

# Search and matching, old Keynesian, and new monetarist models

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# Outline of today's talk

- The Diamond Coconut model
- The Mortensen and Pissaridies model
- Extensions and empirical performance
- New monetarist models
- Warning: some of the notation today may seem a little strange.
  - It is the standard in this literature however.

# Reading for today

- Diamond: “Aggregate Demand Management in Search Equilibrium”
- Diamond and Fudenberg: “Rational Expectations Business Cycles in Search Equilibrium”
  
- Pissarides: “Equilibrium Unemployment Theory”
- Hornstein, Krusell, Violante: “Unemployment and Vacancy Fluctuations in the Matching Model: Inspecting the Mechanism”
  
- Williamson and Wright: “New Monetarist Economics: Models”
  - In: “Handbook of Monetary Economics”

# The Diamond Coconut model (1/5)

- There is a unit mass of people living on a tropical island.
  - They all discount the future at rate  $\rho$ .
- The island has coconut trees on it, from which the people may pick coconuts, by climbing the tree.
  - The trees vary in height, and the taller the tree is, the higher the cost of climbing the tree to pick a coconut.
  - Suppose that the utility cost of climbing a tree picked uniformly at random has a distribution with c.d.f.  $G(c)$ , where  $G(0) = 0$ .
  - However, a taboo prevents people from consuming the coconuts they've picked.
  - Instead, they must search to find another islander with whom to exchange coconuts.
  - Once they exchange, they get a (utility) benefit of  $y > 0$  from consuming the coconut.
- Coconuts are big, so people can only hold one coconut at a time.
  - As a result, at any point in time, people are either holding a coconut and searching for someone to trade with, or not holding a coconut and searching for a coconut tree.
  - Let  $e(t)$  be the number of people holding a coconut.
  - And suppose the probability of finding someone else holding a coconut in the interval  $[t, t + dt)$  is  $b(e(t)) dt$ , where  $b'(e) > 0$ ,  $b''(e) \leq 0$  and  $b(0) = 0$ .
  - Finally, suppose the probability of finding a palm tree in that interval is  $a dt$ .

# The Diamond Coconut model (2/5)

- Let  $V_e(t)$  be the value of holding a coconut at  $t$ , and  $V_u(t)$  be the value of searching for a palm tree at  $t$ .
  - Then:  $V_e(t) = (1 - \rho dt)[(1 - b(e(t)) dt)V_e(t + dt) + (b(e(t)) dt)(y + V_u(t))]$ .
  - So:  $(\rho + b(e))V_e = \dot{V}_e + b(e)(y + V_u)$
- $V_u$  is a bit more difficult, because when a person arrives at a coconut tree, they may decide not to pick the coconut if the tree turns out to be too high.
  - Let  $c^*(t)$  be the climbing cost at which people are just indifferent between climbing the tree and getting the coconut, and searching for another tree.
  - Then  $V_e(t) - c^*(t) = V_u(t)$ , so  $c^*(t) = V_e(t) - V_u(t)$ . And:

$$V_u(t) = (1 - \rho dt) \left[ (1 - a dt)V_u(t + dt) \right]$$

# The Diamond Coconut model (3/5)

- Subtracting the second equation from the first then gives:

$$(\rho + b(e))c^* = \dot{c}^* + b(e)y - a \int_0^{c^*} (c^* - c) dG(c)$$

- The model is closed with the law of motion for  $e$ :

$$e(t + dt) = e(t)(1 - b(e(t)) dt) + (1 - e(t))(G(c^*(t))a dt)$$

- Hence:  $\dot{e} = a(1 - e)G(c^*) - b(e)e$ .

- Let  $c_e^*(e)$  be the  $\dot{e} = 0$  locus, so  $G(c_e^*(e)) = \frac{b(e)e}{a(1-e)}$ .

- Note that the RHS is a bijection of  $[0,1) \rightarrow [0, \infty)$ , so there is a unique  $\bar{e}$  such that  $1 = \frac{b(\bar{e})\bar{e}}{a(1-\bar{e})}$ .

- Now let  $\hat{c}_e^*: [0, \bar{e}] \rightarrow [\underline{c}, \bar{c}]$  be an arbitrary strictly increasing bijection, and suppose that  $G$  were defined by:  $G(c) = 0$  if  $c < \underline{c}$ ,

$$G(c) = \frac{b(\hat{c}_e^{*-1}(c))\hat{c}_e^{*-1}(c)}{a(1-\hat{c}_e^{*-1}(c))} \text{ if } \underline{c} \leq c \leq \bar{c} \text{ and } G(c) = 1 \text{ if } c > \bar{c}.$$

- Note this definition ensures  $G(c)$  is weakly increasing as required.

- Then for  $e \in [0, \bar{e}]$ , we must have  $c_e^*(e) = \hat{c}_e^*(e)$ .

- Thus given  $b$ , there is a  $G$  which will deliver any arbitrary strictly increasing bijection as the  $\dot{e} = 0$  locus.

