, here we set its second derivative to that of Φ_S in the absence of better information.

[11] Note, in Smets & Wouters (2003), the depreciation rate is calibrated, not estimated.

[12] Due to this higher depreciation rate, the economy's effective depreciation rate will be slightly above 0.025. In particular, the steady-state of $\frac{K_{S,t}\delta_S + K_{T,t}\delta_T}{K_{S,t}+K_{T,t}}$ is 0.0251. This is of course well within the sampling error bounds on the depreciation rate target.

consumer habits, locally segmented markets or anti-competitive behaviour. It is plausible that such frictions should be stronger in less advanced nations.

Finally, we set the parameter governing the elasticity of catch-up, τ, to 1, implying that catch-up occurs in levels. It turns out that changing τ does not really help us in meeting our calibration objectives, so we set $\tau = 1$ as a natural intermediate value, in the absence of any better data.

3.2. Calibrated parameters

We set $\log G_{A^*}$ to the mean growth rate (0.0032) of utilisation adjusted TFP for the United States (1947Q2 to 2017Q3) in the data set produced by Fernald (2012),[13] capturing the U.S.'s role as the global technological leader. Similarly, we set $\log G_{P_\$}$ to the mean growth rate (-0.0036) of the price of one dollar in units of the real consumption good from Spain's national accounts (1995Q2 to 2017Q2).[14] We fix the steady-state of $\frac{A_t}{A_t^*}$ to 0.75, which is the level of TFP in Spain relative to that in the U.S. found for 2016 by Bergeaud, Cette, and Lecat (2016). For the steady-state of managerial labour supply, we use U.K. data, since it is more readily accessible. In particular, in the U.K., the July 2016 to June 2017 edition of the U.K. Annual Population Survey reveals that around 10% of the U.K. workforce falls into the categories "corporate managers and directors" or "other managers and proprietors". Thus, we set the steady-state of $L_{M,t}$ to 0.1, remembering that we have defined labour units such that total steady-state labour supply is equal to one. Finally, since it seems reasonable to assume that the 10% of the population who are managers coincides well with the top decile of the income distribution, and that researchers are in the decile below, we set the steady-state of $\frac{W_{T,t}}{W_{S,t}}$ to 2.3, the ratio of the 9th decile of original income in Spain in 2016, to the average of deciles 1 to 9.[15]

It just remains for us to calibrate λ and α_S, along with the the steady-state of $J_t(\cdot)$. We set these instruments so as to approach the following calibration targets:

1. The steady-state of $\frac{W_{M,t}}{W_{S,t}}$ should be around 3.8, since this is the ratio of the top decile of original income in Spain in 2016, to the average of deciles 1 to 9 (and because managers are likely to make up the top 10% of the income distribution).
2. The consumption share of GDP (here taken to include consumption, investment and research)[16] should be around $\frac{0.6}{0.6+0.22}$, the calibration target used by Smets & Wouters (2003) for the same quantity for the Euro-area.

[13] Further details on the source for this and other data is given in Appendix B.

[14] Data on the deflator for Spanish consumption, and the Euro Dollar exchange rate from EuroStat.

[15] Data from EUROMOD. Strictly, we should set $\frac{W_{T,t}(L_{S,t}+L_{T,t})}{W_{S,t}L_{S,t}+W_{T,t}L_{T,t}}$ to 2.3 to match the data, but since $L_{T,t}$ is so small (below 0.001) it makes little difference.

[16] Fixed costs do not appear in GDP, since the statistical agencies are likely to classify them as intermediate consumption.

3. The labour share of GDP should be around 0.7, the calibration target used by Smets & Wouters (2003) for the same quantity for the Euro-area.
4. The research plus entry cost share of GDP plus entry costs should be around 0.2, the fraction of fixed costs in production found by Chen and Koebel (2013).
5. The R&D share of GDP should be around 0.012, the proportion or R&D in GDP for Spain in 2015 according to World Bank data.

Targeting these quantities with a numerical optimisation procedure[17] led us to set $\alpha_S = 0.364$, $\lambda = 0.519$, and the steady-state of $J_t(\cdot)$ to 10.659. These values imply steady-state mark-ups, $\mu_t(\cdot)$, of 0.27. At these values, we exactly hit our targets for $\frac{W_{M,t}}{W_{S,t}}$ and the consumption share, and we almost hit the target for the labour share (0.710 compared to our target of 0.7). We are further off on the fixed cost share and the research share however, with the fixed cost share in our model at 0.176 compared to our target of 0.2 and the research share at only 0.00368 compared to the target of 0.012. This may perhaps be down to the presence of other types of R&D in Spain, such as product innovation, or frontier process innovation. Alternatively, it may be that missing frictions elsewhere in our model, such as imperfect competition in labour markets, are biasing calibrations elsewhere, leading to this wedge.

3.3. Estimation

As a first step, to ease estimation, we separately estimated the stochastic processes which are directly observed, independent of the rest of the model. We estimated a process for R_t^* on the interest rate on three-month U.S. treasury bills, from 1934Q1 to 2017Q3. Using standard criteria, we settled on a specification with 9 lags, which we estimate via maximum likelihood. The estimated coefficients in order are as follows (standard errors in brackets) 1.42 (0.04), −0.80 (0.05), 0.74 (0.07), −0.51 (0.09), 0.27 (0.08), −0.18 (0.09), −0.20 (0.09), 0.36 (0.09), −0.11 (0.04). All were significant at 5%. The estimated shock standard deviation was 0.00126. We estimated a process for $P_{\$,t}$ on the price of one dollar in units of the real consumption good from Spain's national accounts, from 1995Q2 to 2017Q2. Much as before, using standard criteria, we settled on a specification with a single lag, which we estimated via maximum likelihood. The estimated coefficient on the lag was 0.33, with standard error 0.10. The estimated shock standard deviation was 0.0401. Finally, we estimated a process for $G_{A^*,t}$ on utilisation adjusted TFP for the United States from 1947Q2 to 2017Q3 in the data set produced by Fernald (2012). In this case, standard criteria suggested a specification without lags. The estimated shock standard deviation was 0.00824.

We then estimated the remaining parameters (θ, ν_T, ν_M) and the remaining stochastic processes ($\log \Psi_t$, $\log \Omega_t$, $\log \Lambda_{1,t}$, $\log \Lambda_{S,t}$, $\log \Lambda_{T,t}$, $\log \Lambda_{M,t}$, $\log Z_{S,t}$, $\log Z_{T,t}$, $\log \beta_t$) via Maximum a Posteriori estimation, i.e. by maximising the posterior density. On the shock persistence (ρ) parameters, we placed a beta distribution prior with mean 0.5 and standard

[17] In practice, we place weights on the different targets. See the code here: https://github.com/tholden/CatchUpCycles/blob/master/CalibrateModel.m for details.

deviation $\sqrt{0.05}$, i.e. with "α" and "β" parameters both equal to 2. These are the unique values at which the beta p.d.f. is symmetric with finite positive derivative at 0 and 1, implying only weak pressure away from extreme values. On the standard deviation (σ) parameters, we placed a gamma distribution prior with mode 0.01 and standard deviation 1. This is a relatively uninformative prior, centred on a standard level. On the labour supply parameters ν_T and ν_M we also placed a gamma distributed prior, this time with both mode and standard deviation $\nu_S = 2.5$ and standard deviation ν_S. This too is fairly weak, and centring around the value used for ν_S seems natural given that there is no reason why the different labour types should have radically different elasticities. Likewise, on θ we again place a gamma distributed prior, this time with mode and standard deviation equal to 0.01, a moderate value.

