

# Capital Heterogeneity and Investment Prices

How much are investment prices declining?

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# Motivation

Farhi and Gourio: “Accounting for Macro-Finance Trends”

- Study joint evolution of big macro-finance trends:
  - declining interest rates despite stable ROA,
  - roughly stable stock market P/D since 2000,
  - weak investment and weak growth,
  - low labor share, etc.
- Why jointly? Because proposed explanations for one trend have implications for the others
  - e.g. demographics can explain low interest rates
  - but they imply low ROA and high investment...
  - whack-a-mole situation!

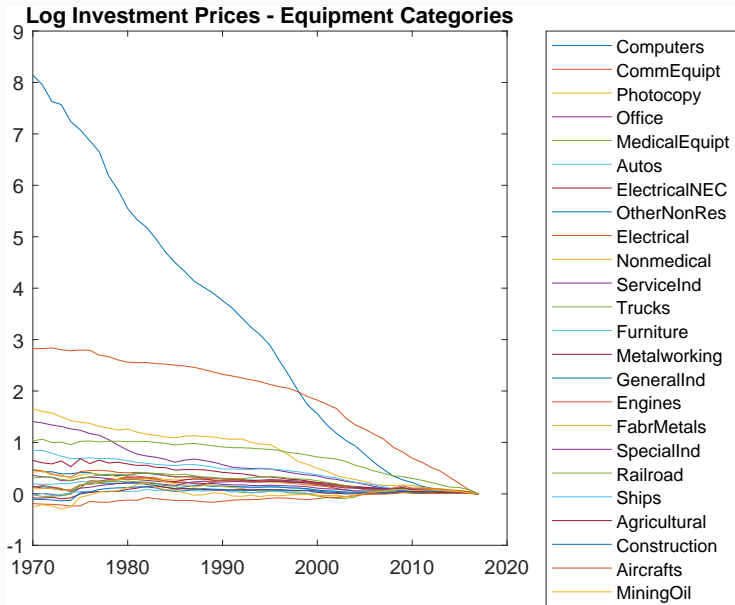
# Motivation

- Use simple extension of neoclassical growth model to measure contribution of several drivers
  - Higher perceived risk since 2000,
  - Why? P/D roughly stable despite much lower rates,
  - Risk helps explain investment, ROA, rates, etc.
  - Also: role of market power, savings demand, TFP growth, ... and **Investment-Specific Technical Change (ISTC)**
- We noticed some puzzling patterns in ISTC data, and wanted to improve our framework's approach to ISTC

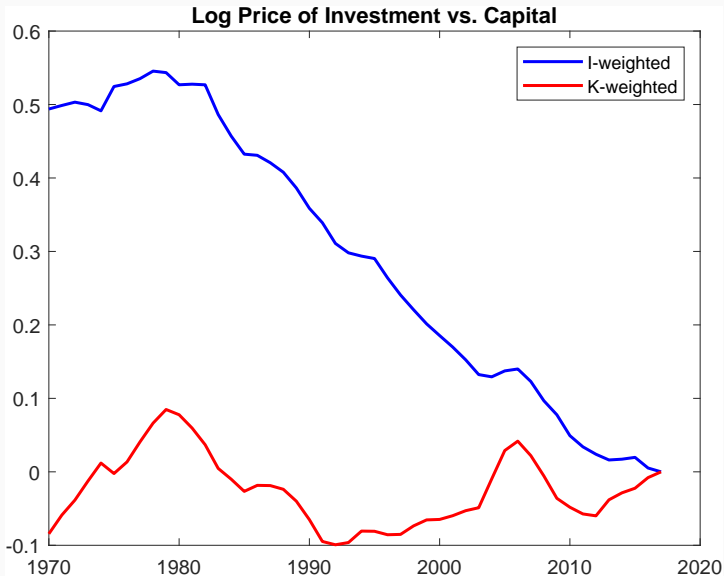
# How important is ISTC?

- Vast literature argues ISTC relevant for key macro facts:
  - Growth ( e.g., Greenwood, Hercovitz and Krusell 1997)
  - Business cycles ( e.g., Fisher 2006)
  - Labor Share ( e.g., Karabarbounis and Neiman 2012)
  - Decline of  $r^*$  ( e.g., Summers 2014, Sajedi and Thwaites 2016)
- ISTC measured using price of new investment goods
- But: huge heterogeneity in price trends - aggregation?

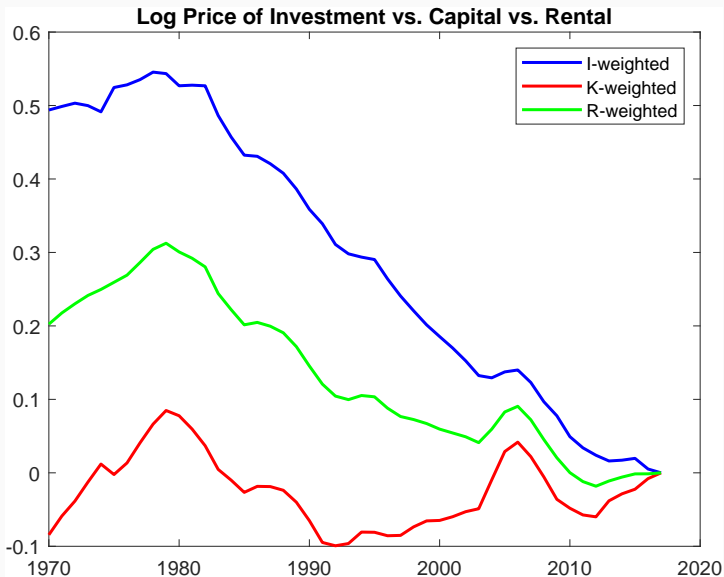
# Heterogeneity in Equipment Price Trends



# Flow- and Stock-weighted Prices



# Flow-, Stock-, and Rental-weighted Prices



# Outline

1. Simple framework
2. Role of ISTC for growth
3. Role of ISTC for “big ratios”
4. More quickly:
  - Role of ISTC for business cycles
  - Role of ISTC for labor share
  - Role of ISTC for  $r^*$



# Simple Framework

## Simple Model

Utility function:

$$U = \int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} dt$$

Production function:

$$Y_t = A_t L_t^{\alpha_L} K_{1t}^{\alpha_{K_1}} \dots K_{nt}^{\alpha_{K_n}}$$

Capital accumulation for each type:

$$\dot{K}_{it} = I_{it} - \delta_i K_{it}$$

Resource constraint:

$$Y_t = C_t + \sum_{i=1}^n p_{it} I_{it}$$

Exogenous:  $A_t, L_t, p_{it}$

# Equilibrium

Euler equation:

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\sigma},$$

Perfect competition capital demand:

$$\alpha_{K_i} \frac{Y_t}{K_{it}} = R_{it} = p_{it} \left( r_t + \delta_i - \frac{\dot{p}_{it}}{p_{it}} \right),$$

