PhD Macro 2 Topic 2 Questions

1) Suppose $A\Sigma_\varepsilon A' = B\Sigma_u B'$. How many restrictions on A and B do we need for all these matrices to be identified when neither A or B are known (assuming that Σ_u is diagonal, and A and B have unit diagonals)?

2) Prove that for a linear SVAR, $\mathbb{E}[x_t | u_{0,i} = 1]$ and $\mathbb{E}[x_t | u_{0,i} = \tilde{u}_{0,i} + 1]$ both give the standard impulse response function.

3) Suppose $x_t = \phi x_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim \mathrm{WNIID}(0, \Sigma_\varepsilon)$. Suppose further that $A\varepsilon_t = Bu_t$, and suppose further that it is known that the first element of u_t has no permanent effect on any variable. By diagonalizing ϕ work out what this implies about the first column of $A^{-1}B$. (Your answer should be in terms of the eigenvalues and eigenvectors of ϕ.)

4) Derive analogues of the four conditions defining the Wiener process for a discrete time random walk.

5) What is the unconditional distribution of the process $\mu t + \sigma W_t$ at t?

6) Derive $\int_0^T W_t \, dW_t$ from first principles, using the definition of the Itō integral.

7) Apply Itō's lemma to $Z_t \coloneqq a(t) + b(t)X_t$, where $dX_t = \mu_t \, dt + \sigma_t \, dW_t$.

8) Derive the mean, variance and auto-covariance function of an Ornstein-Uhlenbeck process started at time $-\infty$. (The Itō isometry will be useful.)

9) Complete the following exercises from "Methods for applied macroeconomic research" (Canova):

a)

Exercise 4.42 *Specify and estimate a bivariate VAR using Euro area GDP and M3 growth. Using the restriction that output growth is not contemporaneously affected by money growth shocks, trace out impulse responses and evaluate the claim that money has no medium-long run effect on output. Repeat the exercise assuming that the contemporaneous effect of money growth on output growth is in the interval [-0.5, 1,5] (do this in increments of 0.1 each). What can you say about the medium-long run effect of money growth on output growth in general?*

b)

Exercise 4.49 *(Giordani) Consider a sticky price model composed of an output gap ($gap_t = gdp_t - gdp_t^P$) equation, a potential output (gdp_t^P) equation, a backward looking Phillips curve (normalized on π_t) and a Taylor rule of the type*

$$\begin{aligned} gap_{t+1} &= a_1 gap_t - a_2(i_t - \pi_t) + \epsilon_{t+1}^{AD} && (4.45)\\ gdp_{t+1}^P &= a_3 gdp_t^P + \epsilon_{t+1}^P && (4.46)\\ \pi_{t+1} &= \pi_t + a_4 gdp_t^g + \epsilon_{t+1}^{CP} && (4.47)\\ i_t &= a_5\pi_t + a_6 gdp_t^g + \epsilon_{t+1}^{MP} && (4.48) \end{aligned}$$

The last equation has an error term (monetary policy shock) since the central bank may not always follow the optimal solution to its minimization problem. Let $var(\epsilon_{t+1}^i) = \sigma_i^2$, $i =$ AD, P, CP, MP and assume that the four shocks are uncorrelated with each other.
(i) Argue that contractionary monetary policy shocks have one period lagged (negative) effects on output and two periods lagged (negative) effects on inflation. Show that monetary policy actions do not Granger cause gdp_t^P for all t.
(ii) Derive a VAR for $[gdp_t,\ gdp_t^P,\ \pi_t,\ i_t]$. Display the matrix of impact coefficients.
(iii) Derive a representation for a three variable system $[gdp_t,\ \pi_t,\ i_t]$ (Careful: when you solve out potential output from the system the remaining variables do not follow a VAR any longer). Label the three associated shocks $e_t = [e_t^{AD}, e_t^{CP}, e_t^{MP}]$ and their covariance matrix Σ_e. Show the matrix of impact coefficients in this case.
(iv) Show that $var(e_t^{AD})$ >$var(\epsilon_t^{AD})$; $var(e_t^{MP}) > 0$ even when $\epsilon_t^{MP} = 0$ $\forall t$ and that $corr(e_t^{MP}, \epsilon_t^P) < 0$. Show that in a trivariate system, contractionary monetary policy shocks produce positive price responses (compare this with what you have in i))
(v) Intuitively explain why the omission of potential output from the VAR causes problems.

d)

Exercise 4.51 *(Quah) Consider a three equations permanent income model*

$$\begin{aligned} c_t &= rWe_t \\ We_t &= sa_t + [(1+r)^{-1}\sum_j (1+r)^{-j} E_t GDP_{t+j} \\ sa_{t+1} &= (1+r)sa_t + GDP_t - c_t && (4.52) \end{aligned}$$

where c_t is consumption, We_t is wealth, r is the (constant) real rate, sa_t are savings and $\Delta GDP_t = D(\ell)\epsilon_t$ is the labor income. Show that a bivariate representation for consumption and output is $\begin{bmatrix} \Delta GDP_t \\ \Delta c_t \end{bmatrix} = \begin{bmatrix} A_1(\ell) & (1-\ell)A_0(\ell) \\ A_1(\beta) & (1-\beta)A_0(\beta) \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{0t} \end{bmatrix}$ where $\beta = (1+r)^{-1}$, e_{1t} is a permanent shock and e_{0t} a transitory shock. Find $A_1(\ell)$ and $A_0(\ell)$. Show that if $\Delta Y_t = \epsilon_t$, the representation collapses to $\begin{bmatrix} \Delta GDP_t \\ \Delta c_t \end{bmatrix} = \begin{bmatrix} 1 & (1-\ell) \\ 1 & (1-\beta) \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{0t} \end{bmatrix}$. Show that the determinant of the matrix vanishes at $\ell = \beta < 1$ so that the MA representation for consumption and income is non-fundamental. Show that the fundamental MA is $\begin{bmatrix} \Delta GDP_t \\ \Delta c_t \end{bmatrix} = b(\beta)^{-1} \begin{bmatrix} (2-\beta)(1-\frac{1-\beta}{2-\beta}\ell) & (1-\beta\ell) \\ 1+(1-\beta)^2 & 0 \end{bmatrix} \begin{bmatrix} \tilde{e}_{1t} \\ \tilde{e}_{0t} \end{bmatrix}$, $var(\tilde{e}_{0t})$=$var(\tilde{e}_{1t}) = 1$.