We estimate the model on the following six series:

1. Interest rates on three-month U.S. treasury bills, from 1934Q1 to 2017Q3.
2. The price of one dollar in units of the real consumption good from Spain's national accounts (via EuroStat), from 1995Q2 to 2017Q2.
3. Utilisation adjusted TFP for the United States from 1947Q2 to 2017Q3 in the data set produced by Fernald (2012).
4. Growth in Spanish consumption per capita from Spain's national accounts from 1995Q2 to 2017Q2.
5. Growth in Spanish investment[18] per capita from Spain's national accounts from 1995Q2 to 2017Q2.
6. Growth in Spanish real wages per hour from the OECD from 1995Q2 to 2017Q2.[19]

For estimation, we linearize the model around the steady-state, with all variables except m_t, $L_{T,t}$ and B_t^* in logarithms. We ignore the occasionally binding constraint during estimation. The results of the estimation are summarised in Table 1 below.

[18] Gross fixed capital formation.

[19] Seasonally adjusted nominal wage data came from the OECD. This was then divided by the consumption deflator from the Spanish national accounts on EuroStat.

Parameter	Prior shape	Prior mode	Prior s.d.	Post. mode	Posterior s.d.
ρ_{Ψ}	beta	0.5	0.2236	0.4998	0.3536
ρ_{Ω}	beta	0.5	0.2236	0.4999	0.3526
ρ_{Λ_1}	beta	0.5	0.2236	0.5011	0.3538
ρ_{Λ_S}	beta	0.5	0.2236	0.9204	0.0204
ρ_{Λ_T}	beta	0.5	0.2236	0.5000	0.3535
ρ_{Λ_M}	beta	0.5	0.2236	0.9792	0.0142
ρ_{Z_S}	beta	0.5	0.2236	0.8606	0.0546
ρ_{Z_T}	beta	0.5	0.2236	0.5000	0.3535
ρ_{β}	beta	0.5	0.2236	0.9131	0.0499
σ_{Ψ}	gamma	0.01	1	0.0103	0.1041
σ_{Ω}	gamma	0.01	1	0.0097	0.0951
σ_{Λ_1}	gamma	0.01	1	0.0014	0.0098
σ_{Λ_S}	gamma	0.01	1	0.0658	0.0074
σ_{Λ_T}	gamma	0.01	1	0.0100	0.0996
σ_{Λ_M}	gamma	0.01	1	0.4052	0.1732
σ_{Z_S}	gamma	0.01	1	0.0188	0.0020
σ_{Z_T}	gamma	0.01	1	0.0099	0.0984
σ_{β}	gamma	0.01	1	0.0024	0.0011
ν_T	gamma	2.5	2.5	3.5218	2.0626
ν_M	gamma	2.5	2.5	4.7297	2.1617
θ	gamma	0.01	0.01	0.0968	0.0203

Table 1: Estimation results.

From Table 1 we see that the data is informative about most parameters, the chief exception being the persistence parameters on $\log \Psi_t$, $\log \Omega_t$, $\log \Lambda_{1,t}$, $\log \Lambda_{T,t}$ and $\log Z_{T,t}$. This is somewhat unsurprising given that we have many more shocks than observables. Essentially, the data does not provide information on whether persistence should come from one shock or another. The data appears quite informative on the estimated structural parameters though. It favours higher values for both ν_T and ν_M, suggesting that research and management labour is less elastically supplied. This is unsurprising given the special skills such labour requires. Furthermore, the data favours a higher value for θ, suggesting that over the period Spain has been less able to stabilise itself via international borrowing than the frictionless benchmark would suggest.

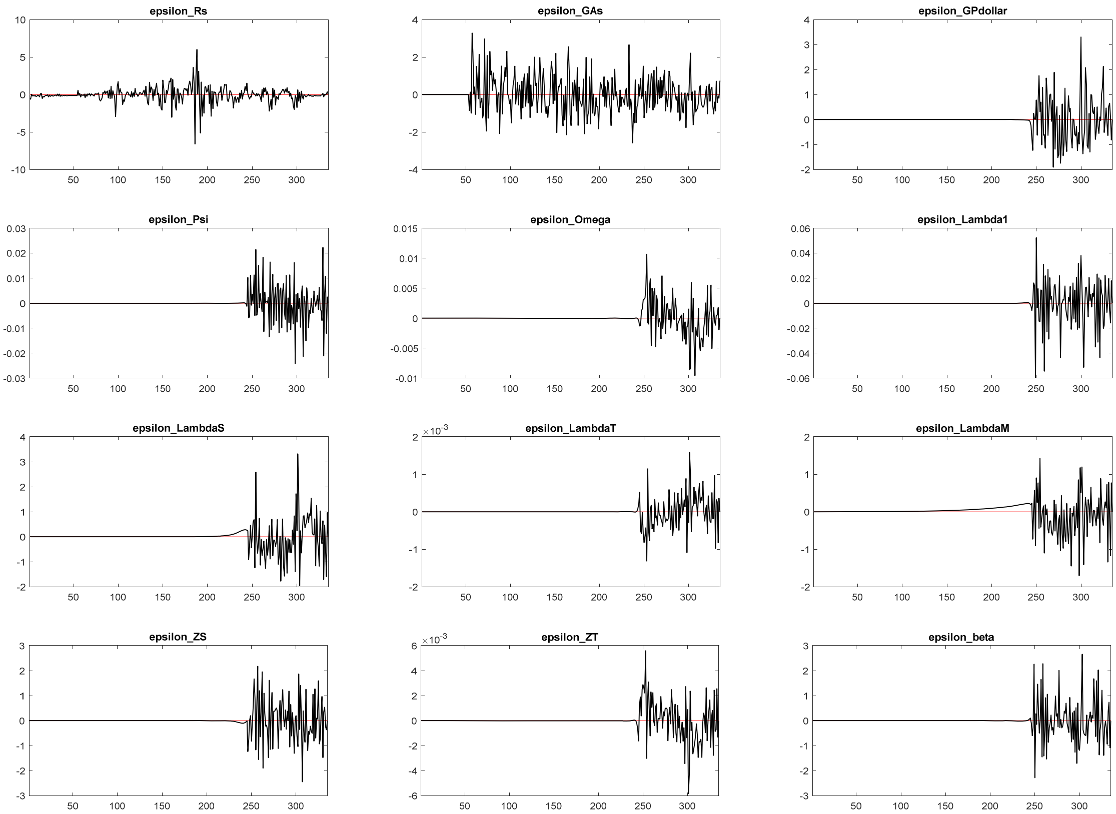

Figure 1: Smoothed shocks. Time on the horizontal axis is measured in quarters.

In Figure 1 we plot the smoothed shocks from our model. Most variables show only minimal persistence, though some lower frequency movements are evident in $\Lambda_{S,t}$ and $\Lambda_{M,t}$ suggesting that there are still lower frequency features of the data that our model cannot match. This is not overly surprising given that this is a relatively simple model, without nominal rigidities, financial frictions or various other standard DSGE features. Figure 2 plots the smoothed variables from our model. Interestingly, before the start of our consumption growth data in 1995, the model predicts consumption growth below -0.1% in 1980, 1981, 1993 and 1994. Now, according to data from the Penn World Tables (Feenstra, Inklaar, and Timmer 2015), Spain's output growth was negative in the following years between 1950 and 1995: 1953, 1959, 1981-1985, 1993. Thus, the model is able to successfully predict the Spanish recessions of the early 1980s and 1990s based only on data about U.S. interest rates and U.S. TFP growth.

