PhD Macro 2 Topic 9 Questions

1) In the Diamond coconut model, suppose that $\rho \coloneqq y \coloneqq a \coloneqq 1$, $b(e) = e$ for all $e \in [0,1]$, and suppose $G(c) = \frac{\left(-1+\sqrt{1+2\log(1-c)}\right)^2}{\sqrt{1+2\log(1-c)}}$ for all $c \in [0, \overline{c}]$, where $\overline{c} \coloneqq 1 - \exp\left(\frac{5}{4} - \frac{3}{4}\sqrt{5}\right)$, with $G(c) = 1$ for $c > \bar{c}$. Verify that $G(0) = 0$ and $G(\bar{c}) = 1$ so G is a valid CDF. Go on to prove that $c_c^*(e) = c_e^*(e)$ for all $e \in \left[0, \frac{1}{2}\sqrt{5} - \frac{1}{2}\right]$, so the model has a continuum of steady-states.

2) [From the 2017 Resit] There is a unit mass of people living on a tropical island, all of whom are risk neutral, and discount the future at rate ρ. The people on the island may pick coconuts by climbing coconut trees. Climbing a coconut tree always has a utility cost of 2 util. Islanders cannot consume their own coconuts however, and must search to find someone with whom to exchange. When they make such an exchange, they immediately consume their "purchased" coconut, obtaining a utility benefit of 5 utils. They cannot climb coconut trees while holding a coconut, so when holding a coconut, they will always search until such a trade happens.

Let $e(t)$ be the mass of people holding a coconut at t, and suppose that while holding a coconut, the chance of meeting another person with whom to trade in the interval $[t, t + dt)$ is $3e(t)dt$. Similarly, while not holding a coconut, the chance of finding a coconut tree to climb for a coconut in the interval $[t, t + dt)$ is $4dt$.

Let $V_e(t)$ be the value of holding a coconut at t, and $V_u(t)$ be the value of searching for a palm tree at t. Finally, define $c(t) \coloneqq V_e(t) - V_u(t)$ and $i(t) = 1$ when $c(t) \geq 2$ and $i(t) = 0$ when $c(t) < 2$.

a) If an islander arrives at a coconut tree at t, under what conditions will they choose to climb the tree to recover the coconut? Hence, derive expressions for $V_e(t)$ and $V_u(t)$, possibly involving $i(t)$. You may assume that they will always climb the tree when they are strictly indifferent.

b) Use the previously derived expressions to find a law of motion for $c(t)$ not containing V_e or V_u, but possibly containing $i(t)$.

c) Now derive the law of motion for $e(t)$, possibly involving $i(t)$.

d) Show that if $c(t) < 2$, then $\dot{e}(t) < 0$. Show that if $c(t) \geq 2$ and $e(t) < \frac{2}{3}$, then $\dot{e}(t) > 0$. Hence, prove that if $e(0) < \frac{2}{3}$, then $e(t) < \frac{2}{3}$ for all t.

d) Plot a phase diagram showing the behaviour of c and e in the vicinity of the point $c = 2$, $e = \frac{2}{9}\rho$. What is special about this point?

f) Use the previous two parts to argue that there are likely to be equilibrium trajectories which alternate between $\dot{e} > 0$ and $\dot{e} < 0$ many times.

3) Modify the Mortensen-Pissaridies model from the lectures as follows. Firstly, assume that new firms start production with productivity given by a draw from the CDF G, where this draw is not known by the firm until they acquire a worker. Secondly, assume that G has full support on the positive real line, rather than being limited to $[0,1]$ as in the lectures. Thirdly, assume that G has mean 1.

a) Do any of the conclusions of the model change?

b) What is the effect of a mean preserving spread of G?

c) Solve the social planner's optimisation problem, assuming that the social planner can choose which firms exit (i.e. R).

4) Extend the Mortensen-Pissaridies model from the lectures as follows. Firstly, assume that unemployed workers have a choice on whether to search or not, and that if they search, they pay a search cost of κ_u. Secondly, assume that employed workers also have a choice of searching for a new job while keeping their present one, and that if they search, they pay a search cost of κ_e. Assuming that $\kappa_u \leq \kappa_e < W(1) - U$ in steady-state, analyse the properties of the model. Do the unemployed always search? Which employed people search?