Rearrange:

$$\frac{p_{it} K_{it}}{Y_t} = \frac{\alpha_{K_i}}{r_t + \delta_i - g_{p_i}}.$$

Implies that on the BGP,

$$g_{K_i} = g_Y - g_{p_i}$$

## Equilibrium growth rate

Production function:

$$g_Y = g_A + \alpha_L g_L + \sum_{i=1}^n \alpha_{K_i} g_{K_i},$$

Use  $g_{K_i} = g_Y - g_{p_i}$  :

$$g_Y - g_L = \frac{g_A - \alpha_K g_{p^R}}{\alpha_L}$$

Where

$$g_{p^R} \equiv \frac{\sum_{i=1}^n \alpha_{K_i} g_{p_i}}{\sum_{i=1}^n \alpha_{K_i}}$$

⇒ Aggregate invt prices using **rental weights**

## Price Indices: Definitions

General (Divisia) index  $p_t^s$  for given shares  $\{s_{it}\}$ :

$$\frac{\dot{p}_t^s}{p_t^s} = \sum_{i=1}^n s_{it} \frac{\dot{p}_{it}}{p_{it}}$$

Flow-weighted (NIPA) (I-w):

$$s_{it}^I \propto p_{it} l_{it}$$

Stock-weighted (FAT) (K-w):

$$s_{it}^K \propto p_{it} K_{it}$$

Rental-weighted (R-w):

$$s_{it}^R \propto R_{it} K_{it}$$

# Rental-shares, Stock-shares, Flow-shares

Rental weights:

$$s_{it}^R = \frac{R_{it}K_{it}}{\sum_{j=1}^n R_{jt}K_{jt}} \propto \alpha_{K_i}$$

Stock weights:

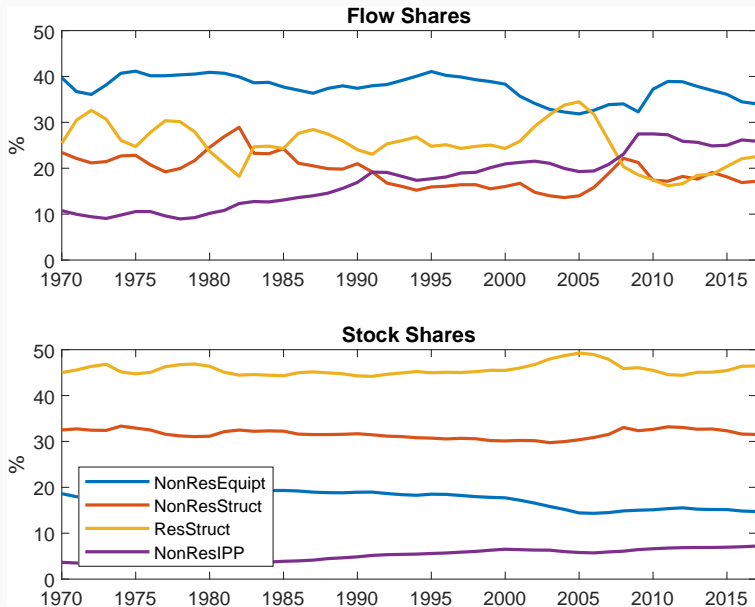
$$s_{it}^K = \frac{p_{it}K_{it}}{\sum_{j=1}^n p_{jt}K_{jt}} \propto \frac{\alpha_{K_i}}{r_t + \delta_i - g_{p_i}}$$

Investment weights **on the BGP**:

$$s_{it}^I = \frac{p_{it}I_{it}}{\sum_{j=1}^n p_{jt}I_{jt}} \propto \alpha_{K_i} \frac{g_Y + \delta_i - g_{p_i}}{r + \delta_i - g_{p_i}}$$

These shares are **very** different!

# I-share and K-share are different!



## How to infer rental shares along the BGP

On the balanced growth path:

$$s_i^R = \omega s_i^I + (1 - \omega) s_i^K$$

where:

$$\omega = \frac{\text{Agg. Invt}}{\text{Agg. Capital Income}}$$

Hence relation between price indices:

$$g_{p^R} = \omega g_{p^I} + (1 - \omega) g_{p^K}$$

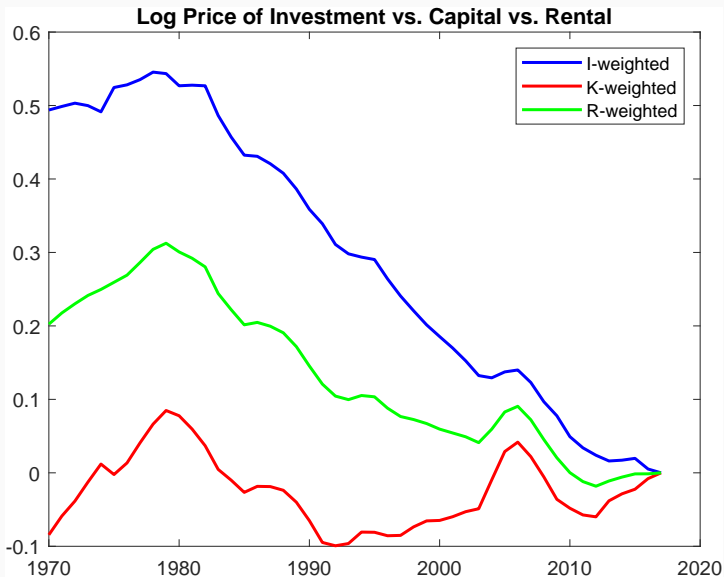
$\implies$  Can infer  $g_{p^R}$  from observables



# **Contribution of ISTC to Growth**

- Fixed Asset Tables: All private fixed assets
- Disaggregation in 57 categories
- Ex.: Invt::NonRes Equipt::Info Processing::Computers
- We use the BEA deflators (not Gordon-Violante-Cummins)

# Flow- and Stock-weighted Prices



# Contribution of ISTC to growth

- GHK: “ISTC contributes 58% to growth”
- Our approach (similar to theirs)
  1. Observe  $\alpha_L, \alpha_K, g_{p^R}, g_Y - g_L$
  2. Infer TFP  $g_A$  from:

$$g_Y - g_L = \frac{g_A - \alpha_K g_{p^R}}{\alpha_L}$$

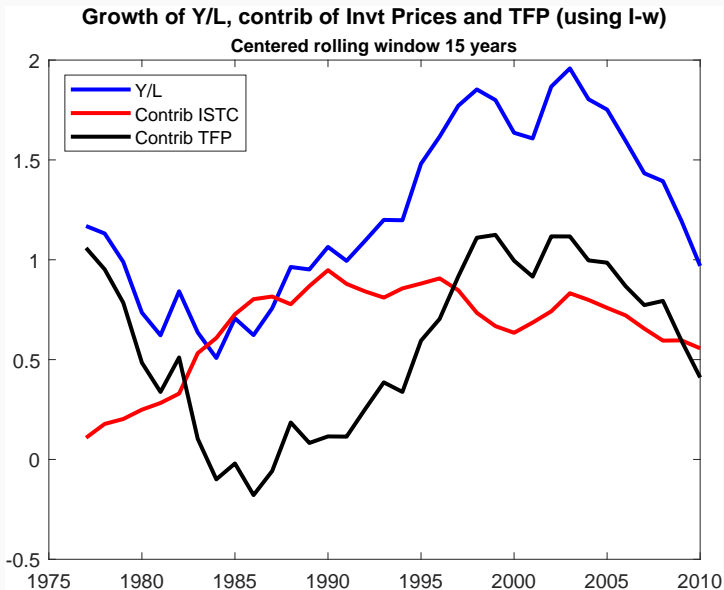
3. Calculate counterfactual growth if  $g_{p^R}=0$
4. What if use  $g_{p^I}$  instead of  $g_{p^R}$