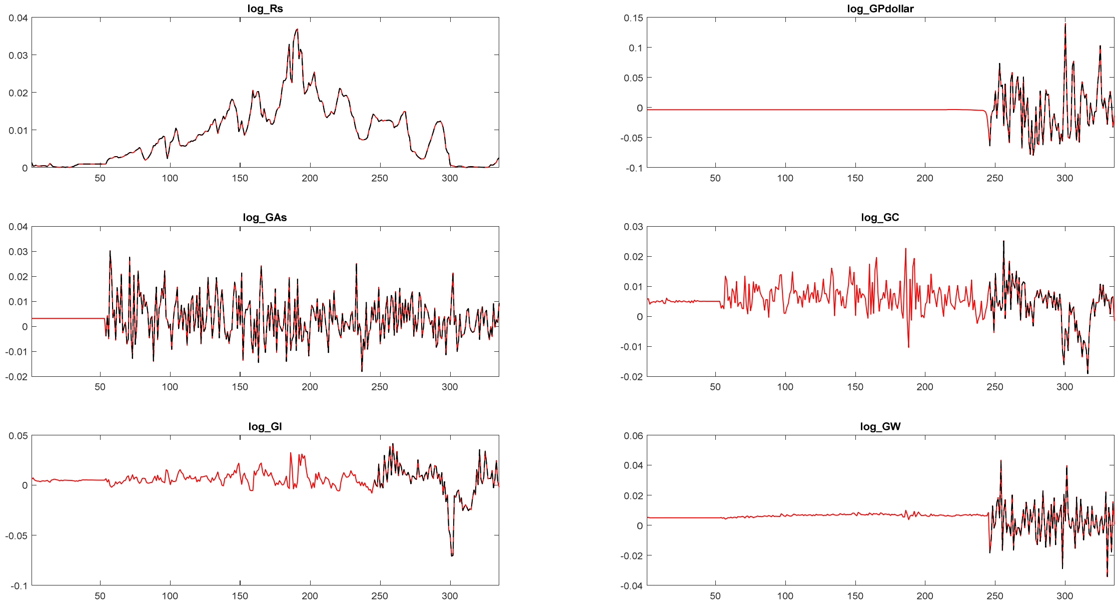

Figure 2: Smoothed variables. Time on the horizontal axis is measured in quarters.

The model's generated forecasts (not shown) have large confidence bands, but the modal forecast at least features both declining productivity relative to trend and consumption relative to trend over the next twenty-five years. Thus, together, the model and the data are telling us that Spain's productivity is currently above its long-term relative level. This may be a surprising prediction—the usual story is of convergence between European nations—but it is the natural conclusion given that growth rates in Spain have been below those in the U.S. over the period.

4. Results

We now analyse the behaviour of our model, using parameters from the posterior mode. Under the same first order approximation used for estimation, the variance decomposition is given in Table 2. The two most important shocks are the shock to frontier (U.S.) productivity growth and the shock to the disutility of managerial labour supply. Amazingly, the identified shock to U.S. TFP explains over 40% of the variance of Spain's consumption (relative to trend), over 30% of the variance of its investment (relative to trend) and around 60%[20] of the variance of its labour supply. Thus, this model features an incredibly strong link between U.S. TFP and Spanish outcomes, via the technological transfer mechanism.

The shock to the disutility of managerial labour supply may be thought of as capturing changes in the amount of entrepreneurship. This may not be entirely structural, and could instead reflect changes in the environment in which these entrepreneurs are operating, coming, for example, from changes in taxes or property rights. These shocks drive almost all

[20] $65.55 \times 0.9 + 3.18 \times 0.1 = 59.313$.

of the variance in the amount of technological transfer that is performed, thus also explaining much of the variance of productivity, consumption and investment.

Additionally, the data finds a role for shocks to productive labour supply, which were also found to be important by e.g. Smets & Wouters (2003). These explain around 20% of the variance of hours, and most of the remaining variance of consumption and investment. Lesser roles were found for exogenous productivity shocks to the productive services sector, and shocks to the discount factor.

Variable	$\varepsilon_{R^*,t}$	$\varepsilon_{G_{A^*},t}$	$\varepsilon_{G_{P_\$},t}$	$\varepsilon_{\Psi,t}$	$\varepsilon_{\Omega,t}$	$\varepsilon_{\Lambda_1,t}$	$\varepsilon_{\Lambda_S,t}$	$\varepsilon_{\Lambda_T,t}$	$\varepsilon_{\Lambda_M,t}$	$\varepsilon_{Z_S,t}$	$\varepsilon_{Z_T,t}$	$\varepsilon_{\beta,t}$
$\log \frac{A_t}{A_t^*}$	0	23.98	0	0	0	0	0.3	0	75.55	0.09	0	0.07
$T_t(\cdot)$	0.01	4.49	0	0.05	0	0	0.41	0	94.79	0.16	0.01	0.07
$\log J_t(\cdot)$	0	3.18	0	0.12	0	0	0.07	0	96.59	0.03	0	0.01
$\log \frac{C_t}{A_t^{*\frac{1}{1-\alpha_S}}}$	0.05	40.23	0.13	0	0	0	5.29	0	50.75	2.03	0	1.51
$\log \frac{I_{S,t}}{A_t^{*\frac{1}{1-\alpha_S}}}$	0.46	31.61	0.08	0	0	0	12.05	0	45.37	5.74	0	4.69
$\log \frac{I_{T,t}}{A_t^{*\frac{1}{1-\alpha_S}}}$	0.05	20.83	0.02	0	0	0	2.51	0	75.18	0.99	0	0.41
$\log L_{S,t}$	0.01	65.55	0	0	0	0	24.43	0	9.2	0.66	0	0.15
$\log L_{T,t}$	0.01	50.84	0.03	0.04	0.01	0	1.26	0.04	47.14	0.51	0.01	0.12
$\log L_{M,t}$	0	3.18	0	0	0	0	0.07	0	96.7	0.03	0	0.01

Table 2: Variance decomposition. Numbers are in percent.

For our remaining results, we increase accuracy by switching to working with a second order approximation to the model, imposing the occasionally binding constraint using the algorithm of Holden (2016b).[21] We note that the theoretical results of Holden (2016c) imply that the model has a unique solution when the constraint binds for 32 periods, and further tests are strongly suggestive of a unique solution when the constraint binds arbitrarily long. Thus, it does not seem that the constraint leads to multiplicity in this model.

[21] To speed up simulation, we do not use "cubature", in the language of Holden (2016b). Therefore, these results are as if people always believed that their mean prediction told them which periods they would be at the bound with certainty. Since the bound is only hit rarely, this is not too costly an approximation.

	Mean	Standard deviation	Skewness	Kurtosis
$\log \frac{A_t}{A_t^*}$	-0.276359	0.086417	-0.424444	0.046284
$T_t(\cdot)$	0.103321	0.058037	1.329472	2.601128
$\log J_t(\cdot)$	2.349445	0.338845	-0.042062	-0.056502
$\log \frac{C_t}{A_t^{*\frac{1}{1-\alpha_S}}}$	-0.582146	0.19231	-0.197354	0.157333
$\log \frac{I_{S,t}}{A_t^{*\frac{1}{1-\alpha_S}}}$	-1.58889	0.275304	-0.288872	0.240104
$\log \frac{I_{T,t}}{A_t^{*\frac{1}{1-\alpha_S}}}$	-7.117093	0.492925	-0.417328	0.19025
$\log L_{S,t}$	-0.113769	0.119949	-0.013873	0.064631
$\log L_{T,t}$	-6.98195	0.149267	-0.24098	0.041737
$\log L_{M,t}$	-2.319422	0.338528	-0.042425	-0.055879

Table 3: Moments from a simulation run of 100,000 periods.[22]

In Table 3 we plot the moments from our estimated model. We see that relative productivity has a remarkably high standard deviation, implying that Spain's productivity could fluctuate between around 65% and 90% of that of the U.S.. Indeed, given the negative skewness we find for relative productivity, recessionary periods in Spain could be associated with even lower than 65% relative productivity. From looking at the mean and standard devation of transfer effort, $T_t(\cdot)$, one might expect that the bound would be hit reasonably often, however, thanks to its high positive skewness, it turns out that this is not the case. Indeed, in our simulation run of 100,000 periods, the constraint was hit only once, and the constraint was not hit even in impulse responses to magnitude ± 10 standard deviation shocks.

