PhD Macro 2 Topic 3 Questions

1) Consider the discrete time Fourier transform, $\mathcal{F}: (\mathbb{Z} \to \mathbb{C}) \to ([0,1] \to \mathbb{C})$. Prove that if $\mathcal{F}(a)$ is viewed as a function on the whole real line, then $\mathcal{F}(a)(\xi) = \mathcal{F}(a)(\xi + 1)$.

2) What is the Fourier transform of the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^{-\alpha|x|}$? (Hint: Split the integral.)

3) What is the Fourier transform of the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^{-\alpha x^2}$? (Hint: Complete the square.)

4) Prove that $S_{XX}(\omega) = S_{XX}(-\omega)$, both for continuous and discrete time processes time processes X_t.

5) Prove that for a weakly stationary continuous time process, $\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, d\omega = \operatorname{var} X_t$. (Hint: The slide on the Dirac Delta "function" may be useful.)

6) Proof that if $\varepsilon_t \sim \text{NIID}(0,1)$ (discrete time), $S_{\varepsilon\varepsilon}(\omega) = 1$.

7) Suppose $\Phi_p(L)y_t = \mu + \Theta_q(L)\sigma\varepsilon_t$, where Φ_p and Θ_q are polynomials of degree p and q respectively, and $\varepsilon_t \sim \text{NIID}(0,1)$. Suppose further that $\Phi_p(z)$ has roots $\rho_{\Phi,1}, \dots, \rho_{\Phi,p}$ and Θ_q has roots $\rho_{\Theta,1}, \dots, \rho_{\Theta,q}$. Express the spectral density of y_t in terms of these roots.

8) Complete the following exercises from "Methods for applied macroeconomic research" (Canova):

a)

Exercise 3.21 *Consider the process $y_t = 10 + 0.4t + e_t$ where $e_t = 0.8e_{t-1} + v_t$ and $v_t \sim$ iid $(0,1)$. Generate y_t, $t = 1, \dots, 200$, and filter it with the HP filter. Repeat the exercise using $y_t = \rho_y y_{t-1} + 10 + e_t$, where $\rho_y = 0.8, 0.9, 1.0$ and $y_0 = 10$. Compare the autocovariance functions of y_t^c in the two cases. Is there any pattern in the results? Why?*

b)

Exercise 3.40 *Consider the DGP used in exercise 3.37. Simulate data for the two components, compute* y_t *1000 times and calculate* $ACF(\tau)$ *of* y_t^c *for* $\tau = 1, \ldots 6$*. For each draw, estimate* y_t^c *using the fixed weight approximate BP filter, the non-stationary, asymmetric BP filter and the simulated* y_t*, and compute* $ACF(\tau)$ *of* y_t^c *for* $\tau = 1, \ldots 6$*. Using the true and the simulated distribution of* $ACF(\tau)$ *for each BP filter, examine which method better approximates the cyclical component of the data.*