PhD Marcro 2 2018 Coursework

Part 1 (Sufficient to pass)

Let $y_t = \begin{bmatrix} \log Y_t \\ \log\left(\frac{P_t}{P_{t-1}}\right) \\ \log(1+r_t) \end{bmatrix}$ where Y_t is real quarterly US GDP, P_t is the quarterly US GDP price deflator and r_t is the quarterly U.S. Federal Funds rate. (The Fed Funds rate should be in units so that a 1% annual interest rate corresponds to $r_t = \frac{1}{400}$, i.e. divide the FRED data by 400.)

Suppose that:

$$u_t = u_{t-1} + \Lambda_u \varepsilon_{u,t},$$

$$v_t = u_{t-1} + v_{t-1} + \Upsilon_{v,u}\varepsilon_{u,t} + \Lambda_v \varepsilon_{v,t},$$

$$w_t = w_{t-1} + \Upsilon_{w,u}\varepsilon_{u,t} + \Upsilon_{w,v}\varepsilon_{v,t} + \Lambda_w \varepsilon_{w,t},$$

$$x_t = \Phi x_{t-1} + \Upsilon_{x,u}\varepsilon_{u,t} + \Upsilon_{x,v}\varepsilon_{v,t} + \Upsilon_{x,w}\varepsilon_{w,t} + \Lambda_x \varepsilon_{x,t},$$

$$y_t = \begin{bmatrix} v_t \\ w_t \end{bmatrix} + x_t,$$

$$\varepsilon_{u,t} \sim \text{NIID}(0,1),$$

$$\varepsilon_{v,t} \sim \text{NIID}(0,1),$$

$$\varepsilon_{w,t} \sim \text{NIID}(0_{2\times 1}, I_{2\times 2}),$$

$$\varepsilon_{x,t} \sim \text{NIID}(0_{3\times 1}, I_{3\times 3}),$$

$$u_0 = m_u,$$

$$v_0 = m_v,$$

$$w_0 = m_w,$$

$$x_0 \sim \text{NID}(0, V_x),$$

where $m_u, m_v, \Lambda_u, \Lambda_v, \Upsilon_{v,u} \in \mathbb{R}$, $m_w, \Upsilon_{w,u}, \Upsilon_{w,v} \in \mathbb{R}^{2\times 1}$, $\Lambda_w \in \mathbb{R}^{2\times 2}$, $\Upsilon_{x,u}, \Upsilon_{x,v}, \Upsilon_{x,w} \in \mathbb{R}^{3\times 1}$, $\Upsilon_{x,w} \in \mathbb{R}^{3\times 2}$, $\Lambda_x, \Phi \in \mathbb{R}^{3\times 3}$, with Λ_w and Λ_x lower triangular, and where V_x solves the Lyapunov equation:

$$V_x = \Phi V_x \Phi' + \Upsilon_{x,u}\Upsilon_{x,u}' + \Upsilon_{x,v}\Upsilon_{x,v}' + \Upsilon_{x,w}\Upsilon_{x,w}' + \Lambda_x \Lambda_x'.$$

1) Show that the law of motion for $\begin{bmatrix} u_t \\ v_t \\ w_t \\ x_t \end{bmatrix}$ may be written in the form $\begin{bmatrix} u_t \\ v_t \\ w_t \\ x_t \end{bmatrix} = \mathrm{P} \begin{bmatrix} u_{t-1} \\ v_{t-1} \\ w_{t-1} \\ x_{t-1} \end{bmatrix} + \Lambda \begin{bmatrix} \varepsilon_{u,t} \\ \varepsilon_{v,t} \\ \varepsilon_{w,t} \\ \varepsilon_{x,t} \end{bmatrix}$, where Λ is lower triangular.

2) Explain why the model would be identified at least were $\begin{bmatrix} u_t \\ v_t \\ w_t \\ x_t \end{bmatrix}$ observed. Discuss the plausibility of the implicit identifying assumptions here.

3) Write MATLAB code to estimate the model using the Kalman filter, only assuming that y_t is observed. Describe how the estimation works in theory, and describe the practical workings of your code. *Hint 1: It may help to come up with a procedure to find a good initial guess for the optimisation algorithm. Hint 2: You may find that the CMAES algorithm is more successful in maximising the objective than fminunc.*

4) Run your estimation code on actual data for the US, for the longest time span available. Show your results, and IRFs to the monetary policy shock, $\varepsilon_{x,3,t}$. Produce standard errors for your estimates and IRFs using a parametric bootstrap (i.e. repeat the following: first, simulate the model with the estimated parameters; second, estimate on the simulated data). Comment on all the results.

Part 2

As before, let $y_t = \begin{bmatrix} \log Y_t \\ \log\left(\frac{P_t}{P_{t-1}}\right) \\ \log(1+r_t) \end{bmatrix}$ where Y_t is real quarterly US GDP, P_t is the quarterly US GDP price deflator and r_t is the quarterly U.S. Federal Funds rate. (Again, the Fed Funds rate should be in units so that a 1% annual interest rate corresponds to $r_t = \frac{1}{400}$, i.e. divide the FRED data by 400.)