The standard deviations of consumption and investment relative to trend are even larger than that of relative productivity. These numbers reflect the fact that the catch-up mechanism generates huge amounts of low-frequency variance, so relative consumption and investment are "close" to following a unit root. Of course, were they true unit root processes, their standard deviations would be infinite. They are also negatively skewed, capturing the fact that large recessions are more common than large booms. The high standard deviation of consumption and investment leads to quite high standard deviations for hours too. Thus, the model is potentially able to explain some of the lower frequency behaviour of labour supply.

Table 4 gives the correlations between variables observed in the simulation run. Notable is the strong comovement between production labour supply and relative consumption and investment, thanks in part to the Jaimovich & Rebelo (2009) style preferences. The number of firms is counter-cyclical though, implying pro-cyclical mark-ups in line with the evidence of Nekarda & Ramey (2010). We also see that technology transfer effort is pro-cyclical, in line with the evidence of the pro-cyclicality of R&D presented by Barlevy (2007).

[22] A 100 period burn-in was discarded.

	$\log \frac{A_t}{A_t^*}$	$T_t(\cdot)$	$\log J_t(\cdot)$	$\log \frac{C_t}{A_t^{*\frac{1}{1-\alpha_S}}}$	$\log \frac{I_{S,t}}{A_t^{*\frac{1}{1-\alpha_S}}}$	$\log \frac{I_{T,t}}{A_t^{*\frac{1}{1-\alpha_S}}}$	$\log L_{S,t}$	$\log L_{T,t}$	$\log L_{M,t}$
$\log \frac{A_t}{A_t^*}$	1.000	0.347	−0.540	0.815	0.785	−0.014	0.255	−0.069	−0.540
$T_t(\cdot)$	0.347	1.000	−0.863	0.262	0.116	0.786	0.200	0.704	−0.862
$\log J_t(\cdot)$	−0.540	−0.863	1.000	−0.296	−0.164	−0.651	0.007	−0.472	0.999
$\log \frac{C_t}{A_t^{*\frac{1}{1-\alpha_S}}}$	0.815	0.262	−0.296	1.000	0.904	0.126	0.671	0.183	−0.296
$\log \frac{I_{S,t}}{A_t^{*\frac{1}{1-\alpha_S}}}$	0.785	0.116	−0.164	0.904	1.000	−0.037	0.626	0.005	−0.163
$\log \frac{I_{T,t}}{A_t^{*\frac{1}{1-\alpha_S}}}$	−0.014	0.786	−0.651	0.126	−0.037	1.000	0.315	0.914	−0.651
$\log L_{S,t}$	0.255	0.200	0.007	0.671	0.626	0.315	1.000	0.539	0.007
$\log L_{T,t}$	−0.069	0.704	−0.472	0.183	0.005	0.914	0.539	1.000	−0.471
$\log L_{M,t}$	−0.540	−0.862	0.999	−0.296	−0.163	−0.651	0.007	−0.471	1.000

Table 4: Correlations from a simulation run of 100,000 periods.

	1	2	3	4	5
$\log \frac{A_t}{A_t^*}$	0.995	0.990	0.985	0.979	0.973
$T_t(\cdot)$	0.988	0.972	0.954	0.932	0.907
$\log J_t(\cdot)$	0.977	0.956	0.934	0.914	0.893
$\log \frac{C_t}{A_t^{*\frac{1}{1-\alpha_S}}}$	0.994	0.989	0.985	0.980	0.976
$\log \frac{I_{S,t}}{A_t^{*\frac{1}{1-\alpha_S}}}$	0.996	0.986	0.974	0.959	0.943
$\log \frac{I_{T,t}}{A_t^{*\frac{1}{1-\alpha_S}}}$	0.990	0.967	0.935	0.897	0.856
$\log L_{S,t}$	0.981	0.963	0.947	0.932	0.918
$\log L_{T,t}$	0.991	0.980	0.966	0.951	0.933
$\log L_{M,t}$	0.978	0.956	0.935	0.915	0.894

Table 5: Sample auto-correlations at lags one to five from a simulation run of 100,000 periods.

Our last table, Table 5, gives the auto-correlations of these variables. As expected, these are extremely high across the board, reflecting the endogenous low frequency behaviour of the model. The sources of this low frequency behaviour will become clear from examining the impulse responses, which we turn to now.

Figures 3 to 14 plot impulse responses to the nine variables we have been considering in these tables to each of our model's shocks. In each case, we plot both the responses to a magnitude two standard deviation positive shock. Time is measured in quarters on the horizontal axis, and the vertical axis is in natural units in each case.

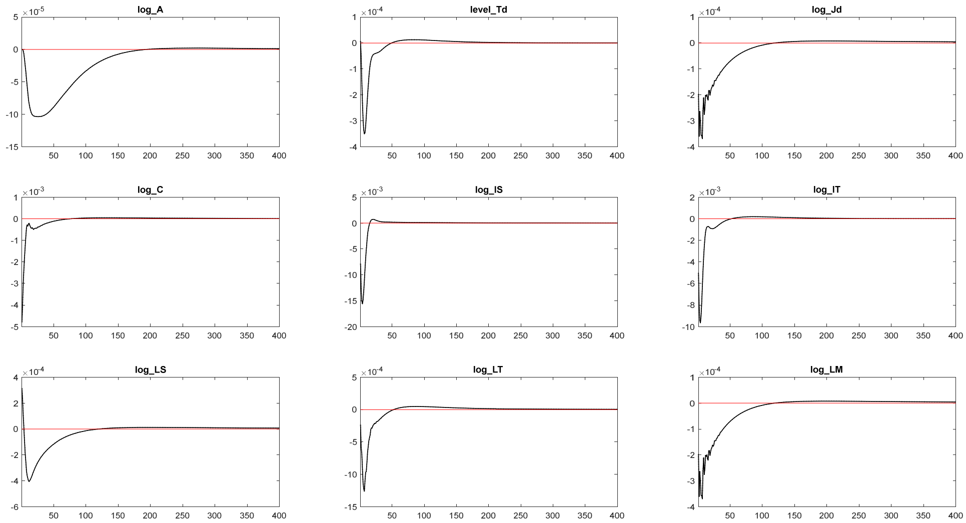

Figure 3: Impulse response to a two standard deviation shock to $\varepsilon_{R^*,t}$. Time on the horizontal axis is measured in quarters.

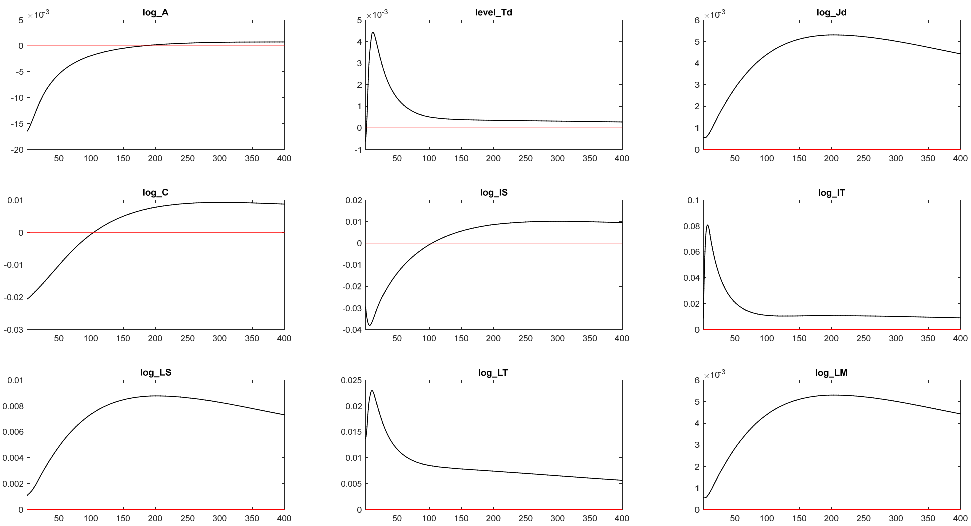

Figure 4: Impulse response to a two standard deviation shock to $\varepsilon_{G_{A^*},t}$. Time on the horizontal axis is measured in quarters.

