PhD Macro 2 Topic 1 Questions

1) Consider a process which takes the values -2, -1, 0 and 1, 2 with probabilities $\frac{1}{5}2^{-t}$, $1-\frac{4}{5}2^{-t}$, $\frac{6}{5}2^{-t}-1$, $1-\frac{4}{5}2^{-t}$, $\frac{1}{5}2^{-t}$ respectively, for $t \in \mathbb{N}$. Prove this is weakly stationary but not stationary.

2) Let $y_{1,t} = e_{1,t} + \theta e_{1,t-1}$, $e_{1,t} \sim \text{NIID}(0, \sigma^2)$, where $\theta \in \mathbb{R}$ and $\sigma > 0$. Let $y_{2,t} = e_{2,t} + \frac{1}{\theta} e_{2,t-1}$, $e_{2,t} \sim \text{NIID}(0, \theta^2\sigma^2)$.

a) Show that $y_{1,t}$ and $y_{2,t}$ have identical auto-covariance functions.

b) If we only observed $y_{1,t}$ and not $e_{1,t}$, could we consistently estimate θ?

c) When is $y_{1,t}$ invertible?

d) When does an $\text{MA}(1)$ process have an $\text{AR}(\infty)$ representation? When is there an $\text{AR}(\infty)$ process with identical auto-covariance function to an $\text{MA}(1)$ process?

3) Suppose that households choose C_t and I_t to maximise $\sum_{s=0}^{\infty} \beta^s u(C_{t+s})$ (where u is an increasing function) subject to the budget constraint $\min\{K_t, A_t\} = C_t + I_t$ and the capital law of motion $K_t = (1-\delta)K_{t-1} + I_t$, where A_t is an $I(1)$ exogenous process. Are K_t, C_t and I_t cointegrated? With what rank? What is the cointegrating matrix? *Hint: You can solve this question without any differentiation.*

4) Complete the following exercises from "Methods for applied macroeconomic research" (Canova):

a)

Exercise 1.4 *Suppose $y_t = e_t$ if t is odd and $y_t = e_t + 1$ if t is even, where $e_t \sim$ iid $(0, 1)$. Show that y_t is not covariance stationary. Show that $y_t = \bar{y} + y_{t-1} + e_t$, $e_t \sim$ iid $(0, \sigma_e^2)$, where $\bar{y}$ is a constant is not stationary but that $\Delta y_t = y_t - y_{t-1}$ is stationary.*

b)

Exercise 1.5 *Suppose $y_{1t} = \bar{y} + at + e_t$, where $e_t \sim$ iid $(0, \sigma_e^2)$ and $\bar{y}, a$ are constants. Define $y_{2t} = \frac{1}{2J+1}\sum_{j=-J}^{J} y_{1t+j}$. Compute the mean and the autocovariance function of y_{2t}. Is y_{2t} stationary? Is it covariance stationary?*

c)

Exercise 1.6 *Consider $y_t = (1 + 0.5\ell + 0.8\ell^2)e_t$, and $(1 - 0.25\ell)y_t = e_t$ where $e_t \sim$ iid $(0, \sigma_e^2)$. Are these processes covariance stationary? If so, show the autocovariance ~~and the autocovariance generating~~ function*

d)

Exercise 1.7 *Let $\{y_{1t}(\varkappa)\}$ be a stationary process and let h be a $n \times 1$ vector of continuous functions. Show that $y_{2t} = h(y_{1t})$ is also stationary.*

e)

Exercise 4.9 *Check if* $y_t = \begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.2 \end{bmatrix} y_{t-1} + \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} y_{t-2} + e_t$ *is stable or not.*

f)

Exercise 4.10 *Consider* $y_t = e_t + 0.9e_{t-1}$ *and* $y_t = e_t + 0.3e_{t-1}$. *Compute the AR representations. What lag length is needed to approximate the two processes? What if* $y_t = e_t + e_{t-1}$?