# The Diamond Coconut model (4/5)

- Let  $c_c^*(e)$  be the  $\dot{c}^* = 0$  locus, so:

$$(\rho + b(e))c_c^*(e) = b(e)y - a \int_0^{c_c^*(e)} (c_c^*(e) - c) dG(c)$$

- Then, since the RHS of this equations tends to 0 as  $e \rightarrow 0$  (as  $b$  does and the LHS is weakly positive),  $c_c^*(e) \rightarrow 0$  as  $e \rightarrow 0$  as well.
- Also, from differentiating this equation:  $c_c^{*'}(e) = \frac{(y - c_c^*(e))b'(e)}{\rho + b(e) + aG(c_c^*(e))} > 0$ .
- So both loci are increasing, and one is arbitrary, so we should expect there to be many steady-states (i.e. intersections of the loci) in general.
  - It does not follow immediately that there are  $G$  and  $b$  such that there are arbitrarily many steady states, as the second loci is a function of  $G$ , and hence of the first.
  - However, if we take the limit as  $\rho = y \rightarrow \infty$ , then  $c_c^*(e) \rightarrow b(e)$ , so at least for large  $\rho$  and  $y$  we can construct  $G$  and  $b$  such that there are arbitrarily many steady states.
- In fact, for specific  $G$  and  $b$  there are infinitely many steady states.
  - For example, let  $\rho := y := a := 1$ ,  $b(e) = e$  for all  $e \in [0,1]$ , and let  $G(c) = \frac{(-1 + \sqrt{1 + 2 \log(1-c)})^2}{\sqrt{1 + 2 \log(1-c)}}$  for all  $c \in [0, \bar{c}]$ , where  $\bar{c} := 1 - \exp\left(\frac{5}{4} - \frac{3}{4}\sqrt{5}\right)$ .
  - Then (tedious algebra) shows that  $c_c^*(e) = c_e^*(e)$  for all  $e \in \left[0, \frac{1}{2}\sqrt{5} - \frac{1}{2}\right]$ .

# The Diamond Coconut model (5/5)

- With this special  $G$  we have complete indeterminacy, and there can be expectations driven business cycles which move or jump along (or even off) the overlapping loci.
  - If everyone believes that at this moment everyone else suddenly decided to pick coconuts, even from tall trees, then everyone indeed wants to start picking coconuts, even from tall trees.
- But there is multiplicity of equilibria quite generally. Indeed, even with  $b(e) = e$  and  $G(c) = c$  there are three equilibria for appropriate  $\rho$ .
  - Diamond and Fudenberg show that there are stable cycles which orbit the intermediate equilibrium.
  - A kind of endogenous business cycle, with alternating wages of optimism and pessimism.
- These features make the model look distinctly Keynesian.
  - Similar ideas underlie Roger Farmer's modern Keynesian models.

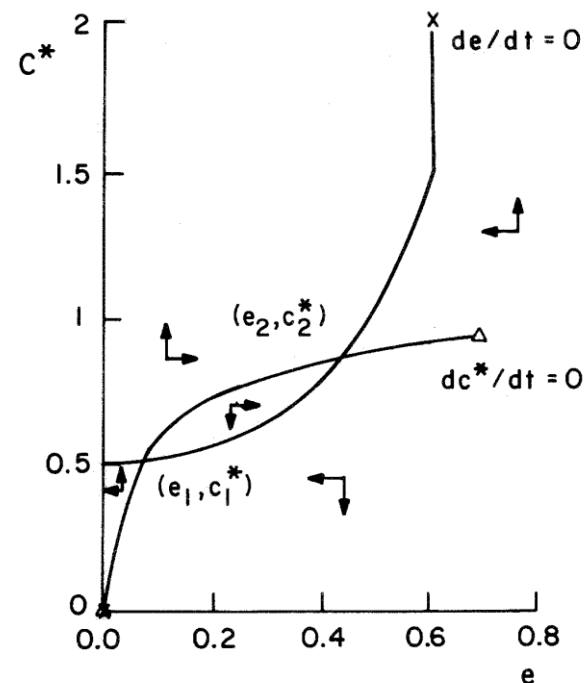


FIG. 1

From Diamond and Fudenberg (1989)



# The Mortensen-Pissarides model (with endogenous job destruction): Setup

- As before there is a unit mass of risk neutral households, who discount the future at rate  $\rho$ .
- Employed workers earn a wage  $w$  per unit of time but cannot search for jobs.
  - Extensions exist that allow for on the job search.
- Unemployed workers search for a new job, and receive the same utility as if they earned a wage of  $b$  per unit of time.
  - $b$  captures leisure, home production and government unemployment benefits.
- Each firm requires precisely one worker and produces  $px$  per unit of time, where  $p$  is an aggregate productivity level and  $x$  is an idiosyncratic one.
  - All firms start production with  $x = 1$ , but after production begins shocks to firm productivity arrive at a Poisson rate  $\lambda_x$ .
  - When a shock hits, a new value for  $x$  is drawn from the distribution  $G(x)$ , where  $G(0) = 0$  and  $G(1) = 1$ .
- There is no cost for firms to enter the market, but firms enter without a worker, and must then search to find one.
- Searching for a worker costs  $pc$  per unit of time.
  - These are the costs of posting a vacancy etc.