An increase in U.S. interest rates (shown in Figure 3) implies consumption must be expected to grow, in order to satisfy the Euler equation. Thus, consumption jumps down. Thanks to the comovement between consumption and hours implied by our preferences, this reduces labour supply, and thus reduces transfer effort, producing a small drop in productivity over the medium-term.

An increase in U.S. TFP (shown in Figure 4) leads to an increase in transfer effort, in order to catch-up to the frontier. This also requires higher transfer investment. It also leads an increase in firm entry, since higher future productivity implies higher profits. Despite this substitution away from consumption, absolute consumption actually rises by about 0.6% on impact. Note that the IRFs plot relative consumption which falls due to the drop of 2.6% in the trend,[23] its denominator. Absolute investment in the productive services sector does fall on impact though, due to the substitution to other types of investment. As productivity increases through transfer effort, so too does consumption. Thanks to our preference specification, this helps support higher labour supply, meaning that all variables overshoot their long-run levels. Indeed, consumption is still 1% above its long-run level 100 years after the initial shock. This shock then is likely to be important in driving the low-frequency movements that Aguiar & Gopinath (2007) found to be so important. This persistence also explains the high variance share of the shock. It is particularly remarkable given that the driving shock is uncorrelated across time.

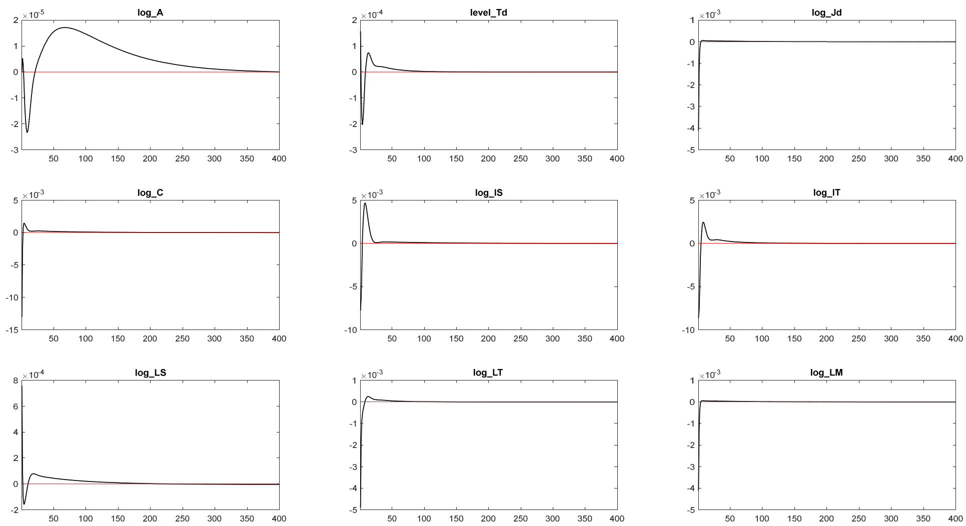

Figure 5: Impulse response to a two standard deviation shock to $\varepsilon_{G_{P_\$},t}$. Time on the horizontal axis is measured in quarters.

[23] $2\frac{\sigma_{G_{A^*}}}{1-\alpha_S} = 0.026$.

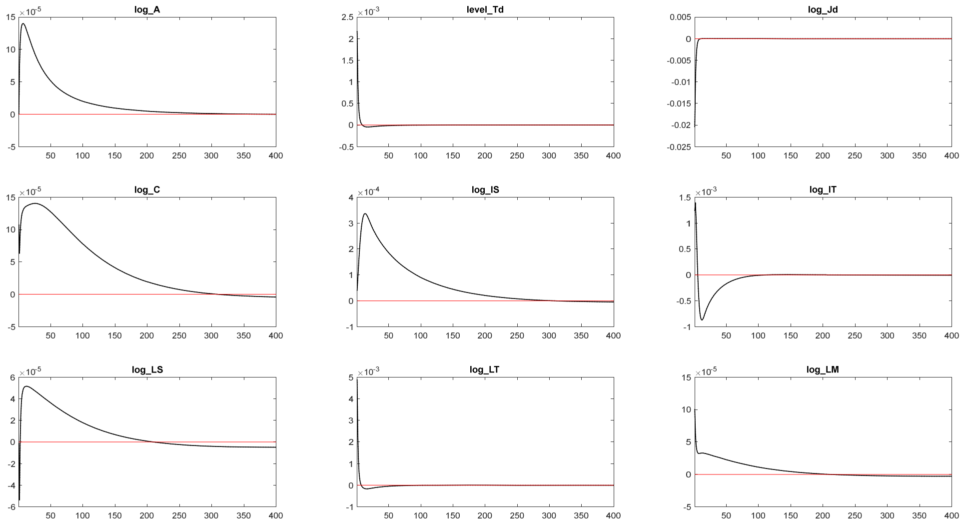

Figure 6: Impulse response to a two standard deviation shock to $\varepsilon_{\Psi,t}$. Time on the horizontal axis is measured in quarters.

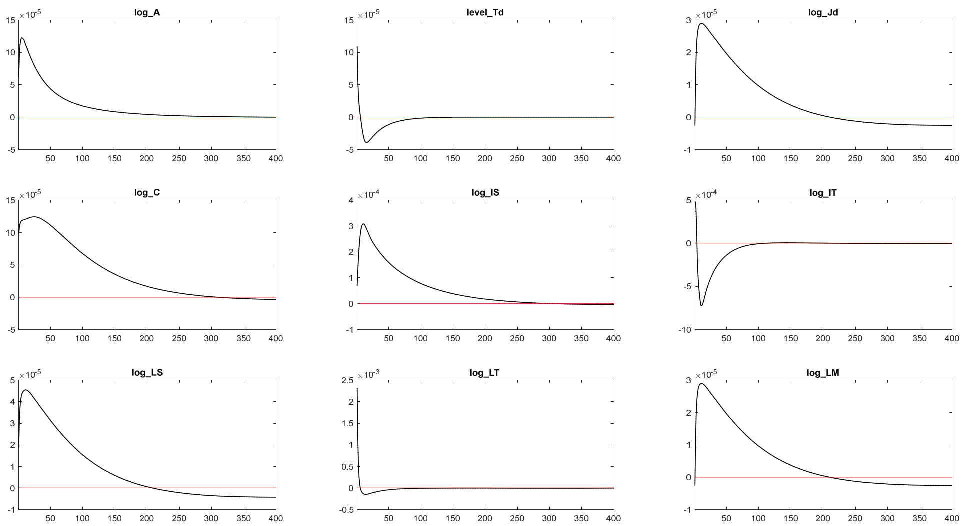

Figure 7: Impulse response to a two standard deviation shock to $\varepsilon_{\Omega,t}$. Time on the horizontal axis is measured in quarters.

In Figure 5 we show the response to a shock to the real price of a dollar, which works much like a shock to the interest rate, since its persistence implies a movement in future real interest rates. On impact though there are some slight windfall effects. Figure 6 gives the impulse response to an increase in the entry cost. This reduces the number of firms, thus increasing each firm's share of profits, thus pushing up transfer incentives. Consequently, there is a small (0.01%) increase in productivity. The response to the shock to the success of technology transfer in Figure 7 is similar. On impact, productivity jumps up as firms are more successful than expected. Then, thanks to the shock's persistence, firms are keen to undertake more transfer for a few more periods, further increasing productivity.