## Smaller ISTC contribution with R-weighting

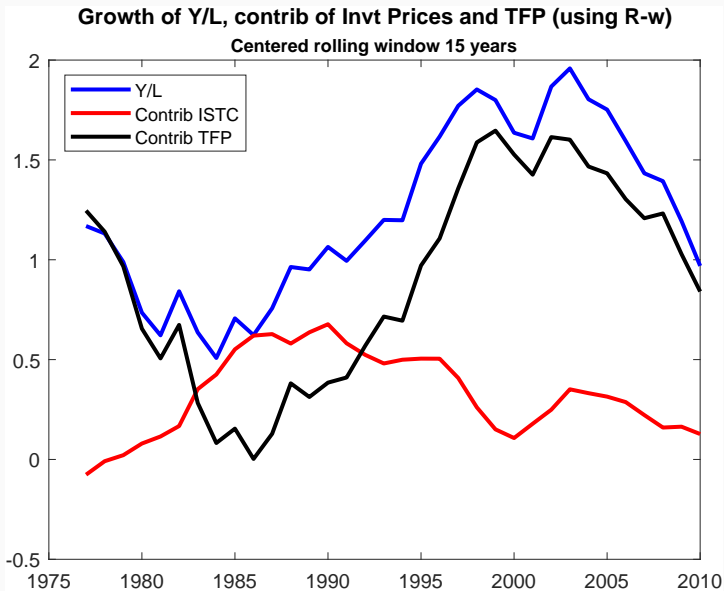
	<b>Data</b>	<b>lw:ITC</b>	<b>lw:TFP</b>	<b>Rw:ITC</b>	<b>Rw:TFP</b>
<b>1970-2017</b>	1.19	0.52	0.66	0.21	0.98
<b>(%)</b>	100.00	43.80	55.91	17.46	82.37

Avg. growth of Y/L and contributions of ISTC and TFP using either I-w or R-w to infer ISTC.

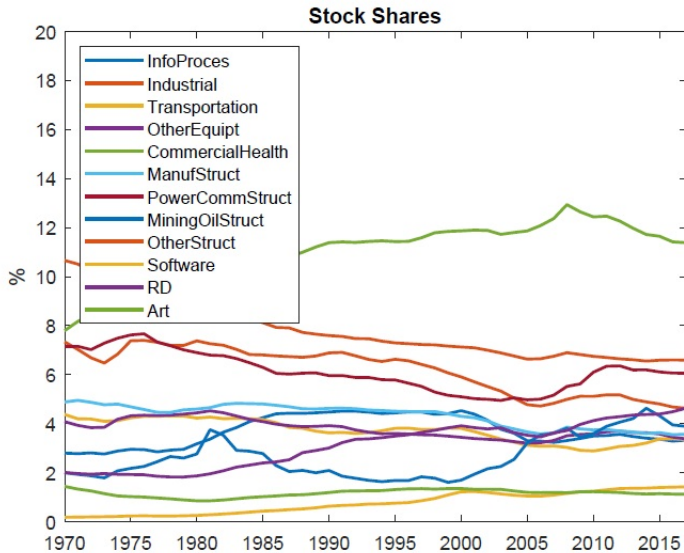
# Contributions to Growth: I-w (GHK)



# Contributions to Growth: R-w

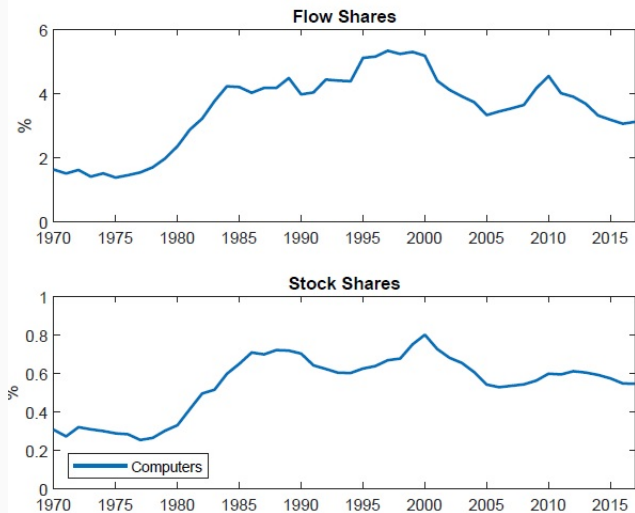


# How stable are shares?





# How stable are shares?



# **ISTC and the Big Ratios**

# Aggregation

Result: along the BGP,

$$\frac{I}{K} = g_Y + \delta^K - g_{p^K}$$

$$\frac{\Pi}{K} = r + \delta^K - g_{p^K}$$

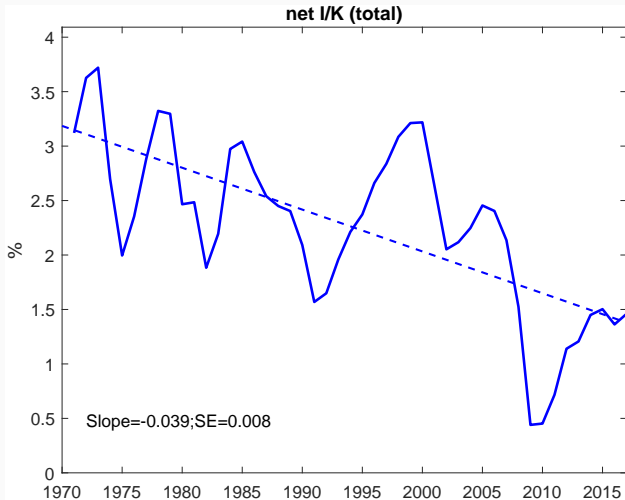
$$\frac{K}{Y} = \frac{\alpha_K}{r + \delta^K - g_{p^K}}$$

where  $I, K, \Pi$  are the (current-cost nominal) aggregates:

$$I = \sum p_i I_i, K = \sum p_i K_i, \Pi = \sum R_i K_i$$

$\implies$  To calibrate one-capital model, use **stock-weighted**

# Application: the decline of investment



## Application: the decline of investment

$$\frac{I}{K} - \delta^K = g_Y - g_{pK}$$

	<b>Net I/K</b>	<b>Contrib <math>g_Y</math></b>	<b>Contrib <math>g_{pK}</math></b>	<b>Residual</b>
<b>1990-2004</b>	2.39	2.51	-0.21	0.10
<b>2003-2017</b>	1.51	1.58	-0.37	0.30
<b>Change</b>	-0.89	-0.93	-0.16	0.20
<b>Change If use PI</b>	-0.89	-0.93	-0.69	0.74