# The MP model: The matching function

- Let  $v$  be the number of vacancies, and  $u$  be the number of unemployed workers.
- For a variety of reasons (coordination, partial information, heterogeneity), matches between vacancies and workers do not happen instantly.
- Instead, we suppose  $m = M(u, v)$  matches happen in any unit of time, where  $M$  is a black-box matching function.
  - By playing with  $M$  you can potentially drastically alter the dynamics of the model.
  - Here though we will assume  $M$  is constant returns to scale (c.r.s.) and increasing and concave in both arguments.
  - The empirical evidence suggests that c.r.s. is a broadly reasonable assumption.
- These matches are randomly distributed over searching workers and firms, thus unemployed workers meet firms at a Poisson rate  $\lambda_w := \frac{m}{u}$  and firms fill vacancies at a Poisson rate  $\lambda_f := \frac{m}{v}$ .
  - Since  $M$  is c.r.s., if we define labour market tightness  $\theta$ , by  $\theta := \frac{v}{u}$ , and if we define the function  $q$  by  $q(\theta) = M(\theta^{-1}, 1)$ , then  $\lambda_w = \frac{v}{u} \frac{u}{v} \frac{1}{u} M(u, v) = \theta M\left(\frac{u}{v} \frac{u}{u}, \frac{u}{v} \frac{v}{u}\right) = \theta q(\theta)$  and  $\lambda_f = \frac{1}{v} M(u, v) = M\left(\frac{u}{v}, \frac{v}{v}\right) = q(\theta)$ .

# The MP model: Job creation and destruction

- Let  $V$  be the value of a firm with an open vacancy.
- Then the free entry condition states that  $V = 0$ .
  
- In the basic MP model, job destruction occurs at an exogenous rate.
- Here, destruction is endogenous, and will only happen when the firm's productivity experiences a shock which takes it below some threshold.
- Let  $J(x)$  be the value of a firm with a worker with idiosyncratic productivity  $x$ .
  - The firm will exit if  $J(x) < 0$ .
  - So define  $R$  as the solution to  $J(R) = 0$ .
  - If  $x < R$  the firm will choose to exit and the job will be destroyed.
  - If  $x \geq R$  the firm will remain in production.
- Then, the probability that the firm will exit in the interval  $[t, t + dt)$  is  $\lambda_x G(R) dt$ .

# The MP model: Unemployment dynamics

- In light of the above, unemployment's dynamics are given by:

$$u(t + dt) = u(t) + (1 - u(t))\lambda_x G(R) dt - u\lambda_w dt$$

- I.e.

$$\dot{u} = (1 - u)\lambda_x G(R) - u\theta q(\theta)$$

- So the  $\dot{u} = 0$  locus gives the “Beveridge-curve” relationship:

$$u = \frac{\lambda_x G(R)}{\theta q(\theta) + \lambda_x G(R)}$$

- Or, alternatively:

$$\lambda_x G(R) = \frac{M(u, v)}{1 - u}$$

- The RHS is increasing in both  $u$  and  $v$ , hence the  $\dot{u} = 0$  locus must be downwards sloping in  $(u, v)$  space, as in the data, at least as long as  $G(R)$  is not too responsive to  $\theta$ .

# The MP model: Value functions for firms

- Let  $w(x, t)$  be the wage in a firm with productivity  $x$  at  $t$ , then:

$$\begin{aligned} J(x, t) &= (p(t)x - w(x, t)) dt \\ &+ (1 - \rho dt) \left[ (1 - \lambda_x dt) J(x, t + dt) + (\lambda_x dt) \int_{R(t)}^1 J(x', t + dt) dG(x') \right] \end{aligned}$$

- I.e.:

$$(\rho + \lambda_x G(R)) J(x) = \dot{J}(x) + px - w(x) + \lambda_x \int_R^1 (J(x') - J(x)) dG(x')$$

- Also:

$$V(t) = -p(t)c dt + (1 - \rho dt) \left[ (1 - \lambda_f dt) V(t + dt) + (\lambda_f dt) J(1, t + dt) \right]$$

- Hence (remembering that  $V \equiv 0$ ):

$$pc = q(\theta) J(1)$$

# The MP model: Value functions for households

- Let  $W(x)$  be the value of being a worker in a firm with productivity  $x$ , and let  $U$  be the value of being unemployed. Then:

$$W(x, t)$$

$$= w(x, t) dt$$

$$+ (1 - \rho dt) \left[ (1 - \lambda_x dt) W(x, t + dt) + (\lambda_x dt) \left[ \int_{R(t)}^1 W(x', t + dt) dG(x') + G(R(t)) U(t) \right] \right]$$

- i.e.:

$$(\rho + \lambda_x G(R)) W(x) = \dot{W}(x) + w(x) + \lambda_x G(R) U + \lambda_x \int_R^1 (W(x') - W(x)) dG(x')$$