Suppose that:

$$u_t = u_{t-1} + \Lambda_{u,t-1}\varepsilon_{u,t},$$

$$v_t = u_{t-1} + v_{t-1} + \Upsilon_{v,u,t-1}\varepsilon_{u,t} + \Lambda_{v,t-1}\varepsilon_{v,t},$$

$$w_t = w_{t-1} + \Upsilon_{w,u,t-1}\varepsilon_{u,t} + \Upsilon_{w,v,t-1}\varepsilon_{v,t} + \Lambda_{w,t-1}\varepsilon_{w,t},$$

$$x_t = \Phi_{t-1}x_{t-1} + \Upsilon_{x,u,t-1}\varepsilon_{u,t} + \Upsilon_{x,v,t-1}\varepsilon_{v,t} + \Upsilon_{x,w,t-1}\varepsilon_{w,t} + \Lambda_{x,t-1}\varepsilon_{x,t},$$

$$y_t = \begin{bmatrix} v_t \\ w_t \end{bmatrix} + x_t,$$

$$\varepsilon_{u,t} \sim \mathrm{NIID}(0,1),$$

$$\varepsilon_{v,t} \sim \mathrm{NIID}(0,1),$$

$$\varepsilon_{w,t} \sim \mathrm{NIID}(0_{2\times 1}, I_{2\times 2}),$$

$$\varepsilon_{x,t} \sim \mathrm{NIID}(0_{3\times 1}, I_{3\times 3}),$$

$$u_0 = m_u,$$

$$v_0 = m_v,$$

$$w_0 = m_w,$$

$$x_0 \sim \mathrm{NID}(0, V_x),$$

where for all t: $m_u, m_v, \Lambda_{u,t-1}, \Lambda_{v,t-1}, \Upsilon_{v,u,t-1} \in \mathbb{R}$, $m_w, \Upsilon_{w,u,t-1}, \Upsilon_{w,v,t-1} \in \mathbb{R}^{2\times 1}$, $\Lambda_{w,t-1} \in \mathbb{R}^{2\times 2}$, $\Upsilon_{x,u,t-1}, \Upsilon_{x,v,t-1}, \Upsilon_{x,w,t-1} \in \mathbb{R}^{3\times 1}$, $\Upsilon_{x,w,t-1} \in \mathbb{R}^{3\times 2}$, $\Lambda_{x,t-1}, \Phi_{t-1} \in \mathbb{R}^{3\times 3}$, with $\Lambda_{w,t-1}$ and $\Lambda_{x,t-1}$ lower triangular, and where V_x solves the Lyapunov equation:

$$V_x = \Phi_0 V_x \Phi_0' + \Upsilon_{x,u,0}\Upsilon_{x,u,0}' + \Upsilon_{x,v,0}\Upsilon_{x,v,0}' + \Upsilon_{x,w,0}\Upsilon_{x,w,0}' + \Lambda_{x,0}\Lambda_{x,0}'.$$

Let:

$$\Theta_t := \left[\Lambda_{u,t}, \Lambda_{v,t}, \left(\operatorname{vech} \Lambda_{w,t}\right)', \left(\operatorname{vech} \Lambda_{x,t}\right)', \Upsilon_{v,u,t}, \Upsilon'_{w,u,t}, \Upsilon'_{w,v,t}, \Upsilon_{x,u,t}, \Upsilon_{x,v,t}, \Upsilon_{x,w,t}, \left(\operatorname{vec} \Upsilon_{x,w,t}\right)', (\operatorname{vec} \Phi_t)'\right]',$$

and suppose that Θ_t evolves according to:

$$\Theta_t = \Theta_{t-1} + D \frac{\partial \log p(y_t | y_{t-1}, \dots, y_1)}{\partial \Theta_{t-1}},$$

$$\Theta_0 = m_\Theta,$$

where $D \in \mathbb{R}^{38\times 38}$ is a diagonal matrix controling the rate of parameter change and $m_\Theta \in \mathbb{R}^{38\times 1}$ gives the initial parameter values. (Note that here $\log p(y_t | y_{t-1}, \dots, y_1)$ is the t^{th} component of the model's log-likelihood as usual.)

1) Explain why the model can still be estimated with the Kalman filter, despite the non-linearity.

2) Write MATLAB code to estimate the model using the Kalman filter, only assuming that y_t is observed. Describe how the estimation works in theory, and describe the practical workings of your code. (To be clear, the parameters to estimate are D, m_Θ, m_u, m_v and m_w.)

3) Run your estimation code on actual data for the US, for the longest time span available. Show your results, and IRFs to the monetary policy shock, $\varepsilon_{x,3,t}$ at various different points in time. (You may produce IRFs as if parameters were not time-varying.) How have parameters and responses changed over time? Produce standard errors for your estimates and IRFs using a parametric bootstrap (i.e. repeat the following: first, simulate the model with the estimated parameters; second, estimate on the simulated data). Comment on all the results.