# **ISTC and Business Cycles**

## Transitional Dynamics (w elastic labor)

$$\max_{C_t, l_{it}, K_{it}} U = \int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} v(L_t) dt$$

s.t. :

$$\dot{K}_{it} = l_{it} - \delta_i K_{it}, i = 1, \dots, N$$

$$Y_t = C_t + \sum_{i=1}^N P_{it} l_{it}$$

$$Y_t = A_t L_t^{\alpha_L} K_{1t}^{\alpha_{K_1}} \dots K_{nt}^{\alpha_{K_n}}$$

for given  $(K_{i0})_{i=1, \dots, N}$ , and  $(A_t, (P_{it})_{i=1, \dots, N})_{t \geq 0}$

## Proposition

Consider a “MIT shock” to the level of investment prices  $P_{i0}$ :  
Before  $t = 0$ , agents expect that

$$P_{it} = P_{i0}e^{g_i t},$$

but after  $t = 0$  they expect

$$P_{it} = P'_{i0}e^{g_i t}.$$

Then, for small changes in prices, the *full dynamics* of aggregates  $(Y_t, L_t, C_t, I_t)_{t \geq 0}$  (in deviation from BGP) depend only on:

$$\xi = s_I g_{p_I} + (1 - s_I) g_{p_K},$$

where  $s_I$  is the aggregate investment share of GDP, and  $g_{p_I}$  and  $g_{p_K}$  are the flow-weighted and stock-weighted changes in prices.

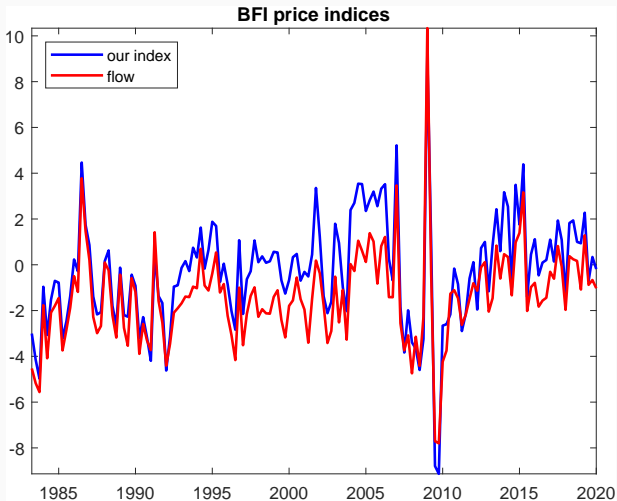


# Business cycle analysis

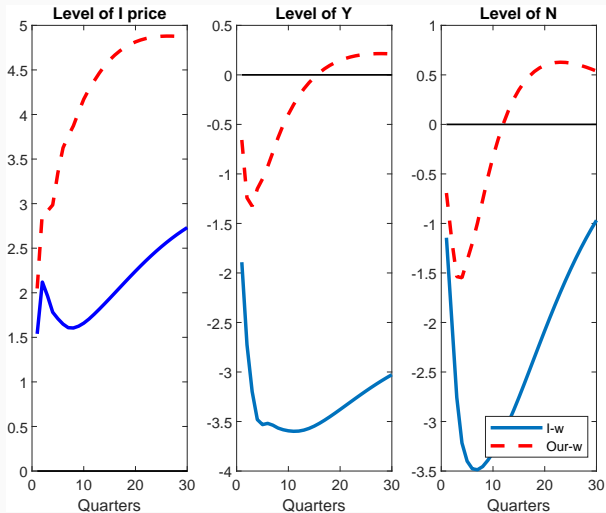
Run Fisher-style VAR:

- 3 variables:  $\text{dlog}(\text{Invt Price})$ ,  $\text{dlog}(Y/L)$ ,  $\log(L/\text{Pop})$ ,
- Long-run restrictions to identify ISTC shock, TFP shock,
- quarterly data, 4 lags, 1982IV-2019IV,
- 14 categories of goods (e.g. info processing),
- Compare I-w and Shock-w.

# Price indices

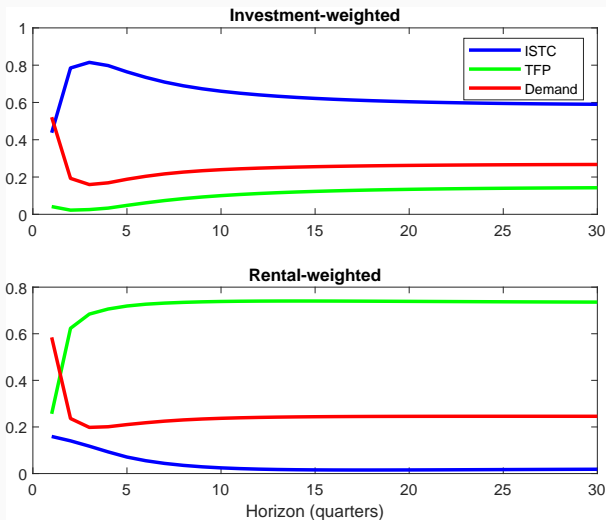


# VAR comparison



# Variance Decomposition BFI

Share of variance of hours due to ISTC / TFP / demand



# **ISTC and the Labor share**

# Labor Share

- If EOS  $K/L$   $\sigma \neq 1$ , chg invt prices affect labor share
- Model extension:

$$Y = (b_K K^{\frac{\sigma-1}{\sigma}} + b_L L^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$$

$$K = K_1^{\gamma_{K_1}} \dots K_n^{\gamma_{K_n}}$$

- Note: nonstationary shares if ISTC
- Consider a permanent small shock to vector  $P_{i0}$ .
- Then change in gross labor share is:

$$(\sigma - 1)\alpha_K \hat{p}^R$$

$\implies$  Relevant price for labor share is **R-weighted**

## Illustration

Implied change in labor share since 1970  
given observed prices changes and assumed EOS:

	<b>lw</b>	<b>Rw</b>
$\sigma = 1.5$	-0.17	-0.07
$\sigma = 1.25$	-0.09	-0.03
$\sigma = 0.75$	0.09	0.03
$\sigma = 0.5$	0.17	0.07

**ISTC and the decline of  $r^*$**



## Decline of $r^*$

- Lower investment price may reduce eqm interest rate by reducing required invt ( e.g. Summers, Sajedi and Thwaites )
- “Lower demand for savings”
- Model extension: upward-sloping savings  $W_t L_t S(r_t)$ 
  - e.g., OLG or Aiyagari
  - Otherwise,  $r^*$  pinned down by preferences
- Equilibrium in asset market:

$$\sum_{i=1}^n p_{it} K_{it} = W_t L_t S(r_t)$$

- Consider a permanent small shock to vector  $p_{i0}$ .
- Then change in  $r^*$  is  $\zeta \widehat{p}^R$
- Correct aggregation for  $r^*$  is **R-weighted**

## Illustration

Implied change in interest rate since 1990  
given observed prices changes and assumed elasticity:

	<b>lw</b>	<b>Rw</b>
$\zeta = .1$	-0.14	-0.06
$\zeta = .2$	-0.27	-0.11
$\zeta = .3$	-0.41	-0.17
$\zeta = .5$	-0.68	-0.28