- Also:

$$U(t) = b dt + (1 - \rho dt) [(1 - \lambda_w) U(t + dt) + \lambda_w W(1, x + dt)]$$

- So:

$$(\rho + \theta q(\theta)) U = \dot{U} + b + \theta q(\theta) W(1)$$

# The MP model: Wage determination

- Since labour is supplied inelastically, in the absence of the matching friction, the equilibrium wage would be zero.
- However, with the matching friction, the firm has something to lose if the worker ever leaves due to dissatisfaction with their wage.
  - Unlike in the competitive case, they cannot immediately replace the lost worker.
- The usual assumption is that the firm and worker undertake (generalized) Nash bargaining over their respective surpluses from the match.
  - As you will know from micro, this can be “micro-founded” via alternating offer bargaining. (Rubinstein 1982; Osborne and Rubinstein 1990)
  - However, in order to get differing bargaining powers, you need differing discount rates, which we will not have here, so the micro-foundation is a little stretched in context.
  - Farmer just omits this equation, and leaves the model indeterminate.

- Given the parties respective outside options, if  $\beta$  is the bargaining power of the worker, the bargaining problem is:

$$\max_{w(x,t)} (W(x,t) - U(t))^\beta (J(x,t) - V(t))^{1-\beta}$$

- From the “discrete-time-esque” value-functions, we see that  $\frac{dW(x,t)}{dw(x,t)} = 1$  and  $\frac{dJ(x,t)}{dw(x,t)} = -1$ . Hence, the first order condition gives:

$$\frac{\beta}{W(x) - U} = \frac{1 - \beta}{J(x) - V}$$

# The MP model: Steady-state

- In steady-state,  $\dot{u} = \dot{j}(x) = \dot{W}(x) = \dot{U} = \dot{V} = 0$  and so equilibrium is characterised by:
  1.  $J(R) = 0$
  2.  $u = \frac{\lambda_x G(R)}{\theta q(\theta) + \lambda_x G(R)}$
  3.  $(\rho + \lambda_x G(R))J(x) = px - w(x) + \lambda_x \int_R^1 (J(x') - J(x)) dG(x')$
  4.  $pc = q(\theta)J(1)$
  5.  $(\rho + \lambda_x G(R))W(x) = w(x) + \lambda_x G(R)U + \lambda_x \int_R^1 (W(x') - W(x)) dG(x')$
  6.  $(\rho + \theta q(\theta))U = b + \theta q(\theta)W(1)$
  7.  $\beta J(x) = (1 - \beta)(W(x) - U)$
- 4 standard equations and 3 functional equations in 4 unknown variables ( $u, R, \theta, U$ ) and 3 unknown functions ( $J(\cdot), w(\cdot), W(\cdot)$ ).



# The MP model: Bargaining at $x = R$ and $x = 1$

- Note that equation 4 implies  $J(1) = \frac{pc}{q(\theta)}$ .
- And equation 6 implies  $W(1) = \frac{(\rho + \theta q(\theta))U - b}{\theta q(\theta)}$ .
- Since equation 7 holds for all  $x$ , it also holds for  $x = R$  and  $x = 1$ .
- Hence, (using equation 1 as well):
  - $0 = (1 - \beta)(W(R) - U)$
  - $\beta \frac{pc}{q(\theta)} = (1 - \beta) \left( \frac{(\rho + \theta q(\theta))U - b}{\theta q(\theta)} - U \right)$
- The first of these implies  $W(R) = U$ , so households are indifferent about the firm exiting at the point at which they exit.
  - Is a model in which people are indifferent about losing their jobs really capturing unemployment?
- The latter of these implies:  $\rho U = b + \frac{\beta}{1 - \beta} \theta pc$ 
  - The more tight are labour markets (so the harder firms find it to fill vacancies), the higher is  $\theta$  and the more valuable being unemployed is.

# The MP model: Solution for wages

- If we multiply equation 3 by  $\beta$ , and equation 5 by  $1 - \beta$ , and then taking their difference, we have:

$$\begin{aligned} & \beta(\rho + \lambda_x G(R))J(x) - (1 - \beta)(\rho + \lambda_x G(R))W(x) \\ &= \beta px - (1 - \beta)\lambda_x G(R)U - w(x) + \beta\lambda_x \int_R^1 (J(x') - J(x)) dG(x') \\ & \quad - (1 - \beta)\lambda_x \int_R^1 (W(x') - W(x)) dG(x') \end{aligned}$$

- Now  $W(x') - W(x) = (W(x') - U) - (W(x) - U) = \frac{\beta}{1 - \beta} (J(x') - J(x))$ , hence, this simplifies to:

$$0 = \beta px + (1 - \beta)(\rho + \lambda_x G(R))U - (1 - \beta)\lambda_x G(R)U - w(x)$$

- I.e.

$$w(x) = (1 - \beta)\rho U + \beta px = (1 - \beta)b + \beta p(x + \theta c)$$

- If labour markets are slack (so the unemployed find it hard to find a job),  $\theta$  is small, and wages are low.

