# PhD Macro 2 Topic 6 Questions

1) Construct the degree $3$ (i.e. two point) Gauss-Laguerre integration rule.

2) Construct the degree $3$ (i.e. two point) Gauss-Legendre integration rule.

3) Evaluate $\mathbb{E}\varepsilon^6$ where $\varepsilon \sim \mathrm{N}(0,1)$, using the Gauss-Hermite quadrature formulas given in the slides.

4) Approximate $\mathbb{E}\log(1+\exp\varepsilon)$ where $\varepsilon \sim \mathrm{N}(0,1)$, using the Gauss-Hermite quadrature formulas given in the slides. How close are you to the true value of 0.806059…?

5) Consider a hypothetical $d$-dimensional cubature rule which evaluates at the point $(0,\dots,0)$ and the points $(z,0,\dots,0)_{FS}$ for some $z>0$. Derive restrictions on $z$ and the weights such that the cubature rule exactly integrates all monomials of degree $3$. Is there an equal weighted rule in this class that exactly integrates all monomials of degree $3$? Is there a rule in this class that exactly integrates the functions $x \mapsto x_i^4$ for $i \in \{1,\dots,d\}$, as well as exactly integrating all monomials of degree $3$? Which rule is likely to behave better?

6) Prove that the degree 5 monomial cubature rule given in the slides is correct.

7) Numerically solve the basic real business cycle model by value function iteration (i.e. guess the value of the value function at a set of grid points over the state space of capital and productivity, plug that into the RHS of the Bellman equation, and update your guess), using Gauss-Hermite quadrature to evaluate expectations.

8) Use MCMC to evaluate $\int_{\mathbb{R}^d} \log\left(1+\sum_{j=1}^d x_j^2\right) f_d(x)\,dx$ where $f_d$ is the p.d.f. of a standard $d$-dimensional multivariate Cauchy distribution (i.e. a multivariate Student-t distribution with $\nu=1, \mu=0_d$ and $\Sigma=I_d$), for a variety of $d$.

9) Use importance sampling to evaluate $\int_{\mathbb{R}^d} \log\left(1+\sum_{j=1}^d x_j^2\right) f_d(x)\,dx$ where $f_d$ is the p.d.f. of a standard $d$-dimensional multivariate Cauchy distribution (i.e. a multivariate Student-t distribution with $\nu=1$, $\mu=0_d$ and $\Sigma=I_d$), for a variety of $d$, using a Gaussian proposal distribution.

10) Complete the following exercises from “Methods for applied macroeconomic research” (Canova):

a)

**Exercise 9.24** *A simple model of returns states that* $R_{it} = R_{Mt}\alpha_i + e_{it}$ *where* $i = 1, \ldots I, t = 1, \ldots T$ *and* $R_{Mt}$ *is a market portfolio (say, the return on the SP500 index). Suppose that, because of risk considerations, the prior for* $\alpha$ *is normal truncated outside the range (-2,2), i.e.* $g(\alpha_i) = \mathrm{N}(\bar{\alpha}, \bar{\sigma}_{\alpha}^2) * \mathcal{I}_{[-2,2]}$. *Describe how to implement an importance sampling algorithm to construct* $g(\alpha|R_i, R_M)$. *How would you select a portfolio composed of assets whose returns are positively correlated with the market?*

b)

**Exercise 9.31** *Consider the model* $y_t = \alpha y_{t-1} + e_t$, $e_t \sim \mathrm{N}(0, \sigma_e^2)$. *The density of* $(y_1, \ldots, y_t)$ *is* $f(y|\alpha, \sigma_e^2) \propto (\sigma_e^2)^{0.5(T-1)} \exp\{-0.5\sigma_e^{-2} \sum_{t=2}^{T}(y_t - \alpha y_{t-1})^2\} \times \exp\{-0.5\sigma_e^{-2} y_1^2(1-\alpha^2)\}$ $[\sigma_e^{-2}(1-\alpha^2)]^{-0.5}$, *where the last term is the density of* $y_1$. *Suppose the only available prior information is* $\alpha < 1$. *Show the form of the posterior for* $(\alpha, \sigma_e^2)$ *and describe how to use a MH algorithm to sample from it.*