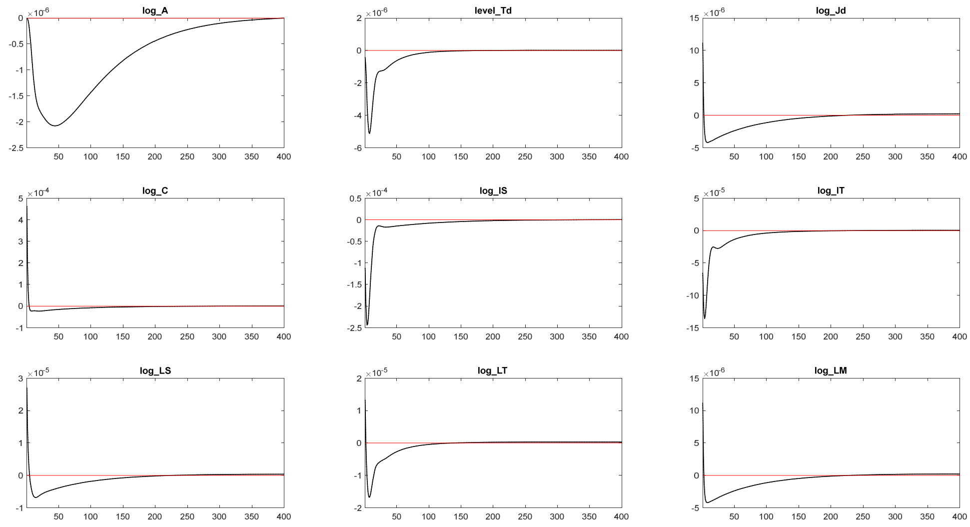

Figure 8: Impulse response to a two standard deviation shock to $\varepsilon_{\Lambda_1,t}$. Time on the horizontal axis is measured in quarters.

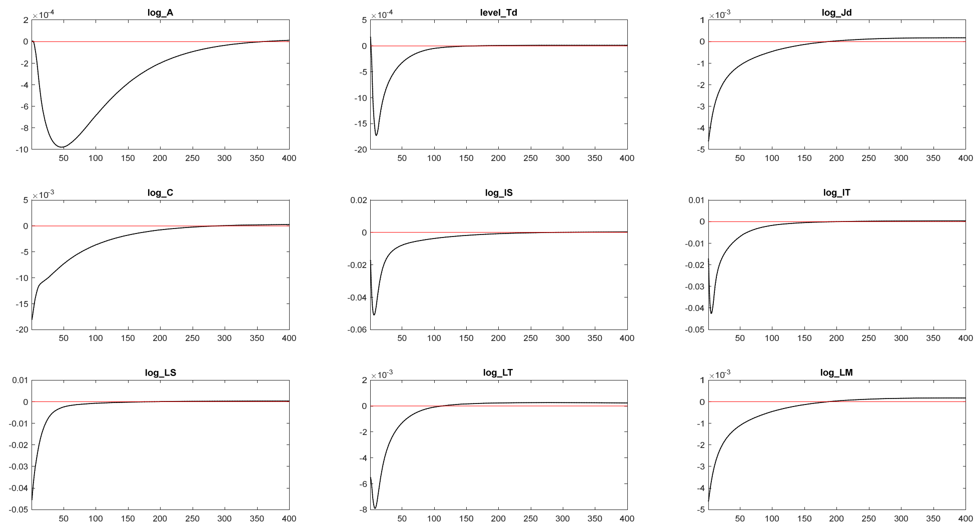

Figure 9: Impulse response to a two standard deviation shock to $\varepsilon_{\Lambda_S,t}$. Time on the horizontal axis is measured in quarters.

A shock to $\varepsilon_{\Lambda_1,t}$ (shown in Figure 8) is akin to a shock to the subsistence level of consumption. As a result, it forces up consumption on impact, and pushes down investment. With investment in transfer lower, productivity naturally suffers over medium-horizons. The shock to $\varepsilon_{\Lambda_S,t}$ (shown in Figure 9) works by a very similar mechanism, except that it also pushes down production labour, which in turn means that even consumption must fall on impact. The same mechanisms are also at work in the IRF to $\varepsilon_{\Lambda_T,t}$ given in Figure 10.

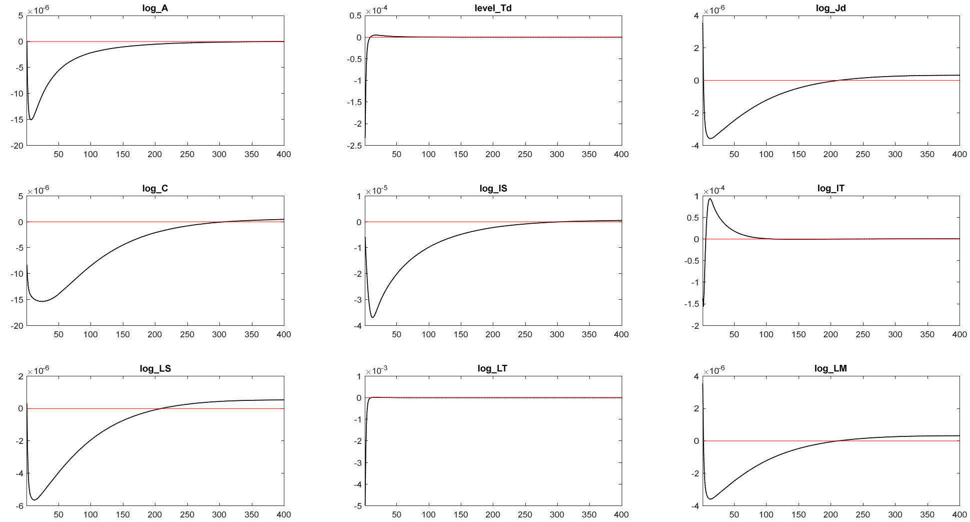

Figure 10: Impulse response to a two standard deviation shock to $\varepsilon_{\Lambda_T,t}$. Time on the horizontal axis is measured in quarters.

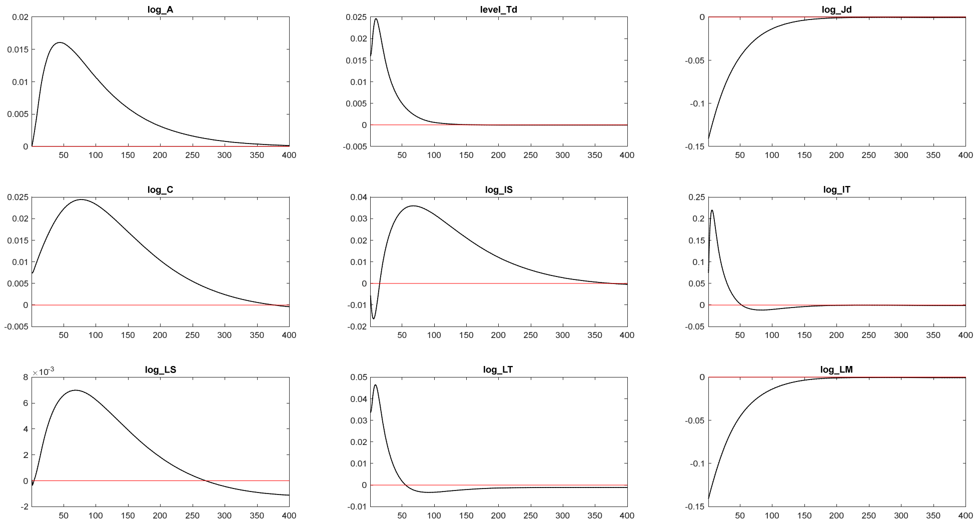

Figure 11: Impulse response to a two standard deviation shock to $\varepsilon_{\Lambda_M,t}$. Time on the horizontal axis is measured in quarters.

The shock to the disutility of managerial labour, $\varepsilon_{\Lambda_M,t}$, in Figure 11 is both more important, and more interesting. With managerial labour now more expensive, firm entry falls. Consequently, firms have stronger incentives for catch-up, which increases productivity over medium-horizons. To provide the input for this transfer effort, investment in the transfer technology must rise, which in the short term comes at the cost of a fall in investment in productive services. However, consumption rises on impact, since the household expects higher future consumption, thus over the medium-term, investment in productive services rises too. At peak, after around 15 years, consumption has risen by 2.5% with labour supply rising around 0.07%, thanks to our choice of preferences. This is a true medium-frequency cycle in action, explaining its high variance share.