# The MP model: Job creation condition

- From equation 3, and our solution for wages:

$$(\rho + \lambda_x)J(x) = (1 - \beta)(px - b) - \beta\theta pc + \lambda_x \int_R^1 J(x') dG(x')$$

- When  $x = R$ , this implies:

$$0 = (1 - \beta)(pR - b) - \beta\theta pc + \lambda_x \int_R^1 J(x') dG(x')$$

- Taking the difference of these two equations then gives:

$$J(x) = \frac{(1 - \beta)(x - R)}{\rho + \lambda_x} p$$

- When  $x = 1$ , this implies:

$$\frac{c}{q(\theta)} = \frac{(1 - \beta)(1 - R)}{\rho + \lambda_x}$$

- This states that the expected hiring cost of creating a new job should be equal to the expected gain from creating it.
- In  $(\theta, R)$  space, it is a downwards sloping curve, as  $q$  is a decreasing function of  $\theta$ . So when labour markets are tight, so hiring costs are high, they will only incur these costs if  $R$  is small which ensures the job will last a long time.

# The MP model: Job destruction condition

- Substituting our solution for  $J(x)$  into the second equation of the previous slide gives:

$$0 = (1 - \beta)(pR - b) - \beta\theta pc + \lambda_x \frac{1 - \beta}{\rho + \lambda_x} p \int_R^1 (x' - R) dG(x')$$

- Hence:

$$R = \frac{b}{p} + \frac{\beta}{1 - \beta} \theta c - \frac{\lambda_x}{\rho + \lambda_x} \int_R^1 (x' - R) dG(x')$$

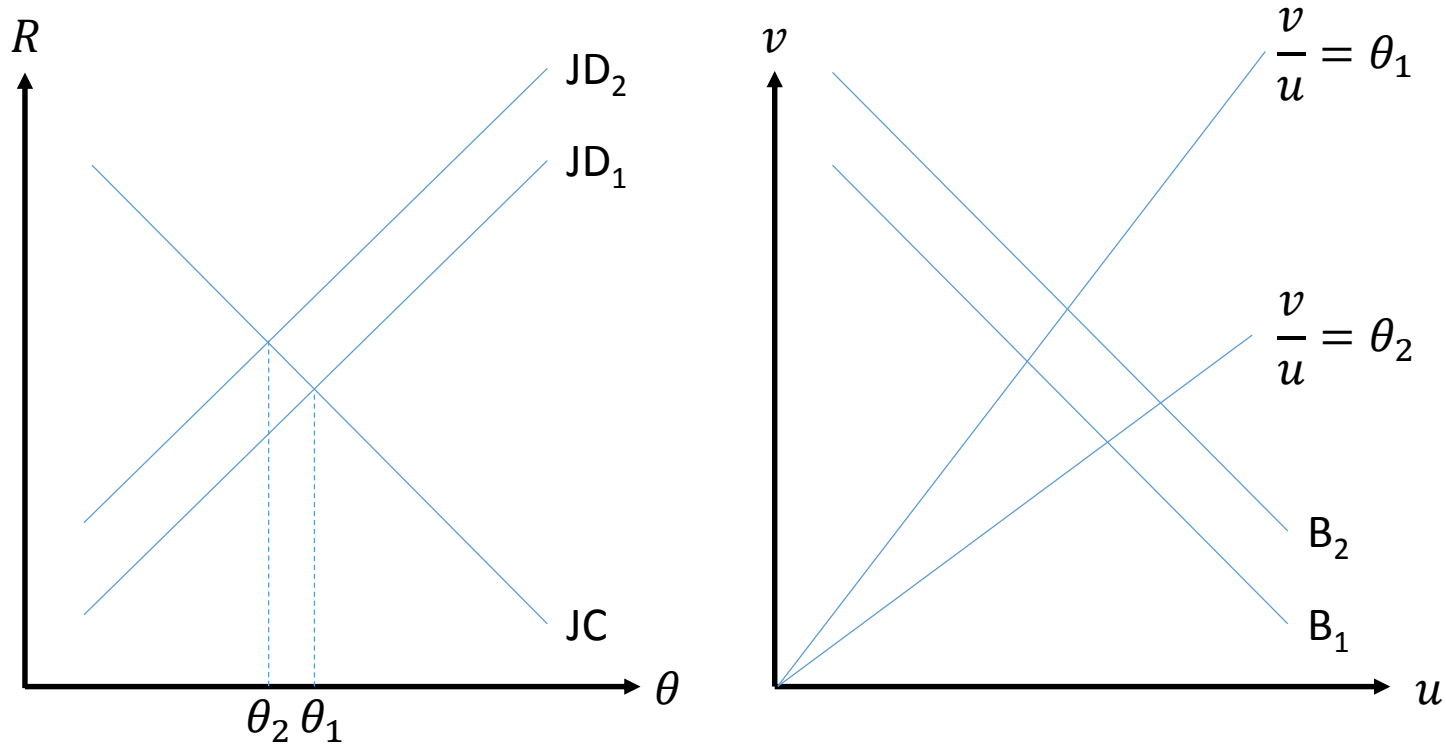
- Since  $\frac{\lambda_x}{\rho + \lambda_x} < 1$  and  $-\frac{d}{dR} \int_R^1 (x' - R) dG(x') = 1 - G(R) \leq 1$ , in  $(\theta, R)$  space, this curve is upwards sloping.