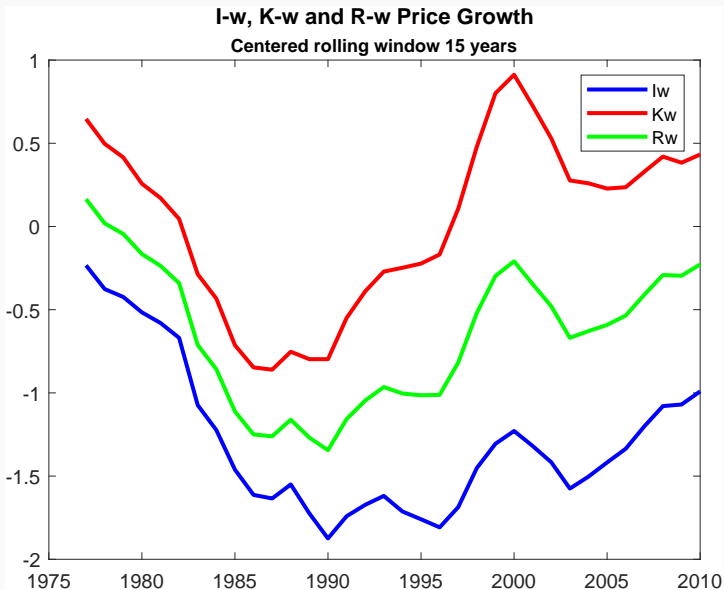
# Conclusion

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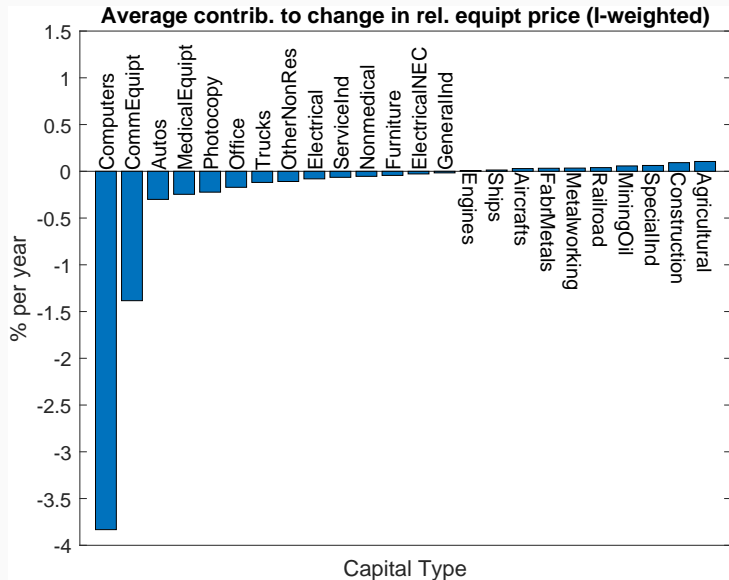
- Methodology: appropriate aggregation depends on question at hand! I-w, K-w, R-w, Shock-w ...
- Simple calculations illustrate this can matter
- In progress: relax some simplifying assumptions (BGP, perfect competition, Cobb-Douglas, ...)