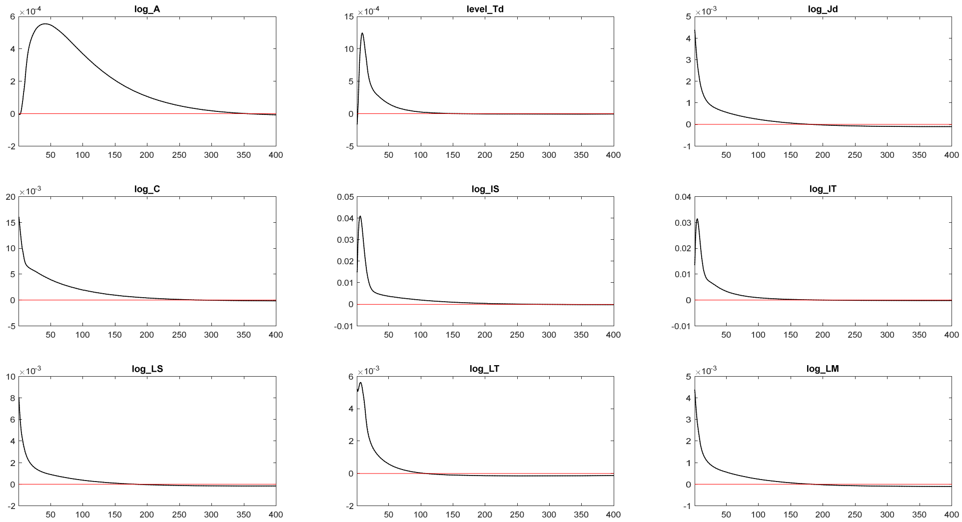

Figure 12: Impulse response to a two standard deviation shock to $\varepsilon_{Z_S,t}$. Time on the horizontal axis is measured in quarters.

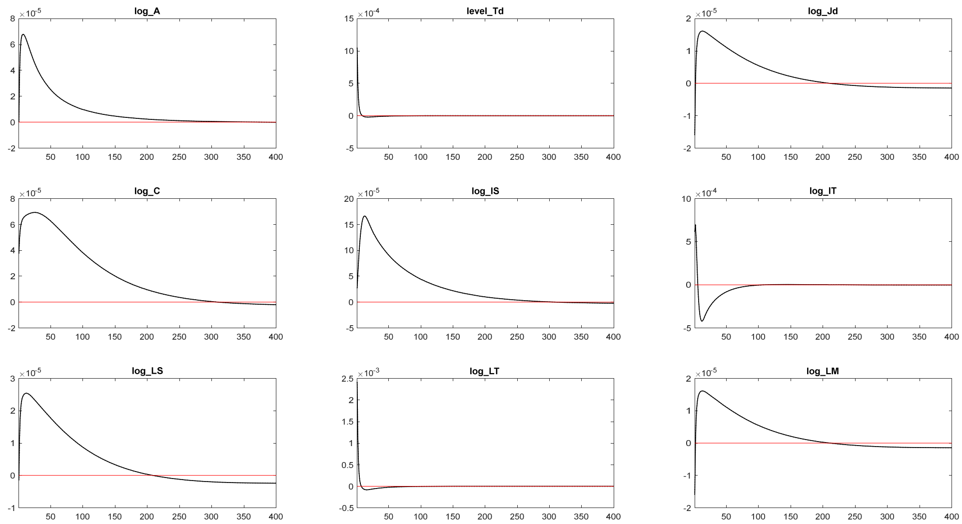

Figure 13: Impulse response to a two standard deviation shock to $\varepsilon_{Z_T,t}$. Time on the horizontal axis is measured in quarters.

The two exogenous productivity shocks (shown in Figure 12 and Figure 13), both lead to an increase in endogenous productivity over the medium term. In the case of an increase in productivity in the productive services sector, this is because higher productivity in productive services leads to an increase in output in that sector. Some of this extra output goes into higher investment in the other sector, thus technology transfer effort increases, pushing up productivity. In the case of an increase in productivity in the transfer sector, the mechanism is more direct.

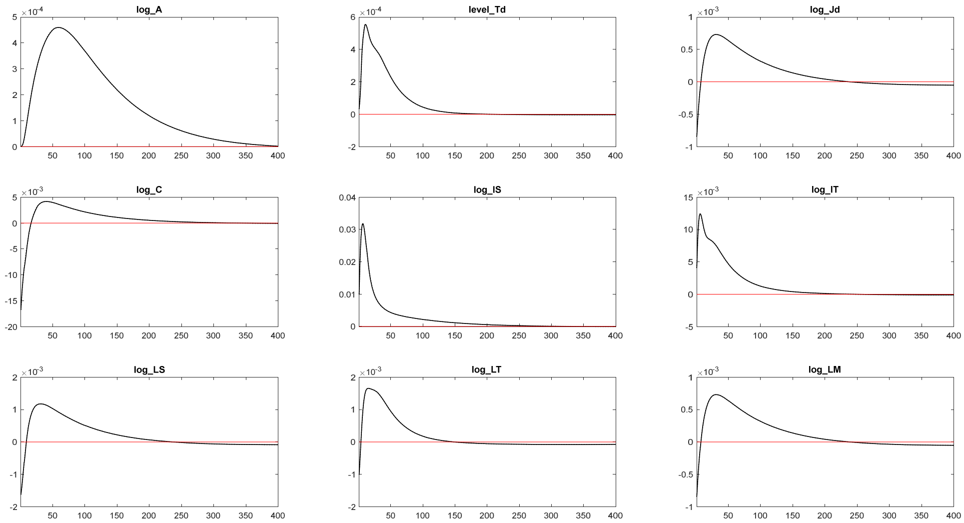

Figure 14: Impulse response to a two standard deviation shock to $\varepsilon_{\beta,t}$. Time on the horizontal axis is measured in quarters.

Finally, we consider the impulse response to an increase in the discount factor, implying higher patience, as shown in Figure 14. Given higher discount factors, the Euler equation implies that consumption must be expected to grow, implying a drop in consumption today. Output is thus substituted away from consumption, and into investment in the two sectors. The investment in the technological transfer sector leads to increased output in that sector, pushing up productivity over the medium term. Thus, the shock leads to a boom in consumption around 10 years after its impact.

5. Conclusions and tentative policy suggestions

This paper has presented and estimated a model of endogenous productivity movements driven by technology transfer. We showed how shocks to the world interest rate that were uncorrelated across time could lead to extremely persistent movements in the productivity, consumption and investment of nations which are lagging technologically. Additionally, we showed that shocks to managerial labour supply are a key driver of the medium-frequency cycle in such countries, with lower managerial labour supply leading to higher productivity. Such shocks should be interpreted as proxying for a variety of forces that might affect entrepreneurship in a country.

While we have not conducted formal policy exercises in this paper, our results are certainly highly suggestive of policies that might help productivity in countries that are lagging in productivity. Perhaps surprisingly, we found that reducing firm entry could significantly increase productivity, as with fewer firms, each firm obtains a higher share of production profits, and thus has greater incentives to increase its productivity. This was despite the fact that mark-ups in our model were endogenous, so higher firm entry reduced the mark-up wedge. Thus, policies aimed at increasing competition might have undesired side effects that far outweigh the benefits through reduced mark-ups. Instead, policy makers should strive to reduce the number of firms in each industry, while keeping mark-ups low. In practice, this could be implemented by removing the many existing measures that unfairly disadvantage large firms, while cracking down on anti-competitive behaviour, and taxing advertising aimed at creating differentiation where none would otherwise exist.