- With high  $\theta$ , workers find it easy to get a new job, so wages are higher, so maintaining an existing job is more expensive, and higher productivity is required.

- Given our solution for  $\rho U$ , this can be rewritten:  $pR = \rho U - p \frac{\lambda_x}{\rho + \lambda_x} \int_R^1 (x' - R) dG(x')$ .

- The first term is not surprising.  $\rho U$  is the flow return to being unemployed, so you might expect that the reservation productivity  $pR$  would equal the flow return to being unemployed.
- In fact  $pR < \rho U$ , implying there is labour hoarding in the model.
- A filled vacancy has a positive option value, since the firm may get lucky with a later  $x'$ .

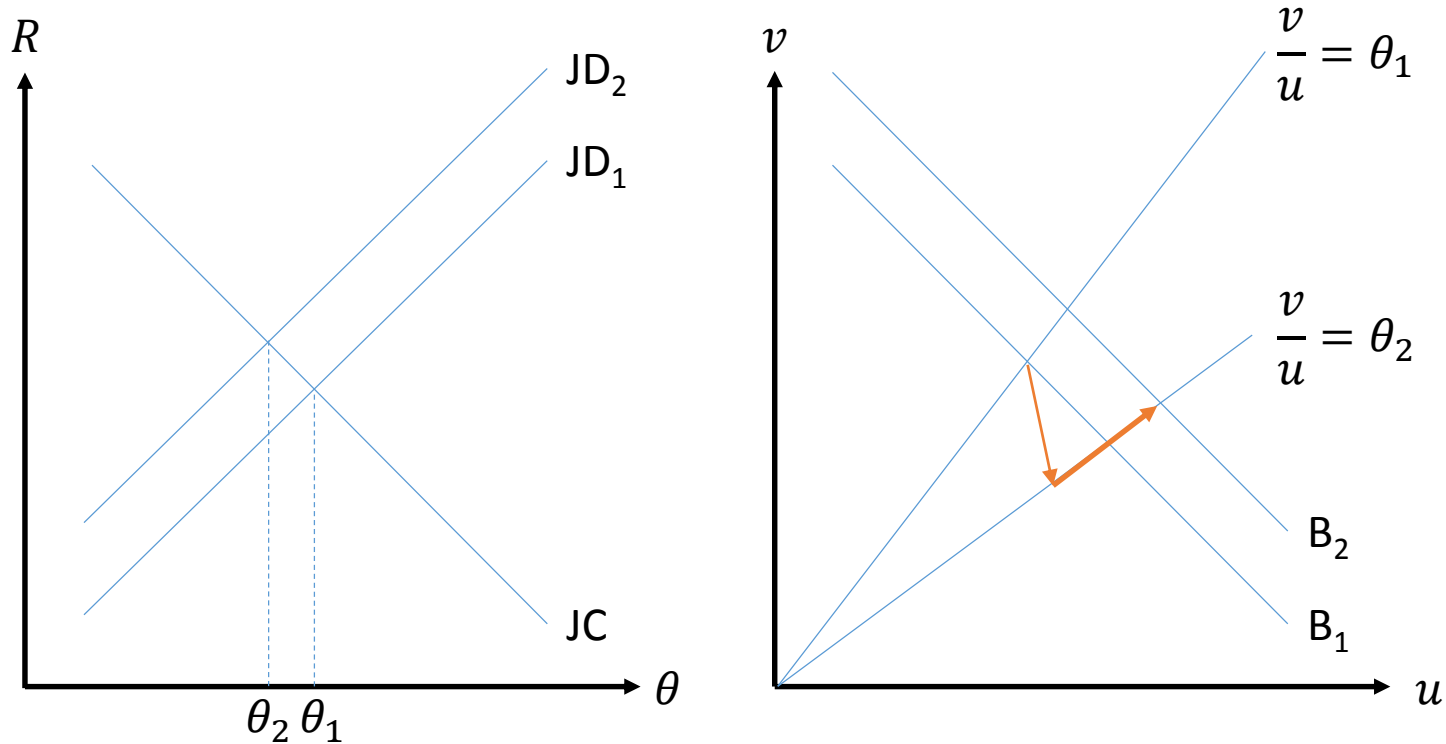
# The MP model: Graphical analysis of an aggregate negative productivity shock