# Backup

# Rolling windows: Price Growth



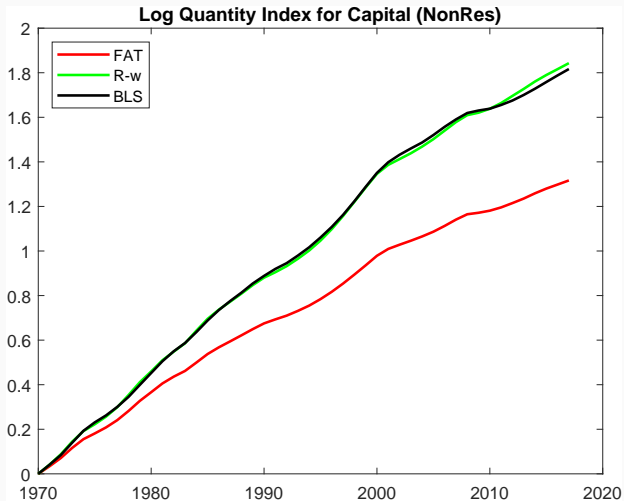
# Contributions to Equipment Deflator



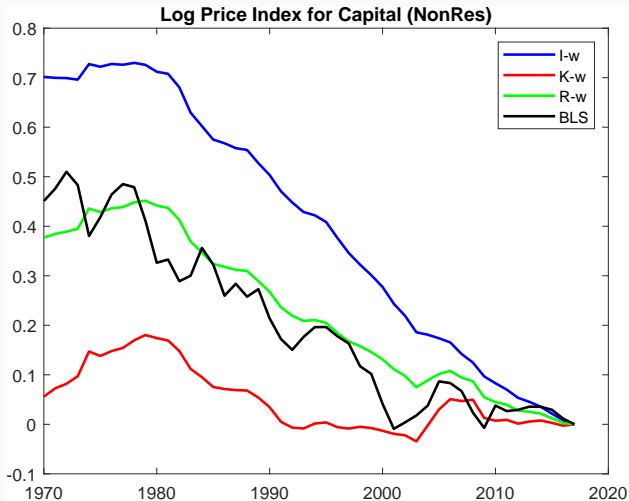




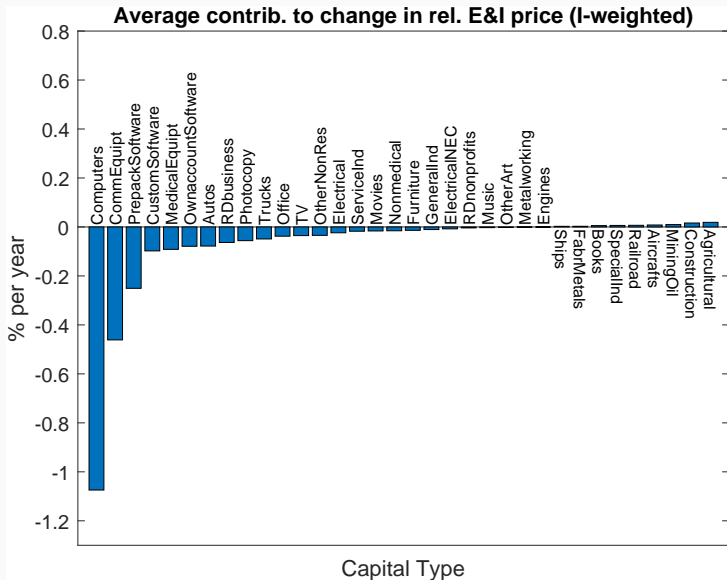
# Comparison with BLS



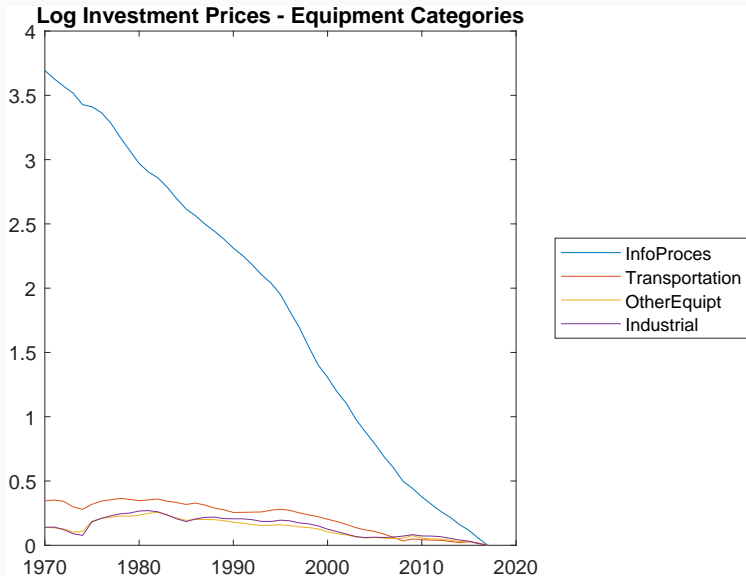
# Comparison with BLS



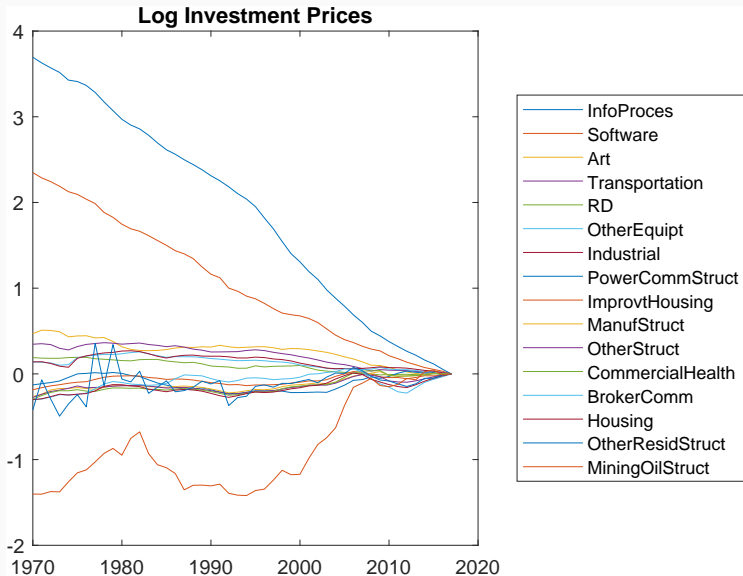
# Contributions to I-w E&I price



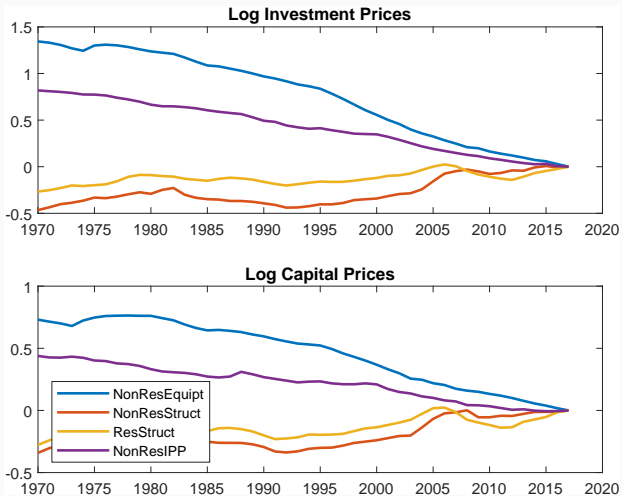
# Prices



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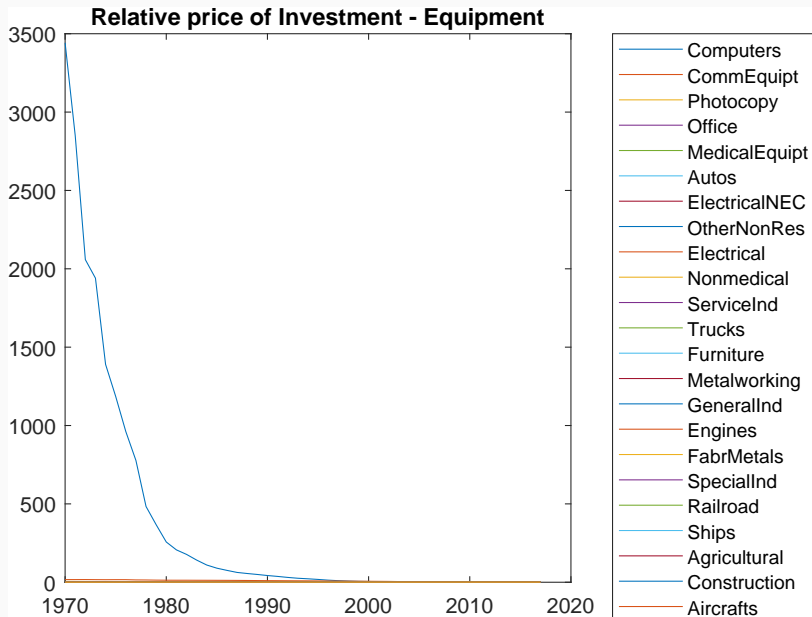
## Proof

$$\begin{aligned}R_i K_i &= (r + \delta_i - g_{p_i}) P_i K_i \\&= (r - g_Y) P_i K_i + (g_Y + \delta_i - g_{p_i}) P_i K_i \\&= (r - g_Y) P_i K_i + P_i I_i\end{aligned}$$

$$\sum R_i K_i = \alpha_K Y = (r - g_Y) K + I$$

$$\begin{aligned}s_i^R &= \frac{R_i K_i}{\sum R_j K_j} \\&= \frac{(r - g_Y) P_i K_i + P_i I_i}{(r - g_Y) K + I} \\&= \frac{P_i K_i}{K} \left(1 - \frac{s_I}{\alpha_K}\right) + \frac{P_i I_i}{I} \frac{s_I}{\alpha_K}\end{aligned}$$

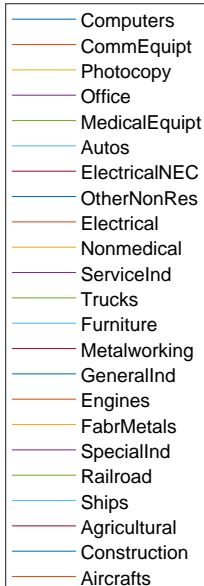
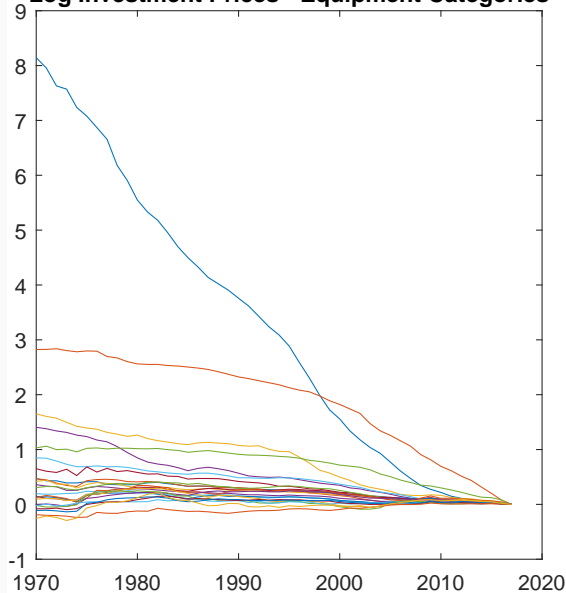
# Relative prices