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Appendix A: Equilibrium conditions

The full set of equilibrium conditions under a symmetric equilibrium are as follows:

$$B_t = 0,$$

$$E_t := P_{\$,t}B_t^* - P_{\$,t}R_{t-1}^*B_{t-1}^*,$$

$$Y_t := C_t + I_{S,t} + I_{T,t} + \frac{\theta}{2}\frac{(P_{\$,t}B_t^*)^2}{A_t^*} + E_t,$$

$$\mu_t := \lambda\frac{\eta J_t(\cdot)}{J_t(\cdot) - (1-\eta)},$$

$$P_{S,t} := \frac{A_t}{1+\mu_{t-1}},$$

$$S_t := \frac{Y_t}{A_t},$$

$$T_t := J_t(\cdot)T_t(\cdot),$$

$$L_{M,t} := J_t(\cdot)\Psi_t,$$

$$\mathcal{R}_{S,t} := \alpha_S P_{S,t}\frac{S_t}{K_{S,t-1}},$$

$$\mathcal{R}_{T,t} := \alpha_T P_{T,t}\frac{T_t}{K_{T,t-1}},$$

$$\omega_t(\cdot) := \frac{J_t(\cdot)(1-\eta)}{\big(J_t(\cdot) - (1-\eta)\big)^2(1+\mu_t)},$$

$$\mathcal{d}_t(\cdot) := 1 - \frac{\omega_t(\cdot)}{1+\omega_t(\cdot)}\frac{(\lambda-\mu_t)(\mu_t - \eta\lambda)}{\lambda(1-\eta)\mu_t},$$

$$U_t := C_t - X_{t-1}\left(\Lambda_{1,t} + \Lambda_{S,t}\frac{L_{S,t}^{1+\nu_S}}{1+\nu_S} + \Lambda_{T,t}\frac{L_{T,t}^{1+\nu_T}}{1+\nu_T} + \Lambda_{M,t}\frac{L_{M,t}^{1+\nu_M}}{1+\nu_M}\right),$$

$$\kappa_{B,t} := U_t^{-\varsigma} - (1-\gamma)\kappa_{X,t}\frac{X_t}{C_t},$$

$$\Xi_{t+1} := \beta_t\frac{\kappa_{B,t+1}}{\kappa_{B,t}},$$

$$W_{S,t} := \frac{1}{\kappa_{B,t}}U_t^{-\varsigma}X_{t-1}\Lambda_{S,t}L_{S,t}^{\nu_S},$$

$$W_{T,t} := \frac{1}{\kappa_{B,t}}U_t^{-\varsigma}X_{t-1}\Lambda_{T,t}L_{T,t}^{\nu_T},$$

$$W_{M,t} := \frac{1}{\kappa_{B,t}}U_t^{-\varsigma}X_{t-1}\Lambda_{M,t}L_{M,t}^{\nu_M},$$

$$S_t = K_{S,t-1}^{\alpha_S}(Z_{S,t}L_{S,t})^{1-\alpha_S},$$

$$T_t = \left(\frac{K_{T,t-1}}{A_t^*}\right)^{\alpha_T}(Z_{T,t}L_{T,t})^{1-\alpha_T},$$

$$W_{S,t}L_{S,t} = (1-\alpha_S)P_{S,t}S_t,$$

$$W_{T,t}L_{T,t} = (1-\alpha_T)P_{T,t}T_t,$$

$$0 = \min\{m_t(\cdot), T_t(\cdot)\},$$

$$A_t = \left[A_{t-1}^{\tau} + ({A_{t-1}^*}^{\tau} - A_{t-1}^{\tau})\frac{\Omega_t T_{t-1}(\cdot)}{1+\Omega_t T_{t-1}(\cdot)}\right]^{\frac{1}{\tau}},$$

$$K_{S,t} = (1-\delta_S)K_{S,t-1} + \left[1 - \Phi_S\left(\frac{I_{S,t}}{I_{S,t-1}}\right)\right] I_{S,t},$$
$$K_{T,t} = (1-\delta_T)K_{T,t-1} + \left[1 - \Phi_T\left(\frac{I_{T,t}}{I_{T,t-1}}\right)\right] I_{T,t},$$
$$X_t = C_t^{1-\gamma} X_{t-1}^{\gamma},$$

$$\frac{1}{J_t(\cdot)}\frac{\mu_t}{1+\mu_t}\mathbb{E}_t \Xi_{t+1} Y_{t+1} \frac{d_t(\cdot)}{\mu_t}\frac{1}{\tau}\frac{\Omega_{t+1}(A_t^{*\tau} - A_t^{\tau})}{A_{t+1}^{\tau}\big(1+\Omega_{t+1}T_t(\cdot)\big)^2} = P_{T,t}\big(1 - m_t(\cdot)\big),$$
$$\frac{1}{J_t(\cdot)}\frac{\mu_t}{1+\mu_t}\mathbb{E}_t \Xi_{t+1} Y_{t+1} = \Psi_t W_{M,t} + T_t(\cdot) P_{T,t},$$
$$1 = \mathbb{E}_t \Xi_{t+1} \frac{\mathcal{R}_{S,t+1} + Q_{S,t+1}(1-\delta_S)}{Q_{S,t}},$$
$$1 = \mathbb{E}_t \Xi_{t+1} \frac{\mathcal{R}_{T,t+1} + Q_{T,t+1}(1-\delta_T)}{Q_{T,t}},$$
$$1 = Q_{S,t}\left(1 - \Phi_S\left(\frac{I_{S,t}}{I_{S,t-1}}\right) - \Phi'\left(\frac{I_{S,t}}{I_{S,t-1}}\right)\frac{I_{S,t}}{I_{S,t-1}}\right) + \mathbb{E}_t \Xi_{t+1} Q_{S,t+1} \Phi_S'\left(\frac{I_{S,t+1}}{I_{S,t}}\right)\left(\frac{I_{S,t+1}}{I_{S,t}}\right)^2,$$
$$1 = Q_{T,t}\left(1 - \Phi_T\left(\frac{I_{T,t}}{I_{T,t-1}}\right) - \Phi'\left(\frac{I_{T,t}}{I_{T,t-1}}\right)\frac{I_{T,t}}{I_{T,t-1}}\right) + \mathbb{E}_t \Xi_{t+1} Q_{T,t+1} \Phi_T'\left(\frac{I_{T,t+1}}{I_{T,t}}\right)\left(\frac{I_{T,t+1}}{I_{T,t}}\right)^2,$$
$$\kappa_{X,t} = \beta_t \mathbb{E}_t\left[\gamma \kappa_{X,t+1}\frac{X_{t+1}}{X_t} + U_{t+1}^{-\varsigma}\left(\Lambda_{1,t+1} + \Lambda_{S,t+1}\frac{L_{S,t+1}^{1+\nu_S}}{1+\nu_S} + \Lambda_{T,t+1}\frac{L_{T,t+1}^{1+\nu_T}}{1+\nu_T} + \Lambda_{M,t+1}\frac{L_{M,t+1}^{1+\nu_M}}{1+\nu_M}\right)\right],$$
$$\kappa_{B,t}\left(1 + \theta\frac{P_{\$,t}B_t^*}{A_t^*}\right) = \beta_t R_t^* \mathbb{E}_t \frac{P_{\$,t+1}}{P_{\$,t}}\kappa_{B,t+1},$$
$$\kappa_{B,t} = \beta_t R_t \mathbb{E}_t \kappa_{B,t+1}.$$

Appendix B: Further details on data sources

- Spanish national accounts from EuroStat here: https://is.gd/SpainNationalAccounts
- Euro-dollar exchange rates from EuroStat here: https://is.gd/EuroDollarExchangeRates
- Spanish population from EuroStat here: https://is.gd/SpainPopulation
- U.S. 3-month treasury bill rates from FRED here: https://fred.stlouisfed.org/series/TB3MS
- Fernald (2012) U.S. TFP data from here: http://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp/
- EUROMOD income deciles from here: https://www.euromod.ac.uk/using-euromod/statistics
- U.K. Annual Population survey accessed via: https://www.nomisweb.co.uk/
- World Bank data on R&D shares from here: https://data.worldbank.org/indicator/GB.XPD.RSDV.GD.ZS