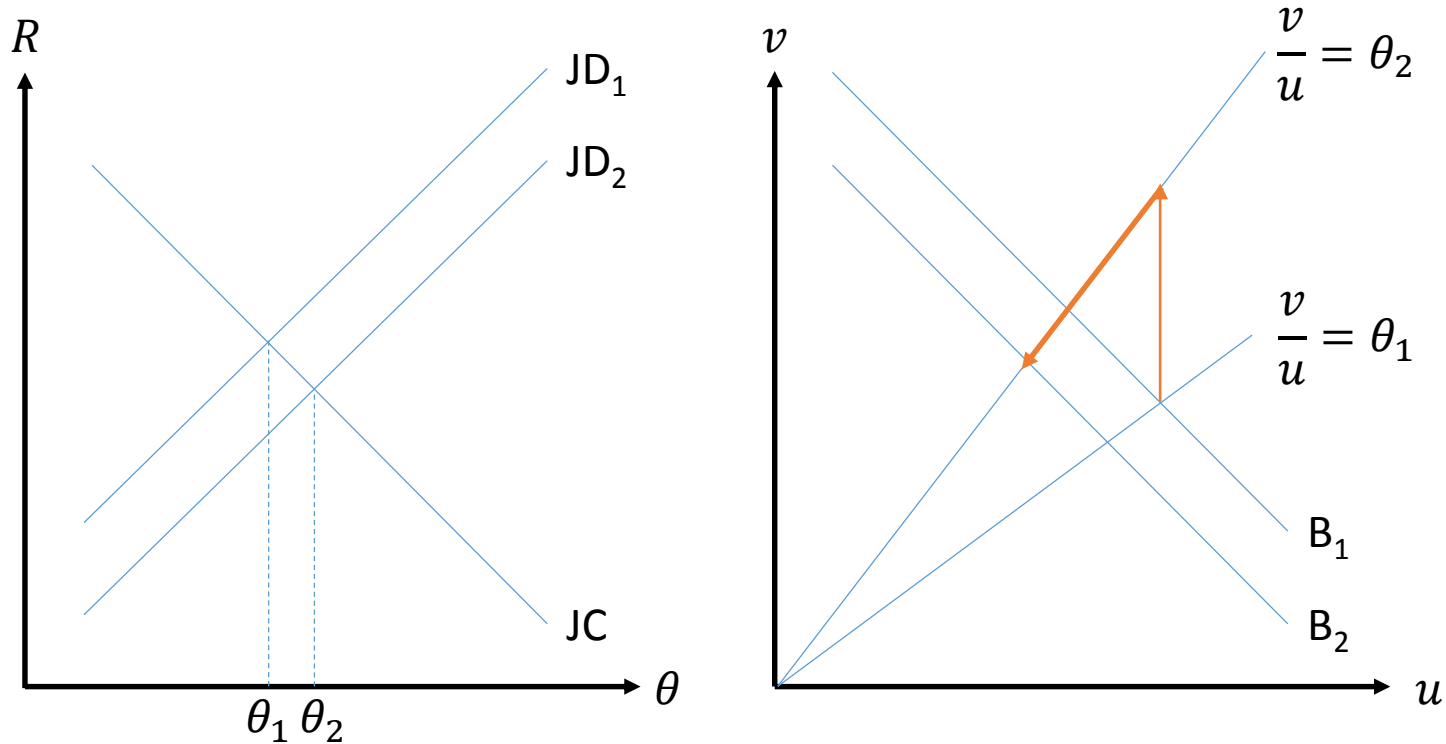
# The MP model: Dynamics

- Since firms enter freely and instantaneously, the number of vacancies can jump.
- Likewise,  $R$  is a choice variable, so can also jump.
- Thus following a jump in productivity,  $\theta$  and  $R$  will jump to the new level required to satisfy the  $JC=JD$  condition.
- $u$  on the other hand is a state variable, and will fall sluggishly as new matches are formed.
  - $u$  can however jump down if  $R$  jumps up, since a jump up in  $R$  results in a positive mass of firms exiting at once.
  - This implies substantial asymmetry in the response to shocks.

# The MP model: Graphical analysis of an aggregate negative productivity shock



# The MP model: Graphical analysis of an aggregate positive productivity shock





# The MP model: The exogenous job destruction special case

- Suppose that  $G(x) = 1$  for  $x > 0$ , so job destruction is no longer endogenous.
- Then for any function  $f$ , if  $R > 0$  then  $\int_R^1 f(x') dG(x') = 0$ , and if  $R = 0$  then  $\int_R^1 f(x') dG(x') = f(0)$ .
- Hence from the job destruction condition:  $R = \frac{b}{p} + \frac{\beta}{1-\beta} \theta c$ .
- So from the job creation condition:

$$(\rho + \lambda_x + \beta \theta q(\theta)) \frac{pc}{q(\theta)} = (1 - \beta)(p - b)$$

# The MP model: Efficient bargaining (1/3)

- Suppose that  $b$  only reflects the value of leisure and/or home production, rather than a government transfer.
- And suppose that  $G(x) = 1$  for  $x > 0$ , so job destruction is exogenous.
- Then a social planner would like to choose  $u$  and  $\theta$  to maximise:

$$\int_0^{\infty} e^{-\rho t} ((1-u)p + u(b - \theta pc)) dt$$

- Subject to:  $\dot{u} = (1-u)\lambda_x - u\theta q(\theta)$ .