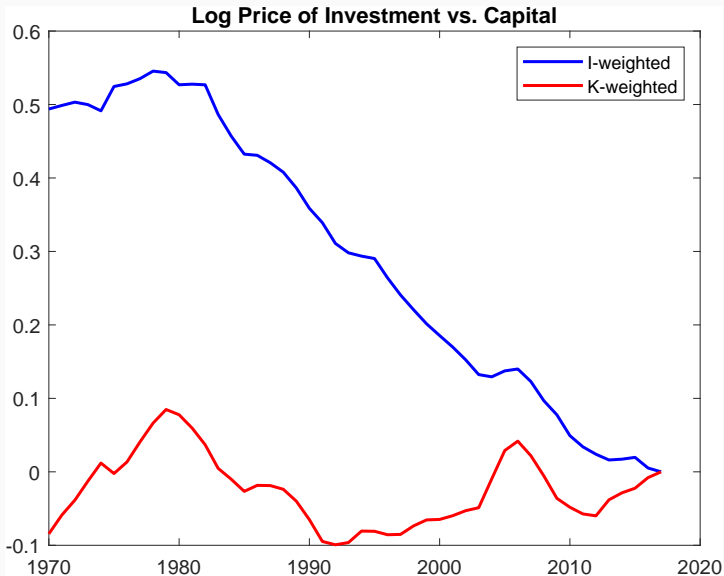


# Log relative prices

## Log Investment Prices - Equipment Categories

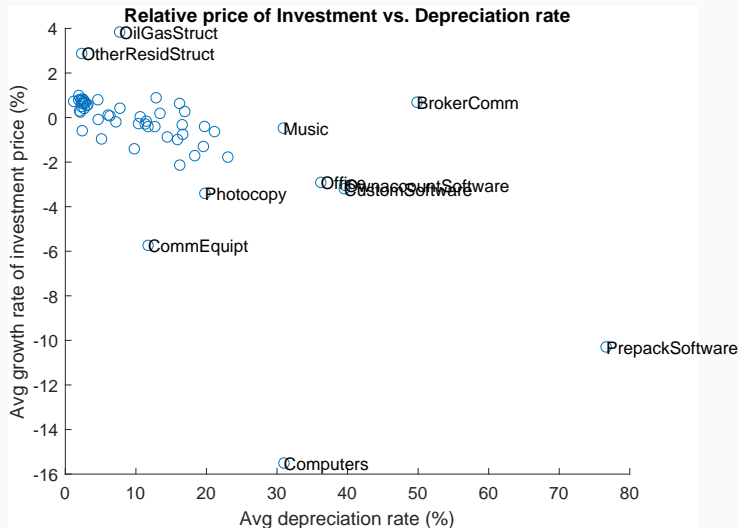


# I-w vs. K-w prices

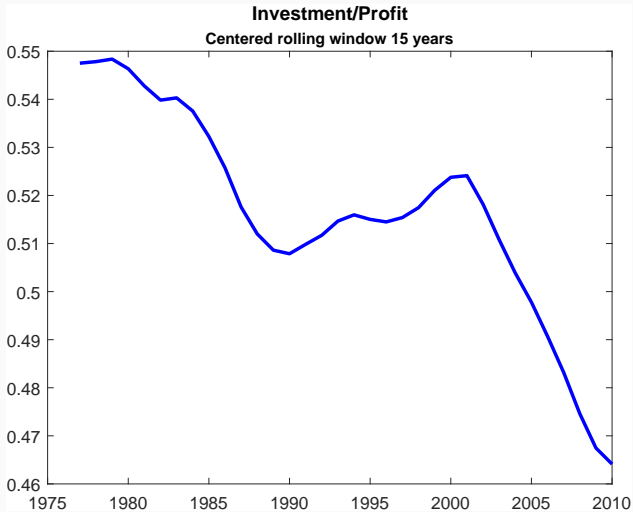




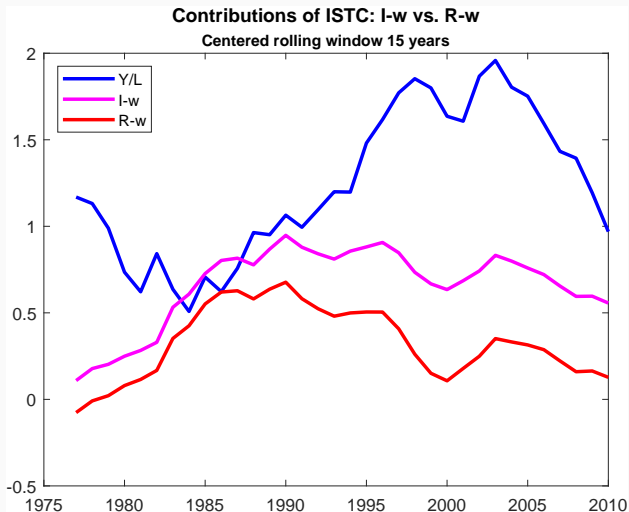
# Depreciation and Price Trend



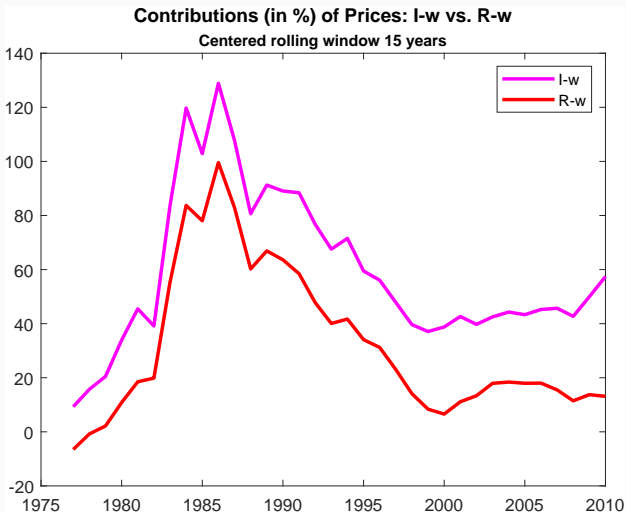
# Investment-Profit Ratio



# Comparison of contribution of ISTC



# Comparison of contribution of ISTC: Percentages

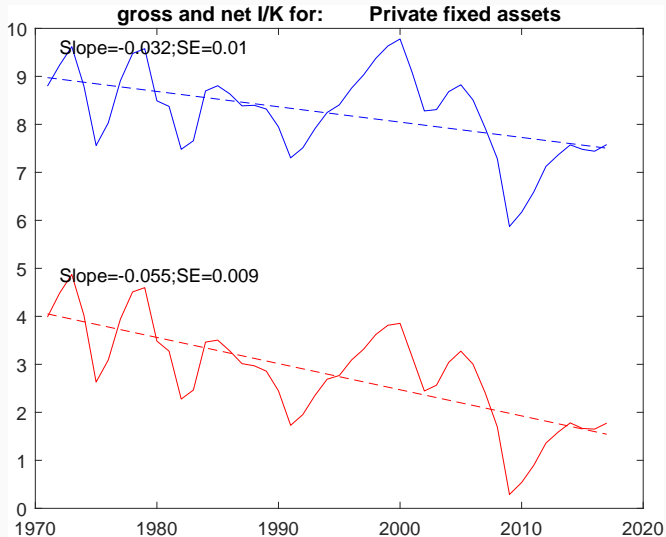


# Macroeconomic Puzzles

- Decline of investment
- High profitability
- Decline of labor share
- Decline of  $r^*$  (TBA)