- Current value Hamiltonian:

$$\mathcal{H}_c(u, \theta, \mu) = (1-u)p + u(b - \theta pc) + \mu[(1-u)\lambda_x - u\theta q(\theta)].$$

- FOCs:

$$\begin{aligned} \rho\mu - \dot{\mu} &= \mathcal{H}_{c,1}(u, \theta, \mu) = -p + b - \theta pc - \mu(\lambda_x + \theta q(\theta)) \\ 0 &= \mathcal{H}_{c,2}(u, \theta, \mu) = -upc - \mu u(q(\theta) + \theta q'(\theta)) \end{aligned}$$

# The MP model: Efficient bargaining (2/3)

- In steady-state, the first condition becomes:

$$\mu = -\frac{(1 + \theta c)p - b}{\rho + \lambda_x + \theta q(\theta)}$$

- Hence from the second condition:

$$pc = (q(\theta) + \theta q'(\theta)) \frac{p - b + \theta pc}{\rho + \lambda_x + \theta q(\theta)}$$

- So, if we define:  $\eta(\theta) = -\frac{\theta q'(\theta)}{q(\theta)} = \frac{uM_1(u,v)}{M(u,v)}$ :

$$(\rho + \lambda_x + \eta(\theta)\theta q(\theta)) \frac{pc}{q(\theta)} = (1 - \eta(\theta))(p - b)$$

- Now, compare this to the decentralized solution:

$$(\rho + \lambda_x + \beta\theta q(\theta)) \frac{pc}{q(\theta)} = (1 - \beta)(p - b)$$

- These agree if and only if  $\beta = \eta(\theta)$ .
  - This is called the “Hosios condition”.
  - No particular reason it should hold, but it provides a useful benchmark.

# The MP model: Efficient bargaining (3/3)

- In the general case,  $\beta = \eta(\theta)$  retains an optimality property.
- To see this, differentiate the job creation condition w.r.t.  $\beta$ :

$$\frac{c\eta(\theta) d\theta}{\theta q(\theta) d\beta} = -\frac{cq'(\theta) d\theta}{(q(\theta))^2 d\beta} = -\frac{1-R}{\rho + \lambda_x} - \frac{1-\beta}{\rho + \lambda_x} \frac{dR}{d\beta}$$

- And then do the same for the job destruction one:

$$\left[ 1 - \frac{\lambda_x}{\rho + \lambda_x} (1 - G(R)) \right] \frac{dR}{d\beta} = \frac{\theta c}{(1-\beta)^2} + \frac{\beta}{1-\beta} c \frac{d\theta}{d\beta}$$

- At an optimum,  $\frac{dR}{d\beta} = 0$ , hence:

$$\frac{\theta c}{(1-\beta)^2} = \frac{\beta}{1-\beta} \frac{\theta q(\theta)}{\eta(\theta)} \frac{1-R}{\rho + \lambda_x}$$

- So from the job creation condition:  $\beta = \eta(\theta)$ .

# Extensions to the MP model and its empirical performance

- Shimer (2005) found that a reasonably calibrated MP model generates insufficient volatility in unemployment, and fails to match the strong pro-cyclicality of the job finding rate.
  - Referred to as “the Shimer puzzle”.
- The problem is that the determination of wages through Nash bargaining ensures wages are highly responsive to productivity, and so the value of posting a new vacancy is insufficiently pro-cyclical.
- Hall (2005) and Shimer (2004) extended the MP model with wage rigidity, and found that this helped address these problems.
  - However, as pointed out by Hornstein, Krusell, Violante (2005), this only works with an implausibly large labour share.
  - They also point out that with wage rigidity, the labour share becomes too volatile.
- Hagedorn and Monovskii (2005) suggested an alternative calibration of the base model which comes closer to matching volatilities, however this calibration only works with implausibly high values for  $b$ .
- Countless other extensions and variants have been produced, with alternative bargaining structures, efficiency wages, capital, endogenous benefits, etc etc.

# The New Monetarist Model

- NK models are concerned with frictions in price setting.
- NM models are concerned with frictions in exchange.
  - They seek to build models in which money emerges “endogenously” as a solution to the double-coincidence-of-wants problem.
- In the simplest model, there are a large number of goods, and each agent produces some subset of that number, but likes consuming a different set (disjoint to the first, for simplicity).
  - When two agents meet, they may both like each other’s respectively produced goods, they may both dislike the other’s produced good, or only one of them may like the other’s produced good.
  - If the latter situation never happened, there would be no role for money.
  - Money would also not be useful if people could perfectly track who they’d met previously, and which goods they owed them.
  - However, with imperfect memory, money is essential to get the “good equilibrium” in which there is trade even in the one-sided situation.
- Richer models have both a “decentralized market” (with matching) and a “centralized” market (with market clearing) in each period.
- The standard results suggest the initial level of money is neutral, and that the Friedman rule is optimal. These models also generate larger costs of inflation than NK models.
- See the review article referenced for a full survey.