# Decline in net I/K



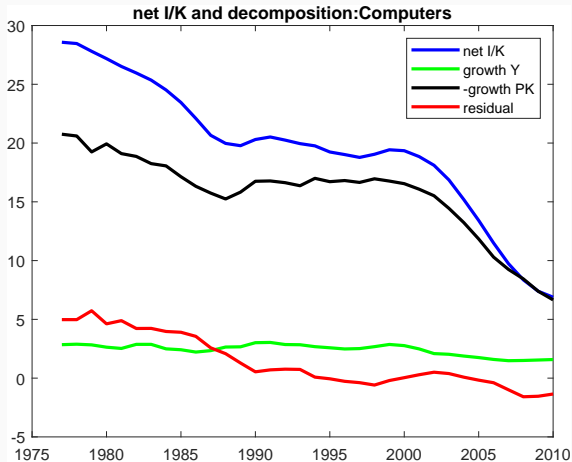
## Decline of I/K

Write BGP condition, adding an error term:

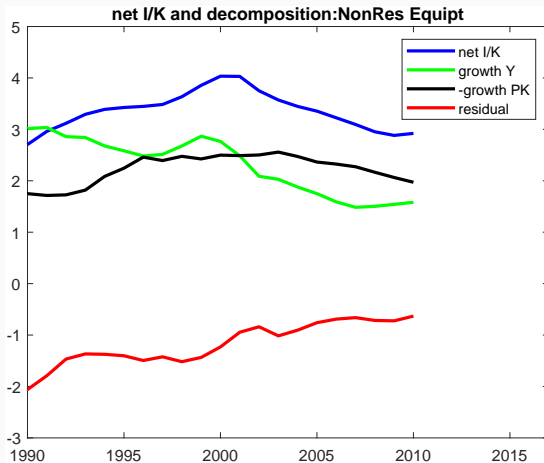
$$\frac{I_{it}}{K_{it}} = \delta_i + g_Y - \frac{\dot{p}_{it}}{p_{it}} + \varepsilon_{it}.$$

True at any level of aggregation (w. stock-weighted indices)

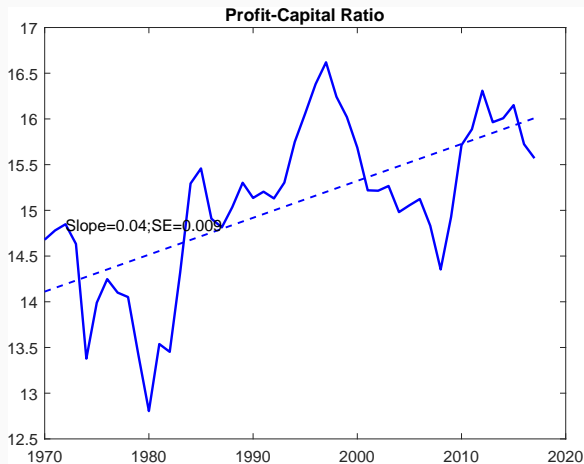
# Evolution of net I/K: computers



# Evolution of net I/K: non-res equipment



# Stability of Profit/K

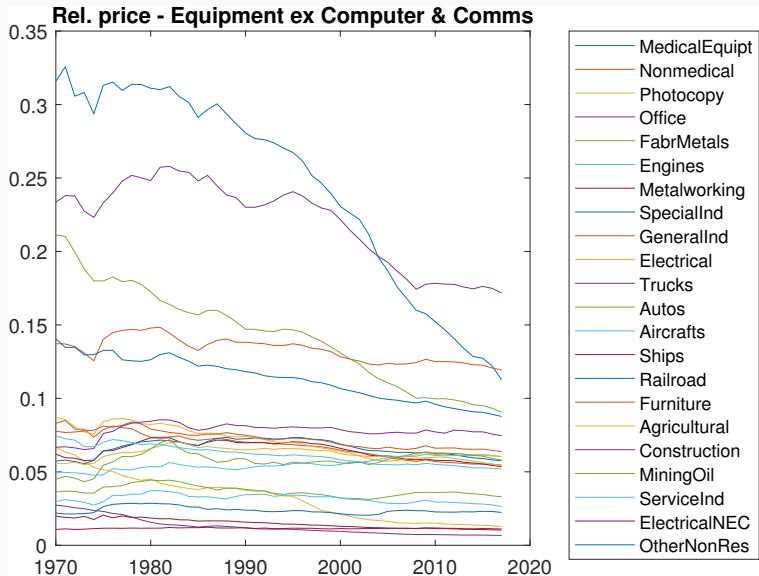


# Data

	<b>DlogY/H</b>	<b>Inv/Prof</b>	<b>Price IW</b>	<b>Price KW</b>	<b>Price RW</b>
<b>1970-2017</b>	1.19	0.51	-1.02	0.23	-0.41
<b>1970-1984</b>	1.17	0.55	-0.23	0.65	0.16
<b>1985-2005</b>	1.49	0.52	-1.49	0.09	-0.73
<b>2006-2017</b>	0.68	0.45	-1.12	-0.01	-0.51

Avg. growth of Y/L, I/Profits, and I-w, K-w, R-w prices

# Heterogeneity in Equipment Price Trends



# Transitional Dynamics

- Single capital state variable: total wealth

$$K_t^W = \sum_{i=1}^N P_{it} K_{it}.$$

- Can characterize equilibrium using standard household FOCs plus:

$$\frac{K_t^W}{Y_t} = \sum_{i=1}^N \frac{\alpha_i}{r_t + \delta_i - g_{it}},$$

$$\dot{K}_t^W = \alpha_L Y_t - C_t + r_t K_t^W,$$

$$Y_t = A_t^{\frac{1}{\alpha_L}} L_t \prod_{i=1}^N \left( \frac{\alpha_i}{P_{it}} \frac{1}{r_t + \delta_i - g_{it}} \right)^{\frac{\alpha_i}{\alpha_L}}.$$



## Intuition

- State variable = total capital relative to BGP,
- The shock shifts BGP to a parallel path,
- Shock also shifts total capital at  $t = 0$ ,
- Overall effect on deviation depends, only on its effect on state variable at  $t = 0$ .

# Graphical Illustration

